Robotics
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Lecture - 13
Robot Kinematics (Contd.)

So, we have seen how to determine the 3 cross 3 matrix corresponding to rotation about Z by an angle theta.

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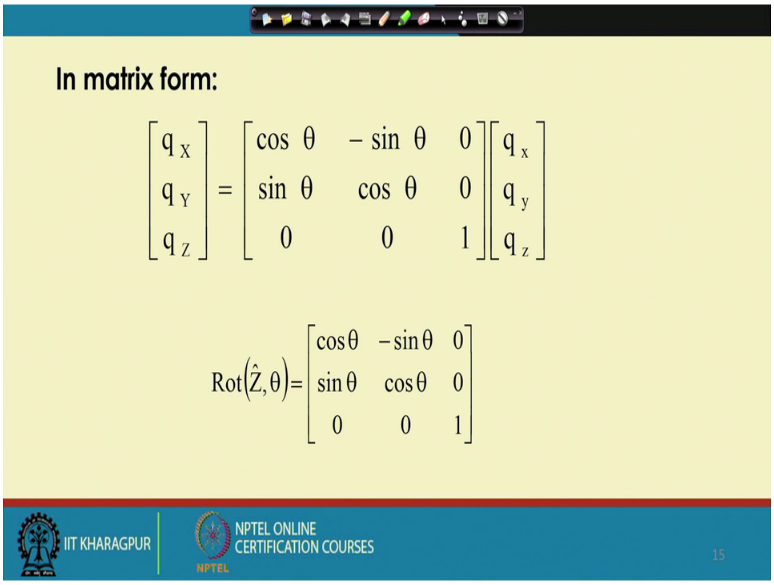

So, this is the matrix.

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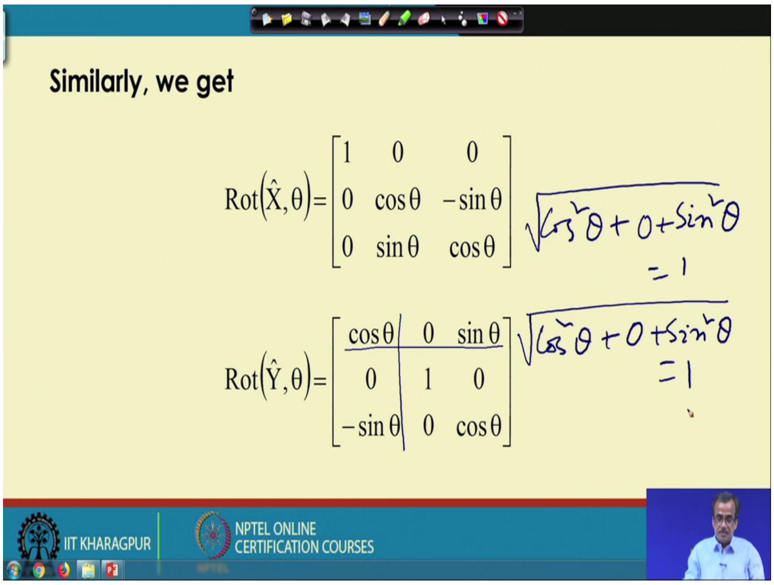

Similarly, we can find out the rotation about X by an angle theta is nothing but 1 0 0, 0 cos theta minus sine theta, 0 sine theta cos theta, then rotation about Y by an angle theta in the anticlockwise sense is cos theta 0 sine theta, 0 1 0, minus sine theta 0 cos theta.

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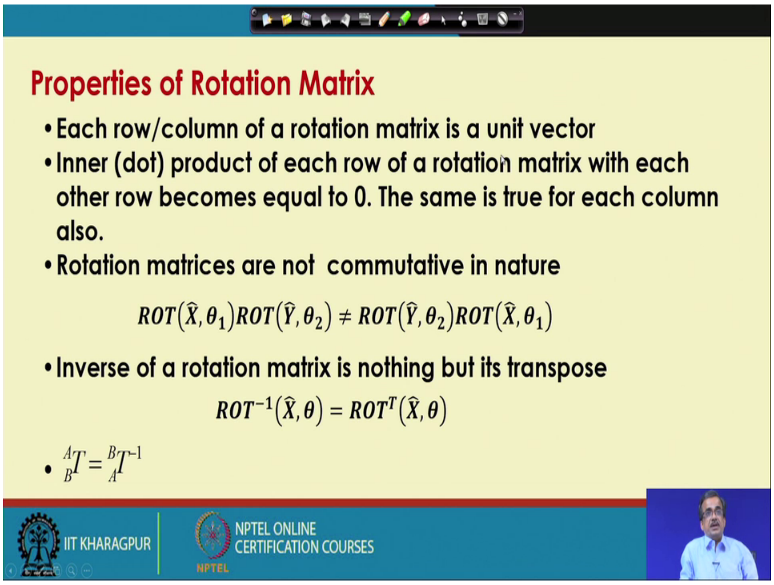

Now, I am just going to discuss the properties the properties of rotation matrix. The first one each row or column of a rotation matrix is a unit vector. That means, if I just concentrate on this particular the rotation matrix like each row. For example, if I consider the first row that is your cos theta 0 sine theta. Now this is the unit vector representation;

that means your square root cos square theta plus 0 plus sine square theta so, this is equal to 1. Similarly, if I concentrate on a particular column then cos theta 0 minus sine theta once again the square root of cos square theta plus 0 plus sine square theta. So, this is equal to 1, this is what you mean by the first property.

The next is the inner product or the dot product of each row of a rotation matrix will become equal to 0. So, if I concentrate on a particular row or if I concentrate on a particular the column: for example, say if I concentrate on. So, this particular the row that is cos theta 0 sine theta and the second row is 0 1 0.

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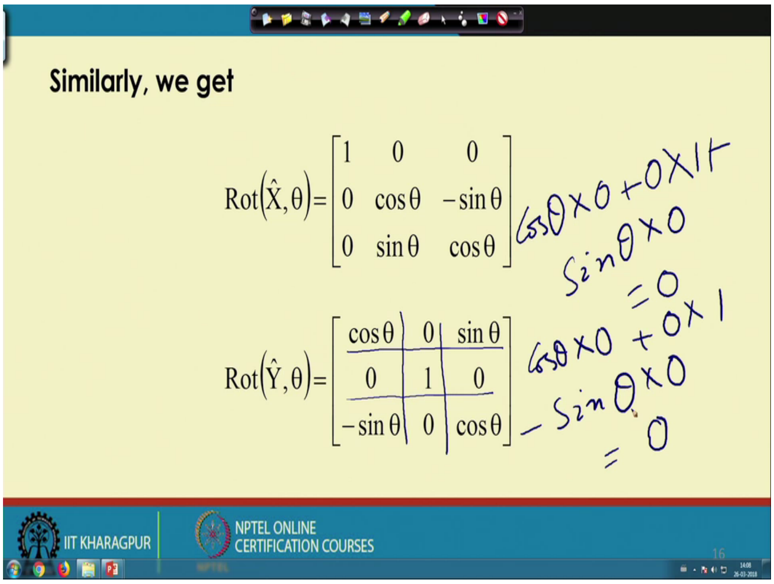

And, if I try to find out the inner product or the cos product so, the inner product of the first row, and the second row it is nothing but cos theta multiplied by 0 plus 0 multiplied by 1 plus sine theta multiplied by 0 and this is equal to 0. Similarly, if I consider the two columns of a particular the rotation matrix like the first column that is cos theta 0 minus sine theta 0 1 0. And if we try to find out the inner product this will become equal to cos theta multiplied by 0 plus 0 multiplied by 1 minus sine theta multiplied by 0 and this is equal to once again 0. So, this is the way actually we can check the second property of this particular the rotation matrix that is a inner product of each row or each column of a rotation matrix with each other row becomes equal to 0.

Next is the rotation matrices are not commutative in nature; that means, it depends on the sequence. So, sequence is very much important while writing down the rotation matrix.

For example, say the rotation the rotation about X by an angle theta 1 then comes rotation about Y by an angle theta 2 is not equal to the rotation about Y by an angle theta 2 rotation about X by an angle theta 1.

Now, if I calculate the left hand side and determine the right hand side separately and try to compare we will see that they are not equal. That means, your this particular rotation matrix depends on the sequence thus the way the sequence along which we are writing down those rotation matrices and they are not commutative in nature. The next property is the inverse of a rotation matrix is nothing but it is transpose. So, if it is a pure rotation matrix then if you want to find out the inverse of that particular rotation matrix it is very easy. So, what you can do is. So, if we can find out the transpose of that particular rotation matrix so, that will be equal to it is your inverse.

For example, say if I see that a rotation X comma theta inverse of that is nothing but the transpose of the rotation matrix X comma theta. So, but this is true only when so, this is a pure rotation matrix. Now, if there is a any such translation term. So, this particular condition will not hold good now here I have written another thing that is the transformation matrix of B with respect to A is nothing but the transformation matrix of A with respect to B inverse, ok.

Now, this is very much essential because say we want to find out a particular joint with respect to the previous and the reverse that is this particular joint with respect to the next and for that what you need is so, this particular inverse we will have to find out and T B with respect to A is nothing but T A with respect to B inverse. So, this particular condition we can use.

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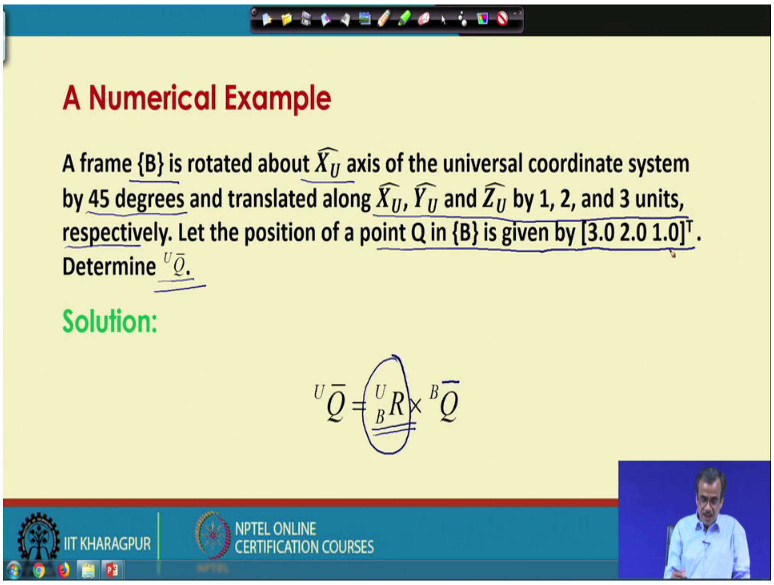

Now, I am just going to solve one numerical example based on the theory which I have already discussed. Now, this numerical example the statement is as follows a frame B. So, B is nothing but a body coordinate frame is rotated about X U that is the X the universal coordinate system X axis by 45 degrees. So, this is positive 45 degrees; that means, your anticlockwise direction and translated along X U, Y U and Z U by 1, 2, and 3 units respectively. Let the position of a point Q in B is given and this is given by 3, 2, 1 transpose then how to determine the position of that same point with respect to the universal coordinate system that is Q with respect to U.

So, here the way we will have to find out it is very simple. So, this Q with respect to U so, this is nothing but the rotation of B with respect to U multiplied by so, Q with respect to B and of course, this is a vector so, we will have to put this vector sign. So, if I can find out this rotation of B with respect to U and I know this Q with respect to B so, very easily I can find out what is Q with respect to U. Now, to determine this rotation matrix that is R B with respect to U.

So, once again let me read that there is a rotation about X U by an angle 45 degree in the anticlockwise sense and there are translation along X U, Y U and Z U by 1, 2 and 3 units. So, using that particular information so, what I will have to do is so, I will have to find out what should be this particular R B with respect to U.

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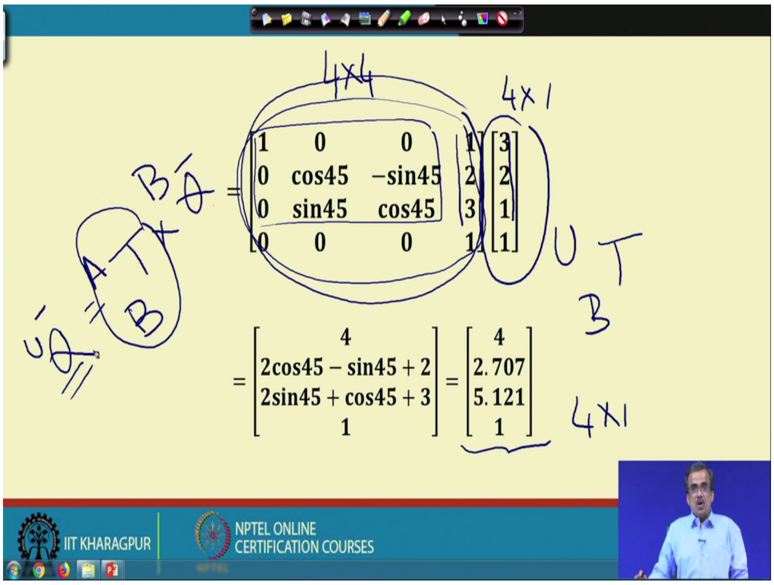

Now, this rb with respect to U if you see the rotation term the rotation terms is nothing but 1 0 0, 0 cos theta minus sine theta, 0 sine theta cos theta, now here theta is nothing but 45 degree. So, this is actually nothing but this particular the rotation term rotation about X U by an angle 45 degree and these are nothing but the translation term.

So, I will be getting the transformation matrix corresponding to that particular rotation and that is nothing but this 4 cross 4. In fact, although I have written there it is R B with respect to U. Truly speaking so, corresponding to that it should be your T B with respect to U. So, if I write T B with respect to U then it will carry actually the 4 cross 4 matrix and this is multiplied by 3 2 1 1 and these 3 2 1 is nothing but is your the position terms that is Q with respect to B.

So, Q with respect to B and if I know actually we will have to find out Q with respect to U and truly speaking this is nothing but T B with respect to A multiplied by actually Q with respect to B. Now, Q with respect to B is this much and this T B with respect to a is nothing but this much and if I just multiply. So, this is a 4 cross 4 matrix and this is a 4 cross 1 matrix and if I multiply then I will be getting this particular the 4 cross 1 matrix and this is nothing but Q with respect to U.

So, very easily we can find out. So, what is this particular the Q U with respect to U. So, Q with respect to U you can find out.

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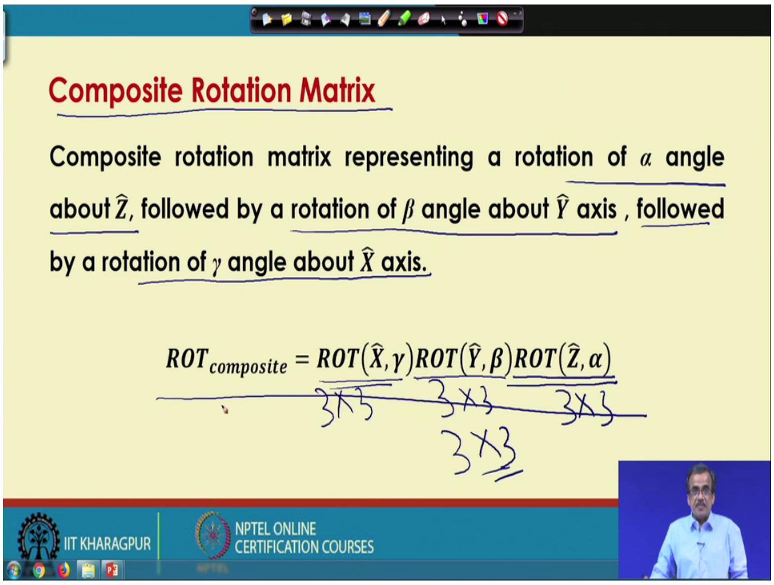

Now, let us try to concentrate on this particular the rotation matrix that is called the composite rotation matrix. Now, I have already discussed that while writing this particular the rotation matrix the sequence is very important and we will have to follow a particular sequence otherwise altogether you will be getting the different the final results and that is why to write down this particular rotation matrix all of us we which who work on robotics we follow one rule that is called the composite rotation matrix rule.

Now, the rule is as follows; the composite rotation matrix representing a rotation of alpha angle about Z axis followed by a rotation of beta angle about Y axis followed by a rotation of gamma angle about X axis is written in this particular the format. The rule is as follows whatever I state first; that means the rotation about Z by an angle alpha. So, that particular thing will be written at the last that is rotation about Z by an angle alpha will be written at the end followed by the rotation of beta about Y axis; so rotation of beta about Y axis followed by the rotation of a gamma angle about X axis. So, rot X comma gamma is written first.

Now, for each of these actually we can find out like what should be the 3 cross 3 matrix. So, this is a 3 cross 3 matrix, this is also a 3 cross 3 matrix and this is also a 3 cross 3 matrix. Now, if I multiply then finally, we can find out one 3 cross 3 matrix and that is nothing but the composite rotation matrix. So, we can find out the 3 cross 3 matrix that is nothing but rot composite that is composite rotation matrix. Now, this particular rule is actually followed by the whole robotics community. So, just to write down whenever there is rotation term.

Now, here I am just going to take some example and I am just going to show you that this particular rule is correct. So, indirectly I am just going to prove that this particular rule that is the rule for the composite rotation matrix is correct.

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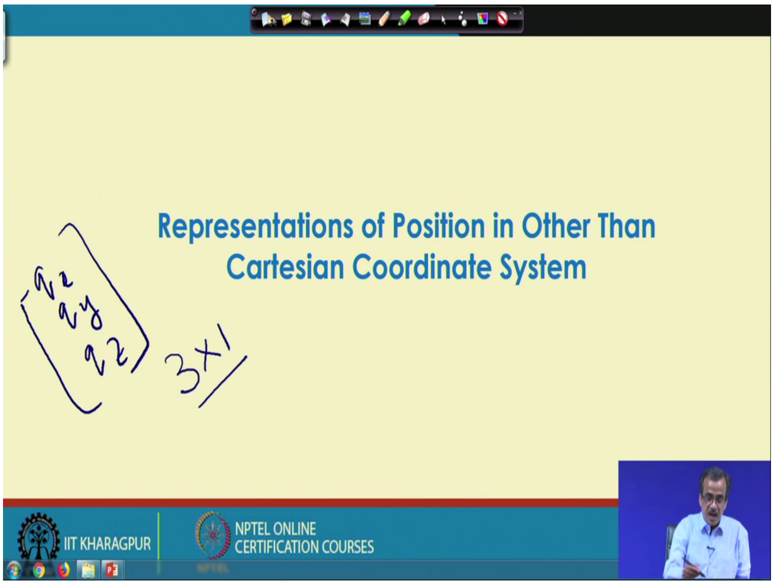

So, here till now whatever we have seen is your we have expressed position in terms of the Cartesian coordinate system and we have seen that in terms of Cartesian coordinate system the position can be expressed in terms of a vector and that is nothing but a 3 cross 1 matrix. For example, say the position vector is nothing but say q x, q y, q z and once again let me repeat that this is nothing but a position vector and in matrix form this is nothing but a 3 cross 1 matrix. Now, the same position the same position can also be expressed in other coordinate system now that I am going to discuss.

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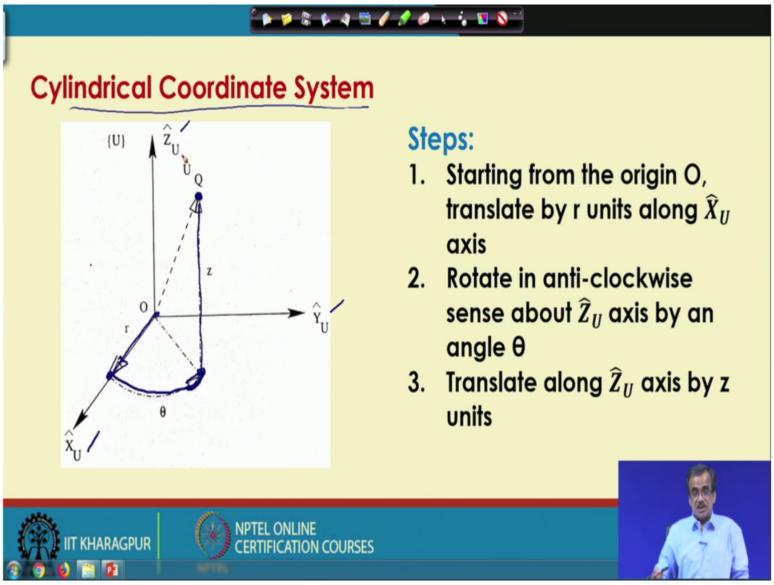

Now, here I am just going to see how to represent the position in cylindrical coordinate system. Now, in cylindrical coordinate system the position of a particular point in 3D can be expressed with the help of actually two translation and one rotation in a particular the sequence.

Now, here supposing that the problem is as follows: so I have got the universal coordinate system and in this particular universal coordinate system. So, this is nothing but X U, Y U, and Z U. Now, supposing that I have represented a particular point that is nothing but Q with respect to U. So, in Cartesian coordinate system if I want to represent so, it is very easy. So, if I know the translations along X direction, if I know the translation along Y direction and if I know the translation along Z direction. So, very easily I can find out this Q with respect to U in Cartesian coordinate system.

Now, here my aim is to reach the same point using the cylindrical coordinate system; so how to reach we take the help of these steps. So, step one we start from the origin O. So, this is the origin O and then we translate by r units along X U. So, from here we translate by r unit along X U. So, I am here the next is your the rotation in a anticlockwise sense about Z U axis by an angle theta. So, I am here now, I am taking rotation about this particular Z U by an angle theta in the anticlockwise sense. So, I am here so, this is my position. Now, the next is translate along Z U axis by an angle Z unit. So, starting from here we translate along this particular Z direction by this small Z unit and I am just going to reach the same point that is nothing but Q with respect to U.

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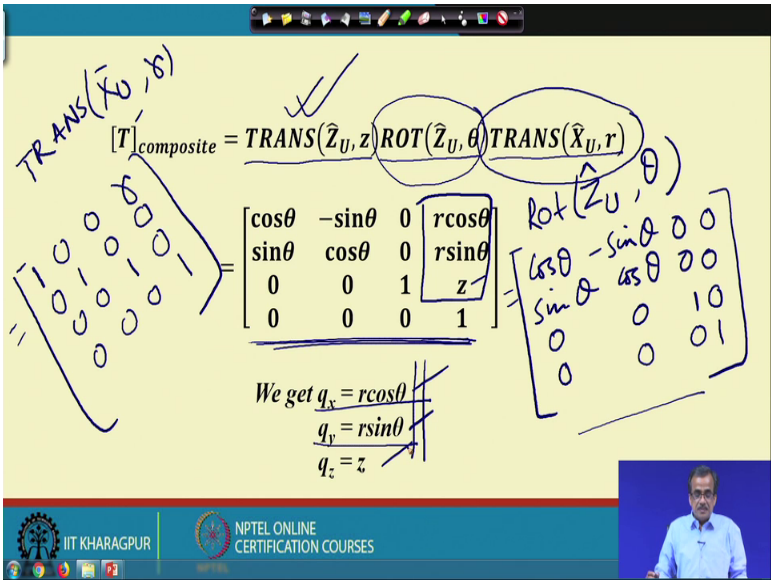

Now, this particular sequence now if I just take the help of like the rule for the composite rotation matrix. So, I can find out that final the transformation matrix or if I want so, I can find out the corresponding rotation matrix also but, here I am interested mostly in position.

So, what I can do is I can find out the transformation matrix. So, T composite that is the transformation matrix corresponding to this particular the composite whatever I stated first that will go to the last that is translation along X U by r unit followed by rotation about Z U by theta followed by translation along Z U by Z. Now, corresponding to each of this particular thing; so we can write down the 4 cross 4 matrix. For example, say corresponding to this rotation about Z U by an angle theta so, I can write down very easily this particular the transformation matrix that is 4 cross 4.

For example, say this will be your the cos theta minus sine theta 0, then comes sine theta cos theta 0, 0 0 1 and this is the pure rotation so, the translation terms will be 0. So, 0 0 0 and of course, I have got the fourth the row that is 0 0 0 1. So, this is nothing but the rotation about Z U by an angle theta in 4 cross 1 matrix form. Similarly, I can also find out the translation along X U by an angle r sorry by r.

Now, this translation it is a pure translation. So, this rotation terms will be nothing but a 3 cross 3 identity matrix. So, very easily I can write down this particular thing for example, say the trans trans X U comma r so, this can be written in the matrix form in a 4 cross 4 form as follows. So, this is 1 0 0, 0 1 0, 0 0 1. So, this is nothing but the rotation term there is a identity matrix. Then along X U, I have got r translation then Y it is 0, Z it is 0 and the fourth row is 0 0 0 1.

So, similarly I can also write down the 4 cross 4 matrix corresponding to this trans Z U comma z. Now, I am getting 3 matrices each having 4 cross 4 dimension and if I just multiply then finally I will be getting so, this particular the 4 cross 4 matrix. Now, in this 4 cross 4 matrix so, these particular terms are going to represent the position terms. For example, say in Cartesian whatever was q x that is nothing but r cos theta, whatever was q y that is nothing but r sine theta. Similarly, your q z is nothing but is equal to z. So, this particular relationship between the Cartesian coordinate system and this particular the cylindrical coordinate system so, very easily I can establish using the rule of the composite rotation matrix.

Now, all of us we know that the relationship between the Cartesian coordinate system and the cylindrical coordinate system is nothing but this that is q x equals to r cos theta, q y equals to r sine theta. So, these are actually very well known relationship between the Cartesian coordinate system and the cylindrical coordinate system. So, the rule for composite rotation matrix whatever we have used so that is correct. So, this is the way actually indirectly we can prove the correctness of this particular the rule for composite rotation matrix.

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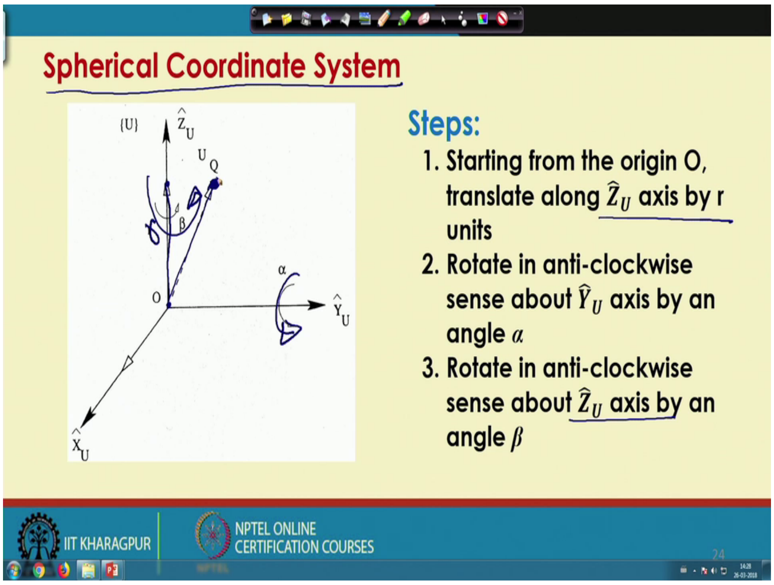

So, now I am just going to discuss another coordinate system and that is called the spherical coordinate system. Now, here our aim is the same point which I represented in Cartesian coordinate system that is this particular X with respect to U. With the help of Cartesian that is q x, q y and q z the same point I want to reach with the help of this spherical coordinate system. So, in spherical coordinate system in fact, there is a one translation and there are 2 rotation in a particular the sequence. So, let us try to check that particular the sequence.

So, step – 1 starting from the origin O, we translate along Z U axis by r. So, this is nothing but the origin of this particular coordinate system. So, starting from here along this particular ZU so, we translate by r unit so, I am here so, this is by r unit. Now, the next is rotate in anticlockwise sense about Y U by an angle alpha. So, this is nothing but my Y U axis so, about a YU. So, I just rotate by alpha by the angle alpha in the anticlockwise sense. So, whatever was here so, this particular thing will be rotated something like this.

So, let me repeat supposing that this is along this particular Z U and here say I have got, so this is nothing but my YU axis. So, I am just rotating about this particular YU so, what will happen this was along this particular Z U. Now, it will be rotated something like this and after that we take another rotation that is rotation in a anticlockwise sense about Z U by an angle beta.

So, now I am rotating about Z U by an angle beta so, there is a possibility that I am going to reach this particular point that is nothing but Q with respect to U. So, what we do, let me repeat first I am just going to translate it, translate along this r unit along the Z by r unit after that I am just going to rotate about Y U by an angle alpha and after that we are going to rotate about Z U. So, this is nothing but Z U so, by an angle beta. And then I will be able to reach this particular point. So, with the help of one translation and two rotation I am just going to reach this particular the point.

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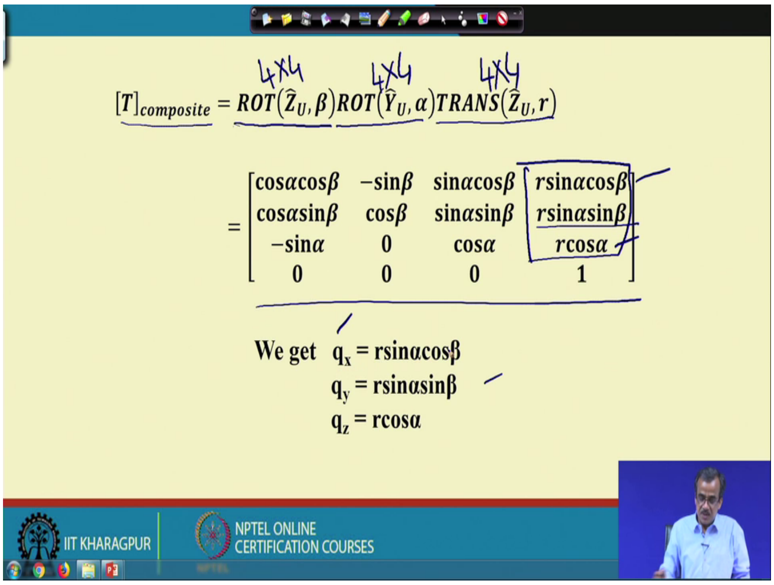

Now, all such translation and rotation if I just write with the help of this composite rotation matrix so, I will be getting so, this particular form that is your that transformation matrix corresponding to this composite. Now, whatever I stated first will go to the last. So, translation along this particular Z U by r unit so, I will have to write at the end because that I stated first, followed by the rotation about Y U by alpha followed by so, so rotation about Z U by an angle this particular the beta.

And, as I discussed this particular the 4 for each of this particular thing like 4 cross 4 matrix I can write down and all of us we know how to multiply like the 2 matrices and so, it is having the dimension 4 cross 4 this is once again like the 4 cross 4. So, these 4 cross 4 matrices we can multiply then you will be getting one 4 cross 4 matrix and that particular 4 cross 4 matrix we can multiply with this then I will be getting this final 4 cross 4 matrix.

Now, in this particular 4 cross 4 matrix actually the position terms are denoted by so, these three that means, in Cartesian. So, what is q x that is nothing but r sine alpha cos beta so, r sine alpha cos beta and q y is nothing but your r sine alpha sine beta and q z is nothing but is your r cos alpha. Now, here so, if I know this Cartesian coordinate system the same point I can also represent in your the spherical coordinate system; that means, this is the known relationship between the Cartesian and the spherical coordinate system. So, once again actually I have a re-derived.

So, this particular the relationship between your the Cartesian and the spherical coordinate system is actually known to us and this is another indirect proof for this particular the rule for composite rotation matrix. That means the position of a point with respect to the universal coordinate system can be expressed in Cartesian coordinate system in cylindrical coordinate system and in spherical coordinate system. And that is why the same robot can be actually controlled either in Cartesian coordinate system or cylindrical coordinate system or in spherical coordinate system.

Now, here actually I am just going to express and I am just going to discuss how to represent the orientation other than the Cartesian coordinate system. So, position we have seen that we can represent in other coordinate system. Now, I am just going to show like how to represent the orientation with respect to the other coordinate system.

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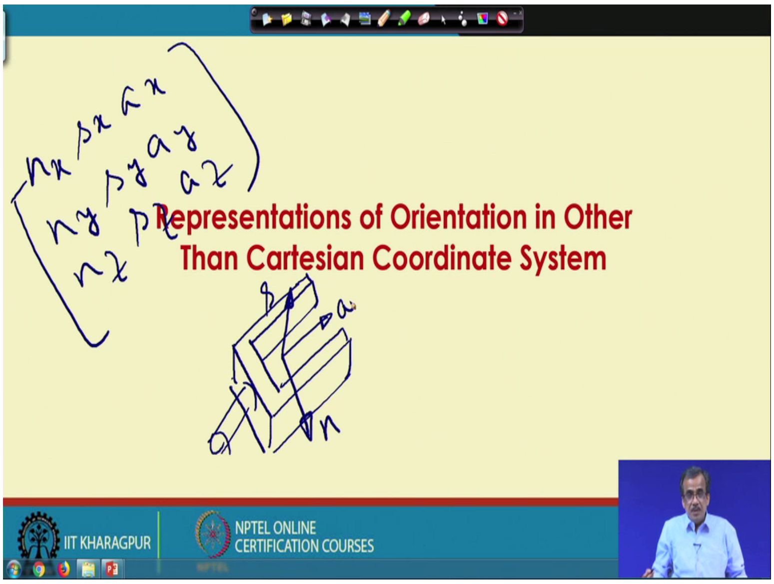

For example, in terms of Cartesian we have already discussed that we take the help of like 3 cross 3 matrix to represent the orientation. Now, if we remember. So, we have seen that with the help of this normal vector, sliding vector and approach vector actually we can represent so this particular orientation. So, n x, n y, n z, s x, s y, s z, a x, a y, and a z. So, this is nothing but the 3 cross 3 rotation matrix in Cartesian to represent the orientation. These I have already discussed in details. Now, this n stands for the normal vector, s stands for the sliding vector and a stands for the approach vector.

Now, if I just draw once again the same picture like one say end effector with two such finger for a particular manipulator so, very easily actually I can represent; so this particular the normal vector and the sliding vector and approach vector. So, if this is the end effector with two finger so, the normal vector is nothing but this and the sliding vector is nothing but this and the approach vector is nothing but this. This is how to represent the orientation in terms of your Cartesian coordinate system.

So now, we will be discussing how to represent the same orientation in other coordinate system.

Thank you.