Robotics
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Lecture - 12
Robot Kinematics (Contd.)

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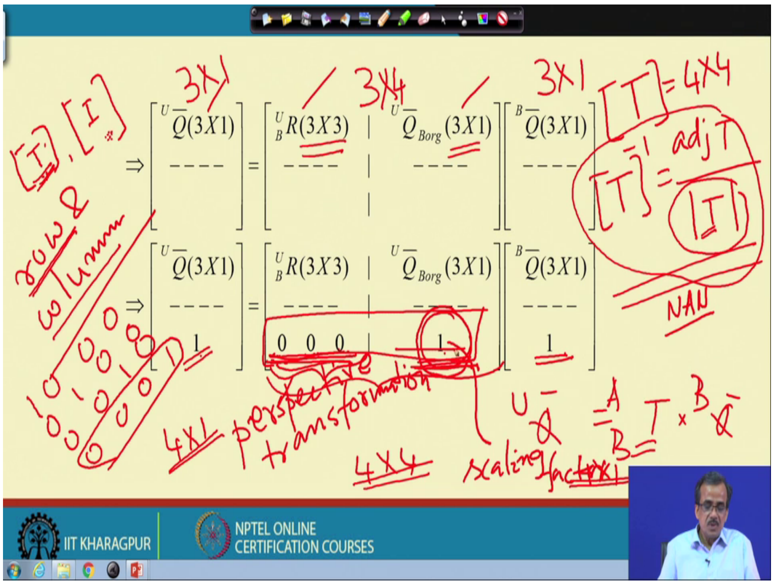

Now, I am going to discuss why do you put three zeros here and one 1 here. Now, to discuss why do you put three zeros and one 1 here. So, I will have to see the definition of the inverse of a particular the matrix and how do you calculate the inverse of a matrix using the computer program.

Now, let me take a very simple example supposing that I have got a matrix say let me take the help of so, this particular T matrix and this T matrix is having the dimension this is 4 cross 4 matrix and these transformation matrix I will have to find out the it is inverse, that means, this transformation matrix has to be invertible that is non singular. Now, by definition the universe of this particular matrix that is nothing but your adjoint of T divided by the determinant of T and we know how to find out the adjoint adjoint is nothing but the transpose of the cofactor. And, we will have to find out the determinant this is by definition how to find out the inverse of this particular the matrix.

Now, if you write down one computer program. So, in the denominator there will be actually the determinant of these particular T matrix. Now, the determinant of the T matrix it may also become equal to 0 sometimes. Now, if T becomes equal to 0 so, this will become adjoint T divided by 0. So, something divided by 0 so, this is going to generate one NAN not a number. So, your computer program is going to give a NAN and that is why to determine the inverse of a particular matrix. So, this particular definition we generally do not use in the company of program instead we follow another method.

Supposing that I will have to find out the inverse of a particular matrix say T and T is nothing but a 4 cross 4 matrix. So, what I do is side by side we just write down one identity matrix I and that is also a 4 cross 4 matrix, that is your 1 0 0, 0 1 0, 0 0 0, 1 0 0, 0 0 1 something like this, ok. Now, we take the help of some row and column operation. So, we take the help of row and column operation. Now, our aim is to convert.

So, this particular T to one identity matrix, ok, we take the help of row and column operation to convert these particular T matrix to the identity matrix and the same set of operation you carry out here on the identity matrix. So, at the end of this particular operation so, I will be getting one identity matrix in place of T and one inverse in place of this particular I. So, whatever matrix I will be getting here, that will be the universe of this particular the T matrix. So, this is the way actually we try to find out the inverse of a matrix in computer program.

Now, ultimately so, this particular T matrix actually has to be invertible and if you want to make it invertible. So, will be in advantageous position if I can keep the fourth row at least which I am going to add here at least it contains 0 0 0 1 because a 4 cross 4 identity matrix is nothing but this. So, if it is something like this the 4 cross 4 identity matrix is something like this so, this particular fourth row. So, very purposefully I am generating this particular the fourth row show that this particular T becomes invertible.

So, I am just going to help that particular transversal matrix to become invertible just by putting 0 0 0 1 here, that is the reason why do you put 0 0 0 and 1 we put there; there is another reason behind that. So, this particular 0 0 0 this is known as perspective transformation if you see the literature this is known as perspective transformation perspective transformation and this particular one is known as the scaling factor.

So, why did why do you call it a scaling factor we call it a scaling factor because in place of one I can write down 5, I can write down 6 and so on. There is no problem if I write 5 here. So, I can take 5 out of the matrix and I can make it 1, and that is why this is known as the scaling factor and I hope I am clear why do you put this particular the fourth row as 0 0 0 1 and now onwards so, we are going to consider that the transformation matrix is nothing but a 4 cross 4 matrix and this is known as the homogeneous transformation matrix.

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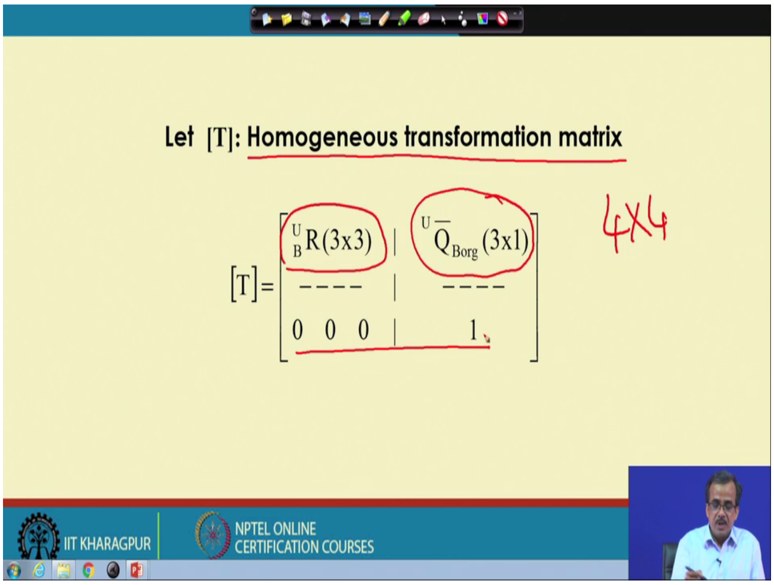

So, this is known as the homogeneous transformation matrix and we will have to determine will have to find out its inverse also because if I just assign one matrix here if I assign one matrix here, my aim is to represent this with respect to the previous and the rivert that means, my aim is to represent this with respect to this. So, this particular thing is possible if and only if you have that particular invertible transformation matrix or in that nonsingular transformation matrix and that is why. So, this particular transformation matrix has to be invertible.

Now, once again if I concentrate this 4 cross 4 homogeneous transformation matrix so, out of these four I have got 3 cross 3 rotation matrix and I have got a position vector. So, this is nothing but 3 cross 1 in matrix form and I have got 0 0 0 1.

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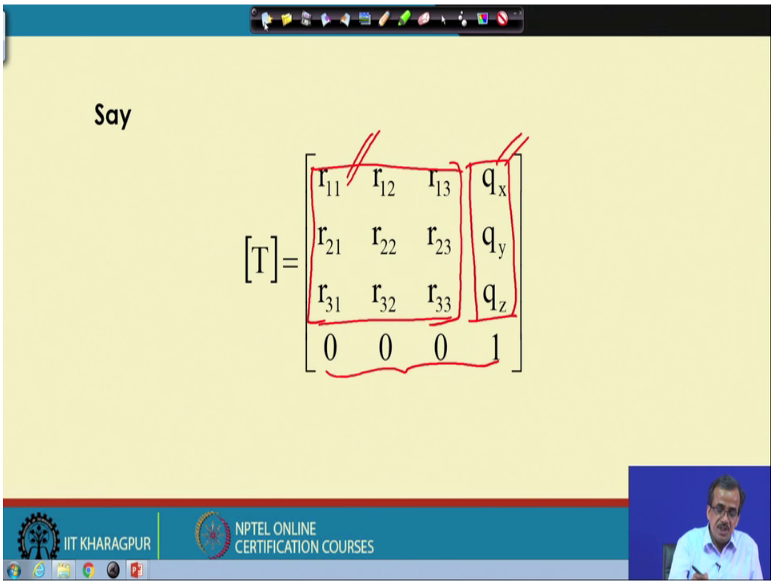

Now, if I take one typical example of one transformation matrix it will look like this. For example, say so, this particular this 3 cross 3 this 3 cross 3 matrix is going to carry information of this particular the rotation. So, this is going to carry information of the rotation and this is going to carry information of the position and the fourth row is your 0 0 0 1. So, this is nothing but a 4 cross 4 homogeneous transformation matrix carrying this particular the rotation matrix and this position vector. So, this is the way actually we represent the transformation matrix.

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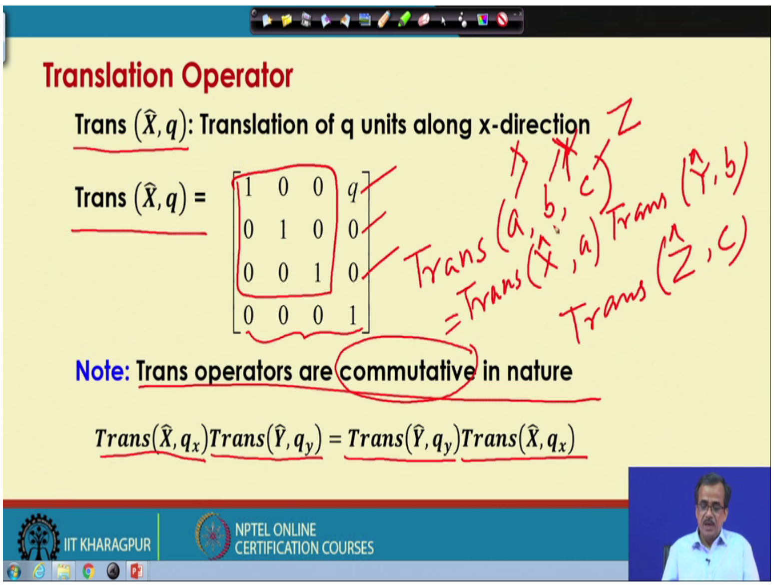

Now, I am just going to concentrate on the translation operator the properties of the translation operator. The translation operator in short is written as trance X comma q; that means, along the X direction the translation is only by q unit and here the rotation matrix is nothing but the identity matrix. You can see that, so this is nothing but the identity matrix 3 cross 3 identity matrix like 1 0 0, 0 1 0, 0 0 1 and along this X direction I have got the position information that is q along y it is 0, along z it is 0 and as usual we have got 0 0 0 1. So, this is nothing but trans X comma q. So, this is the way we can write down these 4 cross 4 matrix.

Now, here I have put one a note the trans operator are commutative in nature that means, it does not depend on the sequence. For example, say I can write down trans X comma q x; that means, along X there is a translation by q x amount, along Y there is a translation by q y amount. So, trans q X comma q x trans Y comma q y is nothing but trans Y comma q y trans X comma q x. So, it does not depend on the sequence and they are commutative in nature. So, trans operator are commutative in nature.

Now, another thing I just want to tell you in some of the literature you will find one notation that notation is something like this trans a comma b comma c. In some of the literature we can find so, this type of notation. Now, it means that translation along X by a unit along Y by b unit along Y by b unit and along this particular Z by c unit and this is equivalent to your trans X comma a, trans Y comma b, trans Z comma c. So, they are equivalent.

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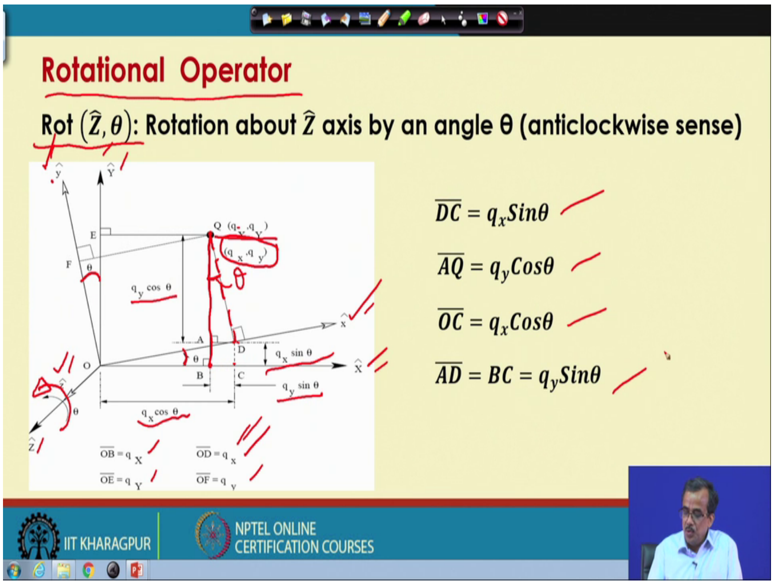

Now, I am just going to start with the rotation operator; that means how to determine that 3 cross 3 that rotation matrix. Now, here I am just going to derive something that is nothing but the 3 cross 3 matrix corresponding to the rotation about Z by theta. So, I am just going to find out the rotation about Z by theta. Now here, so this particular capital X, capital Y and capital Z represents the main coordinate system or the universal coordinate system and small x then comes small y and small z represent the rotated coordinate system and here the rotation is about Z by an angle theta.

So, here you can see that I am rotating about Z by angle theta in the anticlockwise sense. Now, initially this capital X, capital Y and capital Z small x small y and small z they were coinciding. Now, if I take the rotation about capital Z by an angle theta in the anticlockwise sense my small Z will remain same as capital Z, but small X will be different from capital X and this particular rotation will be by the angle theta and small y will be different from capital y and this particular rotation will be theta moreover this particular angle will also become theta, ok.

Now, let us try to concentrate on the main coordinate system or the universal coordinate system first. Supposing that I have got a point here Q in the main coordinate system is coordinates are q x, q y so, this is the coordinate and what about Z? Z is here actually perpendicular to the board and this particular thing as if I am considering on the 2D, but

Z is perpendicular to the board. In fact, and that is why very purposefully I have not written any Z value here on this 2D plane x, y plane. In fact, Z is equal to 0.

Now, with respect to the main coordinate system so, if the coordinate is q capital X, q capital Y so, I can find out so, so this OB; OB is nothing but q capital X and this BQ is equal to OE is nothing but is your q capital Y, ok. Now, I just concentrate on the rotated frame that is your small x, small y and small z, the same point q it is coordinate in the rotated frame is denoted by q small x q small y. That means, so if I just draw this particular if I just draw this particular perpendicular here, ok. So, this particular OD will be q small x. So, OD is q small x and this particular DQ is equal to of is nothing but is your q small y, ok.

Now, if I know this particular that this particular DQ; DQ is how much. So, this particular DQ is nothing but of so, it is of so, this is q y this angle is theta. So, I can find out that this particular AQ is your q y cos theta and similarly this AD which is equal to BC is nothing but q y sine theta. Similarly, this OD this OD is nothing but q x; q x this angle is theta. So, it is cos component that is OC will be your q x cos theta and your CD this particular CD is nothing but q x sine theta, ok. So, those things I have written it here like DC DC is nothing but q x sine theta then AQ; AQ is nothing but q y cos theta then OC; OC is nothing but q x cos theta and ad equals to BC AD equals to BC is nothing but q y sine theta. So, all such things we can find out very easily.

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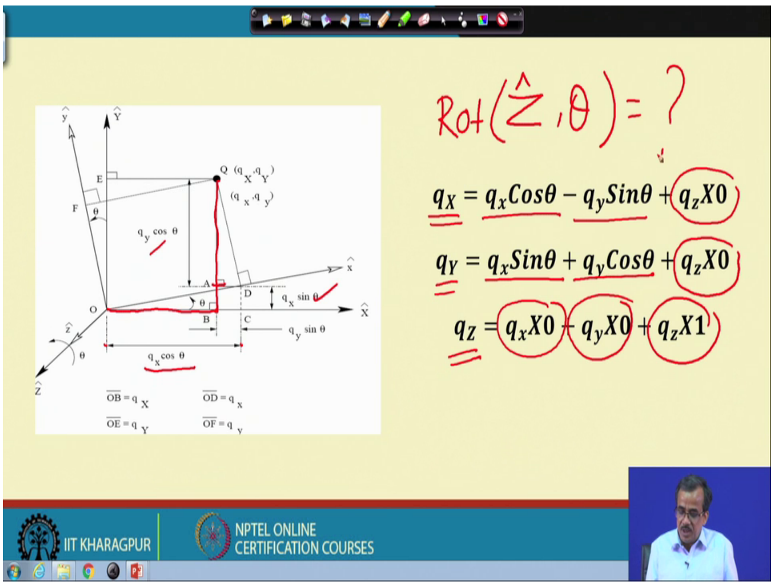

Now, I am just going to write down this particular the q capital X. This q capital X is how much? So, this q capital X is how much? So, up to this is your q capital X, OB. OB is nothing but is your q capital X and that is nothing but q x cos theta. q x cos theta is from here to here minus q y sine theta minus q y sine theta and here the Z component is 0 because this is on the 2D plane. So, I am just writing here q z multiplied 0.

The next is q y. So, what is q y? q y is nothing but is your this BQ, and this BQ is how much that is q x sine theta that is up to this. So, this is q x sine theta plus q y cos theta plus q y cos theta and after that I am adding q z multiplied by 0 and this particular q Z because here Z is perpendicular to the board. So, it is x component will be multiplied by 0, y component it is multiplied by 0 and z is multiplied by 1, ok. So, I am just trying to find out the relationship between the original coordinate system and this particular the rotated frame. That means I am trying to find out the expression for rotation about Z by an angle theta in the anticlockwise set. So, this particular 3 cross 3 matrix I am just going to find out.

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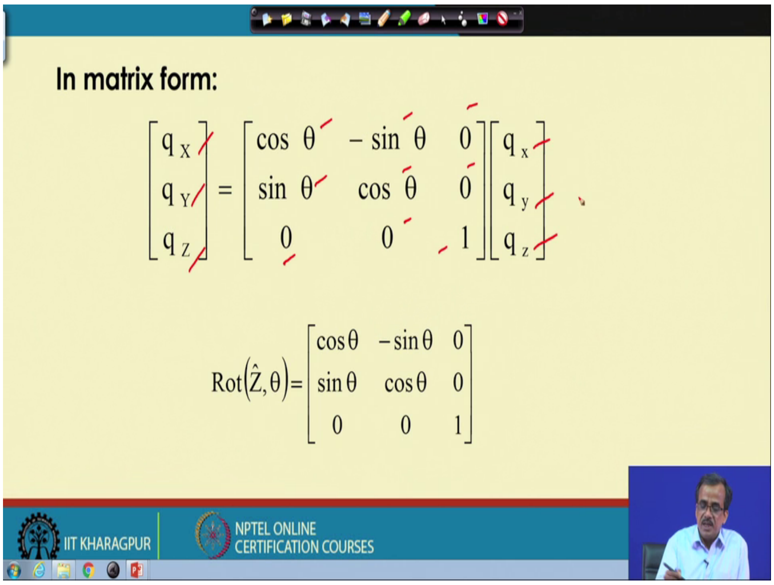

Now, this can be written as I think this can be written as in the matrix form q capital X, q capital Y, q capital Z is nothing but cos theta minus sine theta 0, sine theta cos theta 0, 0 0 1, q small x, q small y, q small x if you just see your the previous one from here I can write down this particular thing.

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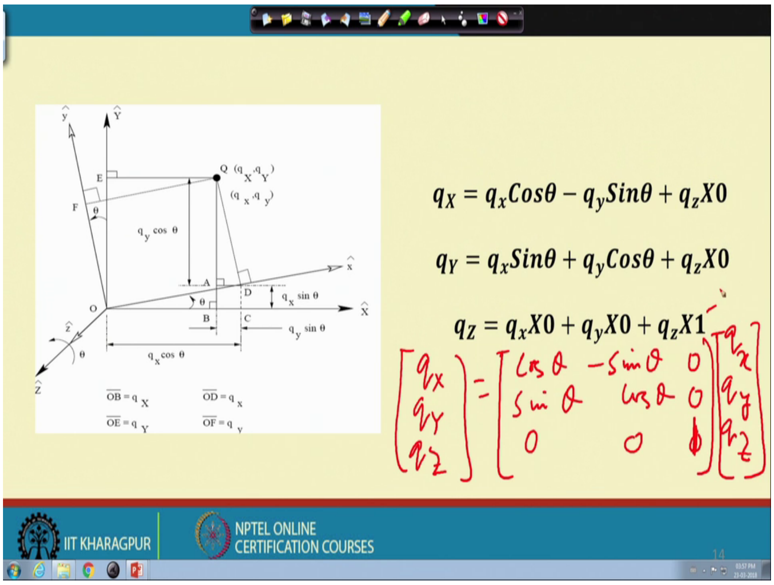

For example, say let me write it here once again so, this q x, q y, q z can be written as your cos theta minus sine theta 0 then comes your sine theta cos theta 0 0 0 1 and this is multiplied by your q small x, q small y, q small z something like this, ok. I am sorry here there will be 1 here, because here we have got 1 here.

So, this is the way actually we can write down this particular the rotation term. So, this is actually your this rotation term the rotation about Z by an angle theta. So, rotation about Z by an angle theta is cos theta minus sine theta 0, sine theta cos theta 0, 0 0 1. So, this is nothing but the rotation about Z by an angle theta.

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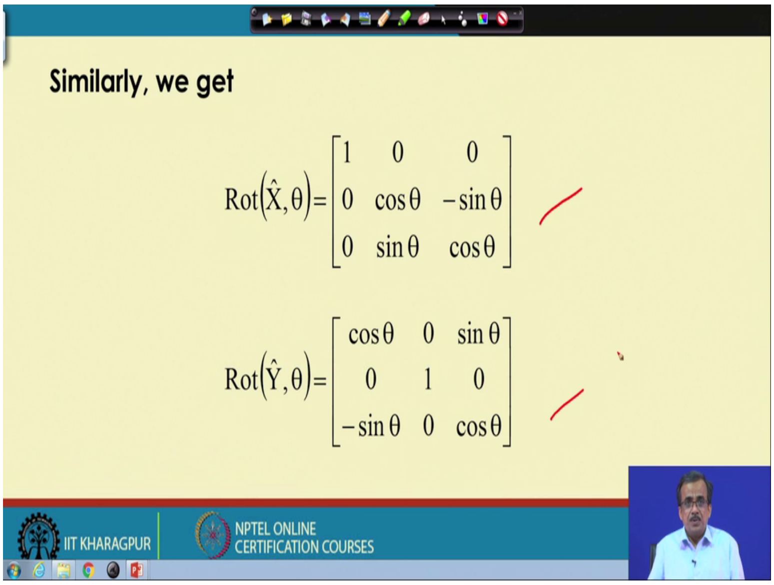

Now, by following the similar procedure the similar procedure in fact, I can find out the rotation about X by an angle theta that is nothing but 1 0 0, 0 cos theta minus sine theta, 0 sine theta cos theta. Similarly, I can also find out rotation about Y by an angle theta is cos theta 0 sine theta, 0 1 0, minus sine theta 0 cos theta. So, using these I can find out rotation about X by an angle theta, rotation about Y by an angle theta.

Thank you.