Robotics
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Lecture - 11
Robot Kinematics

We are going to start with the second topic and this is on Robot Kinematics. Now, here the purpose of kinematics is to study the motion of the robotic link, but here we do not try to find out the reason behind this particular the motion. For example, say if it is a linear movement there must be some force acting, if it is a rotary movement there must be some torque acting, but here in kinematics we do not try to find out what should be the amount of force what should be the amount of torque. But, we study only the motion of the different links the relative motion of the different links and so on.

So, let us see how to carry out this particular the kinematic analysis.

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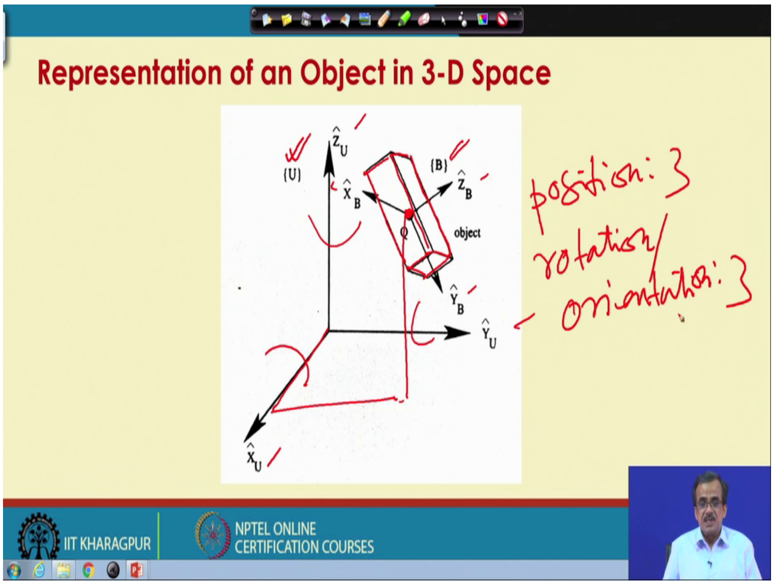

Now, before I start with this kinematic analysis let me start with the very scratch like the very beginning I should say supposing that I have got a 3-D object. So, this is nothing but a 3-D object, now this particular 3-D object so, I will have to represent. So, this is actually the 3-D object which I will have to represent; that means its position and orientation. So I will have to represent in 3-D space.

Now, here so, U indicates the universal coordinate system and it has got the axis like X U, Y U and Z U they are independent, they are mutually perpendicular. Now, we will have to represent. So, this particular 3-D object in this 3-D space. So, how to represent, how to represent it is position, how to represent it is orientation? Now, here to represent these particular position actually what we do is we try to find out the mass center of this particular the 3-D object. So, this is the mass center. So, at this mass center we just draw one coordinate system that is nothing but the B coordinate system and it is having X B, Y B and Z B

Now, here so, if I want to determine the position of this particular mass center. So, what I will have to do is, I will have to move along X I will have to move along y and I will have to move along Z just to find out this particular the position. So, I need three information and if I want to represent the orientation of this particular 3-D object in this 3-D space I will have to consider the orientation or the rotation; rotation about X rotation about Y, rotation about Z. So, I need three more information.

So, for position we need three information that is X, Y and (Refer Slide Time: 00:00) ; and for this particular the rotation or this orientation rotation or orientation, so, we need actually the three more information. So, we need totally six information. So, this is the way actually we can represent the position and orientation of a 3-D object in 3-D space.

Now, let us see like how to represent the position what do you need to represent the position only.

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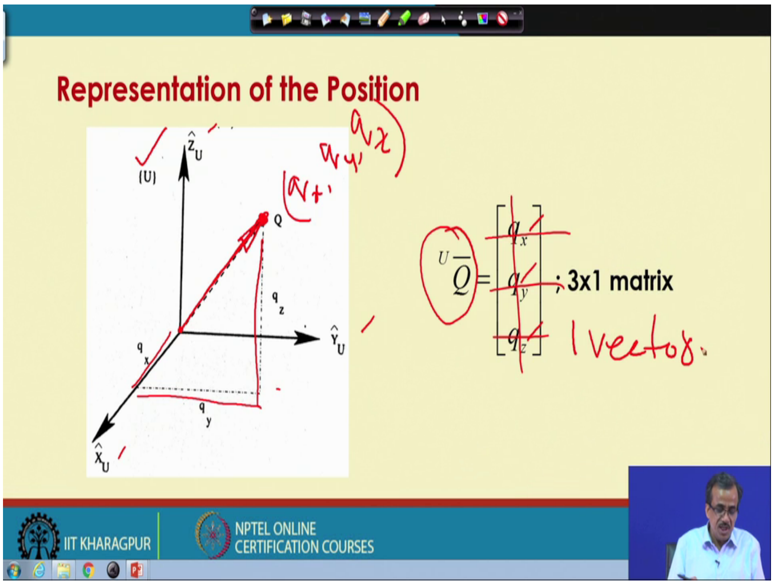

So, representation of the position; that means the position of the mass center of the 3-D object. Now, let me draw once again the same thing. So, I have got the universal coordinate system. So, this is X U, Y U and Z U and q is actually a point whose position is to be determined. So, what I will do is starting from the origin of this particular O. So, I will move along x then I will move along y then I will move along z. So, this particular point will having the coordinate like your q x, q y, q z. So, this is nothing but the coordinate.

And here, so this particular the point this can be considered as a vector as a position vector. So, this is nothing but the position vector and that is denoted by q with respect to u. So, this particular point is actually q with respect to u that is the universal coordinate system and to represent that particular position vector. So, I need the elements like your q x, q y and q z. So, this is a vector and in matrix form this is nothing but a 3 cross 1 matrix. There are 3 rows and 1 column and this is nothing but a 3 cross 1 matrix. So, this is nothing but a 3 cross 1 matrix or a vector. So, to represent the position we need one vector or one 3 cross 1 matrix. So, this is how to represent the position.

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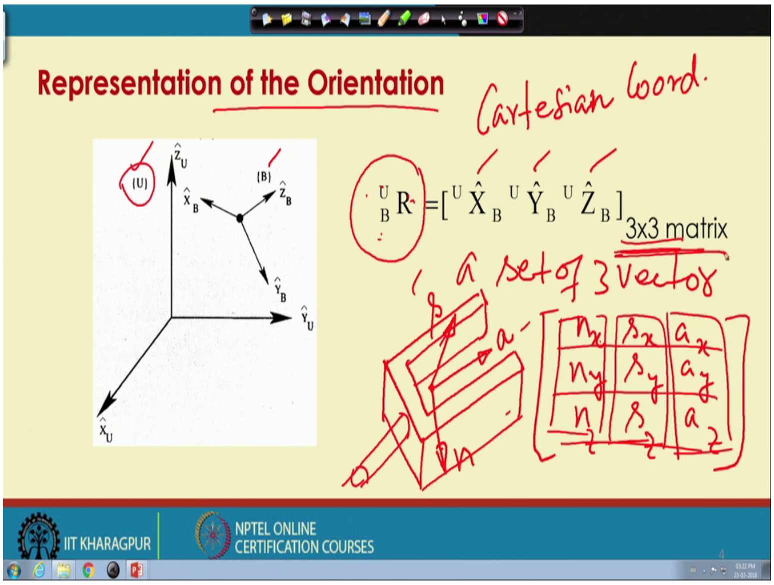

Now, let us see how to represent the orientation. To represent the orientation so, what I do is so, this is once again my U coordinate system, the universal coordinate system and this particular b is the body coordinate system which is attached to the mass center of the body whose orientation I am just going to represent. Now, this particular B you can see that there have been some amount of orientation change. For example, say X B is not parallel to X U Y B is not parallel to Y U. Similarly, Z B is not parallel to Z U that means, there has been some rotation. So, orientation of this particular B has been changed with respect to this particular the U.

Now, here the way we represent so, this particular the orientation or rotation is like this. So, R B with respect to U is nothing but the rotation of B with respect to U that is the rotation of body coordinate system with respect to the universal coordinate system is nothing but is your R B with respect to U. Now, to represent this R B with respect to U we take the help of in fact, three vectors. So, this is one vector, this is another vector, this is another vector. So, we need in fact, a set of three vectors a set of three vectors we need now one set of vector is nothing but a 3 cross 1 matrix and this will become a 3 cross 3 matrix.

Now, how can we visualize so, this particular vector? So, I am just going to prepare one a rough sketch just to understand like what are those the three vectors. Now, let me just prepare one very rough sketch for a robotic hand or a robotic gripper sort of thing. So, this is one actually one 2 finger say very simple gripper. So, this is actually the gripper and. So, I have got two fingers here finger 1 and finger 2. Now here so, with the help of this particular gripper; so, I am just going to grip that particular the 3-D object. Now, if I want to grip the 3-D object the 3-D object may have different orientation and depending on the orientation of the 3-D object so, I will have to change the orientation of this particular the gripper or the finger then only I can grip it and here to represent the orientation actually we take the help of three vectors and this is actually one is called the normal vector that is denoted by n, another is called the sliding vector that is denoted by s, another is called the approach vector that is denoted by a, ok.

So, we have got actually normal vector sliding vector and approach vector. So, if I just write down in the form of the matrix n, s, a this is x, x, x; n, s, a y, y, y; n, s, a z, z, z. So, in the matrix form this particular orientation will be represented like this and this is in Cartesian coordinate system. So, I am now, discussing here the Cartesian coordinate system Cartesian coordinate system how to represent that particular the orientation.

Now, here this n x, n y and n z are the elements of this particular the normal vector. So, these are the elements of the normal vector, these are the elements of the sliding vector so, this one and these are the elements of these particular the approach vector. So, I need a set of three vectors and this is nothing but actually the 3 cross 3 matrix. In the matrix form so, we have got 3 rows and 3 columns. So, this is nothing but a 3 cross 3 matrix. So, to represent the orientation we need a set of three vectors or we need a matrix and that is nothing but actually a set of in a 3 cross 3 matrix.

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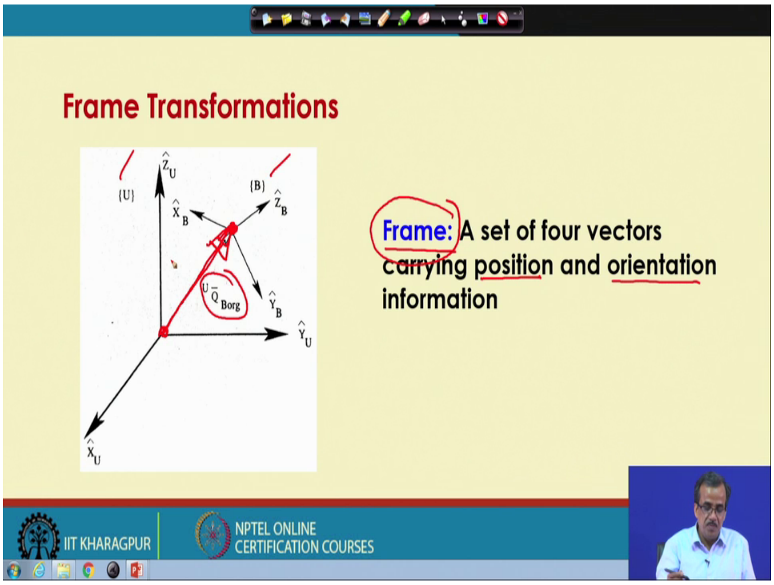

Now, I am just going to define this particular the frame. Now, frame is actually a set of four vectors which carry information of the position and orientation. We know that to represent the position we need only one vector and to represent the orientation we need in fact three vectors, a set of three vectors; that means, to represent both position as well as orientation we need a set of four vectors. Now, this set of four vectors is known as the frame. Actually, what we do is we try to assign frame at each of the joints.

Now, here in this schematic view if you see so, this is nothing but the universal coordinate system and once again B is actually the body coordinate frame which is attached to the mass center of the body. Now, this particular coordinate system X B, Y B and Z B is having some translation with respect to U. So, this is the amount of translation, so this particular origin has been shifted to this particular point and this particular translation is nothing but Q B origin with respect to U. So, in the vector form I can represent like this. And here there are some rotation and that is why X B is not parallel to X U, Y B is not parallel to Y U and Z B is not parallel to Z U and this is the way actually we can represent the position and orientation with the help of actually the four vectors.

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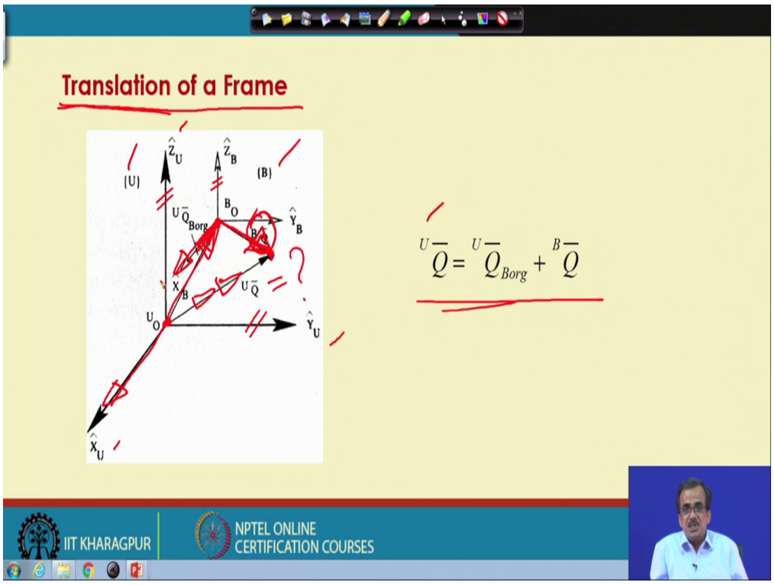

Now, I am just going to concentrate on the frame transformation. Now, frame transformation means it includes frame translation and frame rotation. So, here actually I am just going to concentrate on the translation of a frame first. So, only translation the pure translation I am just going to consider.

Now, once again so, this is the universal coordinate system X U, Y U, Z U are the universal coordinate system, B is the body attached coordinate system and here I am just going to consider only translation. So, there is no rotation of B with respect to your X U and that is why you can see that so, this particular X B this is actually the X B is parallel to your X U then comes your. So, this Y B is parallel to Y U and Z B is parallel to your Z U, but there is has been a shifting of the origin from here. So, previously the origin was here now, the origin has been shifted to this and here I am just going to show it with the help of one position vector.

So, this is the situation. So, the frame B or the coordinate system B has been translated only with respect to the universal coordinate system, but there is no such rotation. Now, supposing that the position of a particular the point with respect to the B is known supposing that this particular vector is known, ok. So, if this particular vector is known and this particular translation is also known then how to find out this Q with respect to U; so that is our aim.

So, our aim is to determine the position of these are the same point which is lying on this particular body B with respect to the universal coordinate system that means, I am trying to find out. So, this particular the position vector and very easily I can find out that is Q with respect to U. So, Q with respect to U is nothing but Q B origin with respect to U. So, this is nothing but Q B origin with respect to U plus Q with respect to B. So, this is your Q with respect to B. So, I will be getting that is Q with respect to U.

That means if I know the position with respect to the body coordinate frame very easily you can find out the position information with respect to the universal coordinate system and here I am just going to consider I have consider only the translation.

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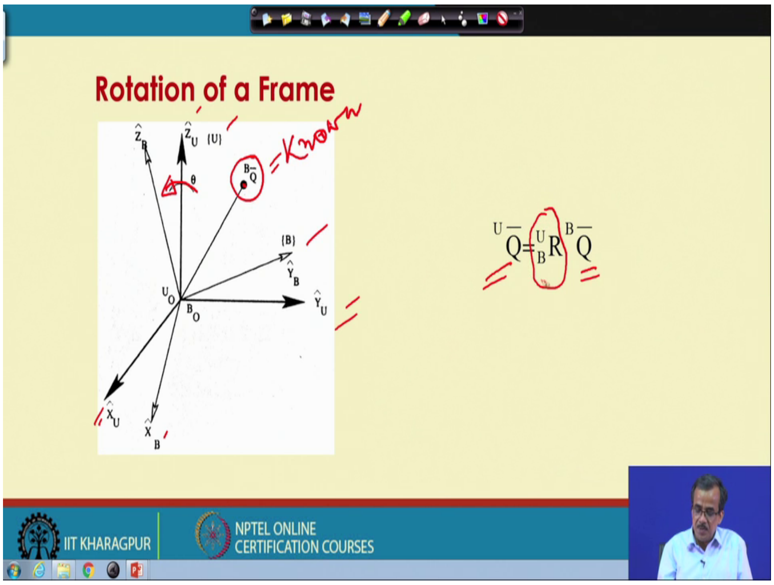

Now, I am just going to consider only rotation that is pure rotation. Now, once again let me consider that universal coordinate system U having X U, Y U and Z U and there is pure rotation with respect to the universal coordinate system by some angle say theta in the anticlockwise sense. Now, if I rotate by an angle theta in the anticlockwise sense so, my Z B that is the Z axis of the body coordinate system. So, ZB will be different from Z U initially they were coinciding. Now, Z B will be different from Z U, X B will be different from X U and Y B will be different from Y U, ok.

And so, we will be getting this particular rotated frame and supposing that the position of a particular point with respect to the rotated frame B is known to us. So, this is known. So, once again let me repeat the body coordinate system that has been rotated with respect to the universal coordinate system and I know the position that is Q with respect to B and our aim is to find out Q with respect to U the position with respect to the universal coordinate system; so that we will have to determine.

Now, this Q with respect to U is nothing but the rotation of B with respect to U multiplied by Q with respect to B, so this particular Q with respect to B is known and if I can find out. So, this particular rotation matrix that is a R B with respect to U. So, very easily I can find out Q with respect to U; now, how to determine that RB with respect to U? That I am going to discuss after some time.

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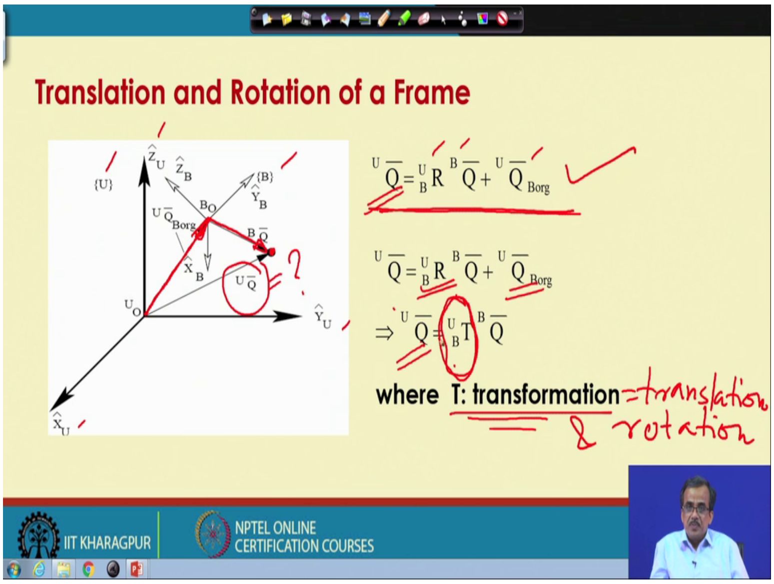

Now, I am just going to consider a more complicated situation where I am just going to consider both translation as well as rotation and once again let me repeat that. So, this is the universal coordinate system X U, Y U and Z U and B is the body coordinate system there has been some translation that means the origin has been shifted. So, from here to this particular point and this is actually the position vector that is Q B origin with respect to U. And so, this particular B the B coordinate system has got some rotation with respect to this particular the universal coordinate system U and that is why XB is not parallel to X U, Y B is not parallel to Y U and Z B is not parallel to Z U. So, this is the situation. So, there has been both translation as well as rotation.

Now, supposing that so, this particular point that is Q with respect to B is known. So, this particular point is known and what is our aim? Our aim is to find out say Q with respect to U. So, that is our aim. So, this is known that is Q B origin with respect U. So, this particular vector is known this particular position vector is known moreover the rotation matrix of B with respect U say that is also known then how to find out this Q with respect to U.

Now, to find out this Q with respect to U so, I will have to find out like this. So, I will have to find out so, rotation of B with respect U multiplied by so, Q with respect to B. So, both the things are known plus Q B origin with respect to U. So, this is also known so, very easily we can find out Q with respect to U. Now, this Q with respect to U is nothing but this expression. Now, here inside the expression there are two things; one is actually the translation of the origin of B, another is actually the rotation of B with respect to U. So, there are two things one is the translation, another is your the rotation.

So, now, what I am going to do is I am just going to use a particular term which includes both translation as well as rotation and that particular term is nothing but transformation. So, this particular transformation it includes both translation both translation and rotation. So, translation and rotation are taken together inside this particular the transformation.

Now, here so, this particular expression I am just going to write down in terms of the transformation matrix that is Q with respect to U is nothing but the transformation of B with respect to U. So, in place of this rotation and this position now, I am using this particular the transformation; so transformation of B with respect to U multiplied by Q with respect to B. Now, if I do this transformation of B with respect to U now, I can multiply. So, this Q with respect to B and you will be getting this Q with respect to U. So, this is the way actually we can find out if we have both translation as well as rotation.

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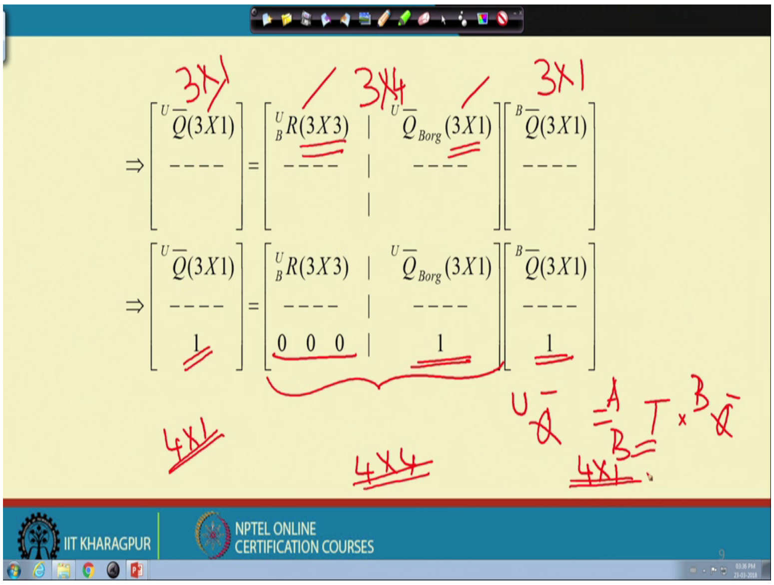

Now, let us try to check the dimension of this particular the dimension matching of this particular the matrix. Now, if you see in the last slide we wrote the equation that is Q with respect to U is nothing but the transformation of B with respect to A multiplied by actually Q with respect to B. Now, here this Q with respect to U if you see in terms of matrix. So, this is nothing but a 3 cross 1 matrix. Now, this particular transformation matrix has got two things; one is called the rotation matrix and we have got the position vector. Now, this rotation matrix is a 3 cross 3 matrix and position vector is nothing but a 3 cross 1 matrix.

So, taken both the things together that is rotation as well as translation will be actually 3 cross 3 and 3 cross 1. So, this will become your 3 cross 4 matrix. So, there will be 3 rows and 4 columns and this is nothing but 3 cross 1 and this particular Q with respect to B is nothing but your 3 cross 1.

So, that means, I will have to multiply one 3 cross 1 matrix by one 3 cross 4 matrix just to get a 3 cross 1 matrix which is which is not possible. Now, to solve this particular problem to make it possible actually what I do is, so, here we do some corrections some modification sort of thing. So, here on this position term Q with respect to U we add one and here just below the rotation matrix on the fourth the row we use 0 0 0 three zeroes we add and here we add 1 and here we add 1 and now, this particular transformation matrix will have the dimension that is your 4 cross 4. So, this was 3 cross 4.

Now, I have added one more row here. So, this will become your 4 cross 4 and this particular thing will become 4 cross 1 matrix and this will also become 4 cross 1 matrix. Now, if I just multiply. So, this 4 cross 1 with 4 cross 4 so, I will be getting this 4 cross 1 matrix. So, it is matching. Now, my question is like why do you put 1 here, why do you put 1 here, why 1 here and why do you put these three zeros? That I am going to discuss.

Thank you.