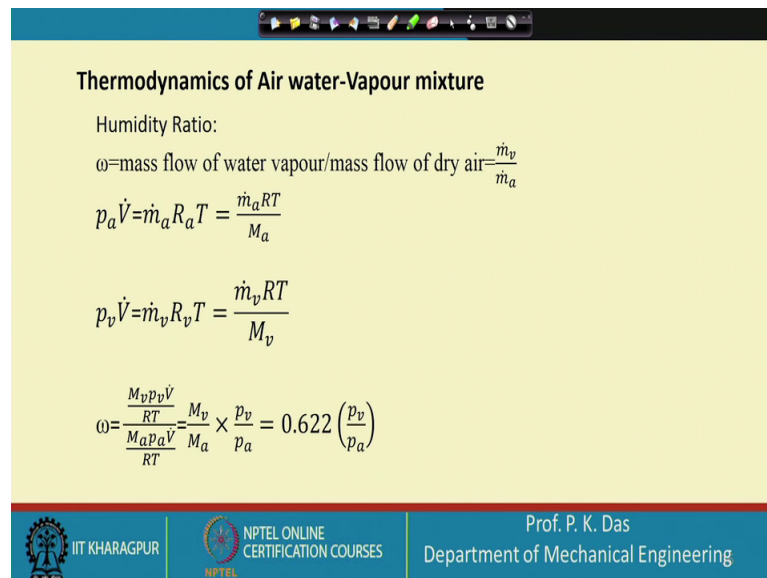


Heat Exchangers: Fundamentals and Design Analysis.
Prof. Prasanta Kumar Das
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 61
Direct Contact Heat Exchanger (Contd.)

So, we were, friends we were what, discussing cooling tower, which is a Direct Contact Heat Exchanger and as I have told psychrometry is very important and in a cooling tower we can theoretically get a cold water temperature as low as wet bulb temperature. So, basically it is like this.

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Thermodynamics of Air water-Vapour mixture

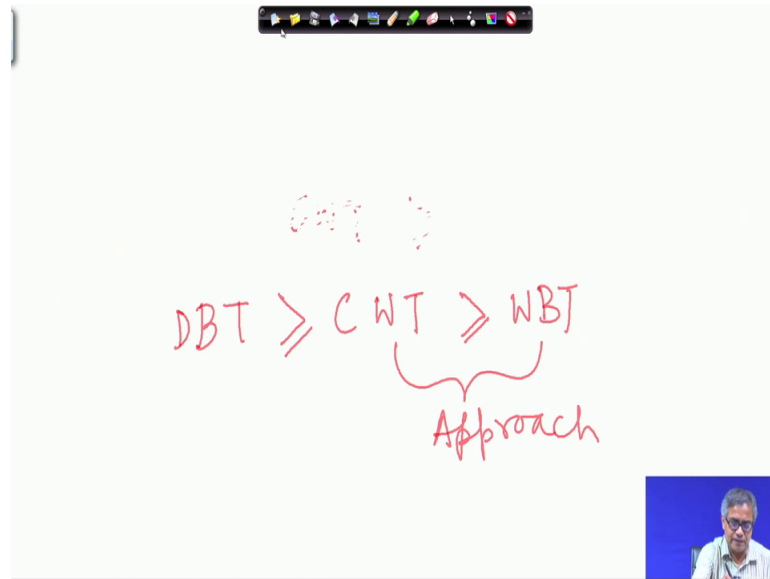
Humidity Ratio:

$\omega = \text{mass flow of water vapour} / \text{mass flow of dry air} = \frac{\dot{m}_v}{\dot{m}_a}$

$$p_a \dot{V} = \dot{m}_a R_a T = \frac{\dot{m}_a R T}{M_a}$$
$$p_v \dot{V} = \dot{m}_v R_v T = \frac{\dot{m}_v R T}{M_v}$$
$$\omega = \frac{\frac{M_v p_v \dot{V}}{R T}}{\frac{M_a p_a \dot{V}}{R T}} = \frac{M_v}{M_a} \times \frac{p_v}{p_a} = 0.622 \left(\frac{p_v}{p_a} \right)$$

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If we see that C W T, is greater than equal to, greater than equal to another. Let us, put it in slightly different manner. It is like this, we can put it in a slightly different manner. Let us, a dry bulb temperature DBT is greater than equal to Cold Water Temperature in a cooling tower and Cold Water Temperature is greater than equal to Wet Bulb Temperature.

So; that means, cold water temperature will be in between your dry bulb temperature and wet bulb temperature and this difference we are calling as Approach. So, with these let us go back to our slide and what we want to do that little bit of air water vapour, mixture thermodynamics these are all known things. I have given this only for a recapitulation. So, here we have defined the Humidity Ratio and Humidity Ratio one can get in terms of partial, pressure of vapour and pressure of air. So, with this relationship any book of thermodynamics will give you. I am not spending time on this.

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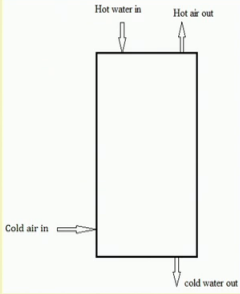
SSSF Model for Cooling Tower



Conservation of mass for dry air:
 $\dot{m}_{a,in} = \dot{m}_{a,out} = \dot{m}_a$

Conservation of mass for water:
 $\dot{m}_{w,in} + \omega_{a,in} \dot{m}_a = \dot{m}_{w,out} + \omega_{a,out} \dot{m}_a$

First Law Analysis (Air+Water):

$$\dot{m}_{cw,in} c_w T_{w,in} - \dot{m}_{cw,out} c_w T_{w,out} = \dot{m}_a \{ (c_{p,a} + \omega_{out} c_{p,steam}) T_{a,out} - (c_{p,a} + \omega_{in} c_{p,steam}) T_{a,in} + (\omega_{out} - \omega_{in}) h_{fg} \}$$



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Now, let us say we are, doing a steady state steady flow model of a cooling tower. So, you can go through this one. I will again not explain much, but what we can see that, Hot water is entering here cold water is going out similarly Cold air is coming in and hot, hot air is going out hot and humid air we should say hot and humid air is going up now conservation of mass for dry air. So, psychrometry, I would request all of you to go through psychrometry. So, in psychrometry we are considering air water vapour mixture.

But in that dry air quantity that remains constant. So, in certain quantity of dry air the amount of water vapour that goes on changing. So, most of the properties are all the property almost all the properties are moist air or air water vapour mixture is defined based on the mass of the dry air. So, this is also one important thing you have to keep it in mind and dry air quantity through the cooling tower that remains constant then conservation of mass of water. So, what is there that, water be, water is coming in along with moisture, air some amount of water vapour is coming in then, water is going out and along with, along with, air some amount of water vapour is going out.

So, this is the conservation of water. Generally very small amount of water is evaporated. So, the amount of water that is evaporated it has got really very negligible effect on the total mass flow rate of water, but it has got a very significant effect in the heat and mass transfer. So, far air side this has got significance. Then First Law of, Analysis this is also

very, I mean simple I mean easy to understand that water there will be change in temperature.

Because hot water is getting cooled and you see for water I have considered constant, constant mass flow rate which is not exactly true, but for all practical purposes it is true. For air when air is entering. So, air plus water vapour that is there, it has got some temperature. So, C P into all these things that will give the enthalpy of, air and similarly for outgoing air also we can get it. So, we can make some sort of a balance of what is happening ok. So, some sort of first law analysis we can do and, with some simplification we can get these results.

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$$\dot{m}_{cw,in}c_w T_{w,in} - \dot{m}_{cw,out}c_w T_{w,out} = \dot{m}_a \{ (c_{p,a} + \omega_{out}c_{p,steam})T_{a,out} + \omega_{out}h_{fg} - (c_{p,a} + \omega_{in}c_{p,steam})T_{a,in} - \omega_{in}h_{fg} \}$$

$$\dot{m}_{cw,in}c_w T_{w,in} - \dot{m}_{cw,out}c_w T_{w,out} = \dot{m}_a \{ (c_{p,a}T_{a,out} + \omega_{out}(c_{p,steam}T_{a,out} + h_{fg})) - (c_{p,a}T_{a,in} + \omega_{in}(c_{p,steam}T_{a,in} + h_{fg})) \}$$

$$\dot{m}_{cw,in}c_w T_{w,in} - \dot{m}_{cw,out}c_w T_{w,out} = \dot{m}_a(h_{a,out} - h_{a,in})$$

Where,

$$h_{a,out} = (c_{p,a}T_{a,out} + \omega_{out}(c_{p,steam}T_{a,out} + h_{fg}))$$

$$h_{a,in} = (c_{p,a}T_{a,in} + \omega_{in}(c_{p,steam}T_{a,in} + h_{fg}))$$

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I am not going detail into details of it because these are simple thermodynamics and you can get it. Only one thing, I like to bring your attention that some amount of water vapour will get into air. So, when that is going it is bringing some amount of latent heat that h f g is coming in ok. So, this is very important because h f g having a, high value we will see this will matter much in our heat transfer.

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Local Heat and Mass Transfer in Air-Water System

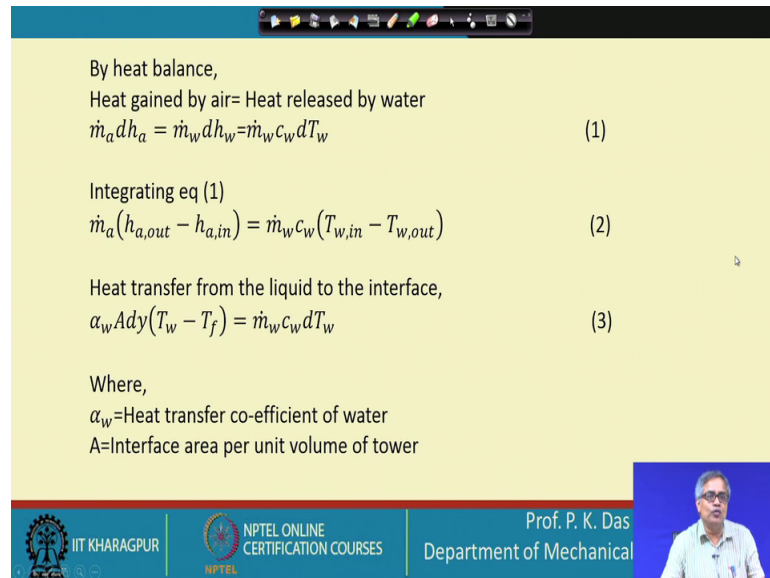
$$\left(\dot{m}_w + \frac{d\dot{m}_w}{dy} dy \right), \left(T_w + \frac{dT_w}{dy} dy \right) \quad \left(h_a + \frac{dh_a}{dy} dy \right), \dot{m}_a \left(1 + \omega + \frac{d\omega}{dy} dy \right)$$

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Next, we are what we are doing, for the bulk of the or the for the total of the heat exchanger, cooling tower we have done First Law Analysis. Now, we are trying to do some sort of energy balance for a small control volume. You can see that, the, we have taken the height of the tower, a small portion of the cooling tower with a height dy the area is dA . What is this area? This area is nothing, but the heat transfer area it is not the, any physical area. It is the interfacial area which the air and water is having. So, for all practical purposes, it is very difficult if not impossible to determine it, but in our analysis it will enter ok, in our analysis it will enter. So, what we can find that this is the amount of water that is coming in \dot{m}_w and T_w with T_w temperature.

Let us say, it has entered with this much amount of because there will be change in the, water flow rate. So, higher flow rate of water is entering here and higher temperature hot water. So, it is higher temperature. Similarly, air this is some enthalpy and this is the change of enthalpy and this is the change of mass flow rate because some amount of evaporation is there.

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By heat balance,
Heat gained by air = Heat released by water
$$\dot{m}_a dh_a = \dot{m}_w dh_w = \dot{m}_w c_w dT_w \quad (1)$$

Integrating eq (1)
$$\dot{m}_a (h_{a,out} - h_{a,in}) = \dot{m}_w c_w (T_{w,in} - T_{w,out}) \quad (2)$$

Heat transfer from the liquid to the interface,
$$\alpha_w A dy (T_w - T_f) = \dot{m}_w c_w dT_w \quad (3)$$

Where,
 α_w = Heat transfer co-efficient of water
A = Interface area per unit volume of tower

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So, again one can do some sort of energy balance which one can do and you see basically the enthalpy change of, water that is equal to the enthalpy change of air water vapour mixture and by integration you can get this kind of a thing.

Now, what we can do some sort of a Heat transfer from the liquid at the interface, we can have if, α_w is the Heat transfer co-efficient for water side we can get this kind of a relationship. α_w is the Heat transfer co-efficient of water and you see, A we have taken, A we do not know, A we cannot measure, but only we are thinking that at the interface area that is A and A into delta y that is for the height delta y the amount of area. So, A is the Interfacial area per unit, volume because we can also multiply with some sort of A width of the cooling tower or we can take that it is per unit height of the cooling tower.

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Similarly, the heat transfer from bulk air to interface is given by,

$$\alpha_a A dy (T_f - T_a) = \dot{m}_a (c_{p,a} + \omega_{in} c_{p,steam}) dT_a \quad (4)$$

Where, α_a = Heat transfer co-efficient of air

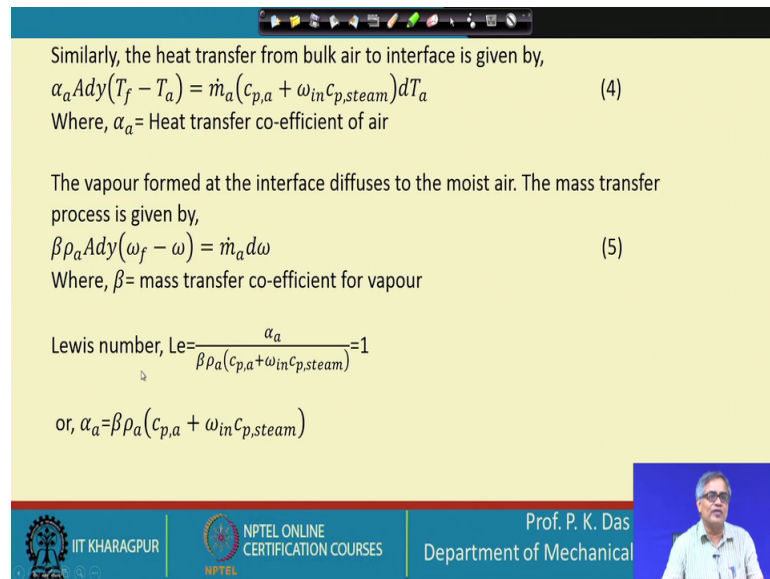
The vapour formed at the interface diffuses to the moist air. The mass transfer process is given by,

$$\beta \rho_a A dy (\omega_f - \omega) = \dot{m}_a d\omega \quad (5)$$

Where, β = mass transfer co-efficient for vapour

Lewis number, $Le = \frac{\alpha_a}{\beta \rho_a (c_{p,a} + \omega_{in} c_{p,steam})} = 1$

or, $\alpha_a = \beta \rho_a (c_{p,a} + \omega_{in} c_{p,steam})$



So, with all these things we proceed. And, then what we do similarly heat transfer from the, bulk to air interface for air also we get this one. Now, vapour formed at the interface diffuse in the moist air the mass transfer process is given like this. So, rho beta as is some sort of a mass transfer co-efficient, beta into rho a into area that gives mass transfer co-efficient, some sort of a bulk mass transfer coefficient, but beta we can take as if it is some sort of a mass transfer coefficient and we get this. Then, we bring some sort of Lewis number because you see; there is both heat transfer and mass transfer. We have to combine these two things. So, fortunately we can define one number Lewis number, definition has also already been given and Lewis number is equal to 1 for air water system.

So, the Heat transfer co-efficient for air that comes that now becomes very easy because we can have this kind of a relationship. Ok, as Lewis number is equal to 1. So, this is one advantage, with this.

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Hence from eq(4)

$$\beta \rho_a (c_{p,a} + \omega_{in} c_{p,steam}) A dy (T_f - T_a) = \dot{m}_a (c_{p,a} + \omega_{in} c_{p,steam}) dT_a \quad (6)$$

Multiplying eq(5) by h_{fg} ,

$$\beta \rho_a A dy h_{fg} (\omega_f - \omega) = \dot{m}_a h_{fg} d\omega \quad (7)$$

Adding equations (6) and (7)

$$\dot{m}_a (h_{fg} d\omega + (c_{p,a} + \omega_{in} c_{p,steam}) dT_a) = \beta \rho_a A dy \left((c_{p,a} + \omega_{in} c_{p,steam}) T_f + h_{fg} \omega_f \right) + \beta \rho_a A dy \left((c_{p,a} + \omega_{in} c_{p,steam}) T_a + h_{fg} \omega \right)$$

Or,

$$\dot{m}_a dh_a = \beta \rho_a A dy h_f - \beta \rho_a A dy h_a = \beta \rho_a A dy (h_f - h_a)$$

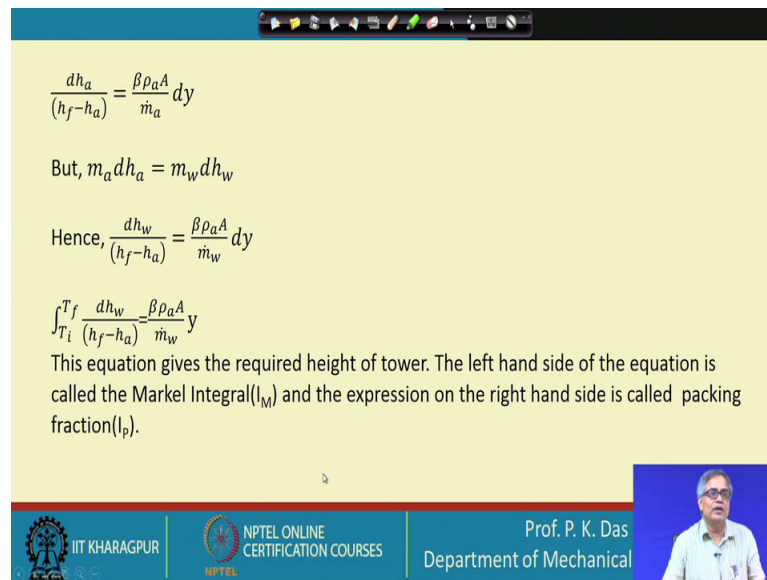
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Now, we can again go for the energy balance, this is the temperature difference this is the Heat transfer co-efficient you see, in the transfer co-efficient the mass transfer is also there and, this is your, for air what is the enthalpy change? Ok, within the small control volume. Then some sort of manipulation has been done and, h_f that is the, enthalpy difference that has been brought in.

And, the difference in the specific humidity that has also been brought in, with this, finally, we can get $\dot{m}_a dh_a$, \dot{m}_a is the air flow rate dh_a is the difference in enthalpy and that is equal to this mass transfer co-efficient into dy into h_f into h_a ; h_f is the enthalpy of the fluid, h_a is the enthalpy of air, what does it mean? It means the enthalpy increase in the air that is equal to that we are, equating with the, energy transfer um, due to, basically which is due to, mass transfer and, what will be the energy transfer due to mass transfer? h_f is the enthalpy of water and h_a is the enthalpy of air.

So, basically from h_f ; that means, from water some amount of moisture will go to air. So, this is the enthalpy difference and this is the mass transfer co-efficient and this will increase the overall enthalpy of air. So, that is why we are getting dh_a . So, this has got lot of similarity if you see how the heat exchanger equations have been developed. So, it has got lot of similarity ok.

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$$\frac{dh_a}{(h_f - h_a)} = \frac{\beta \rho_a A}{\dot{m}_a} dy$$

But, $\dot{m}_a dh_a = \dot{m}_w dh_w$

Hence,
$$\frac{dh_w}{(h_f - h_a)} = \frac{\beta \rho_a A}{\dot{m}_w} dy$$

$$\int_{T_i}^{T_f} \frac{dh_w}{(h_f - h_a)} = \frac{\beta \rho_a A}{\dot{m}_w} y$$

This equation gives the required height of tower. The left hand side of the equation is called the Markel Integral (I_M) and the expression on the right hand side is called packing fraction (I_p).

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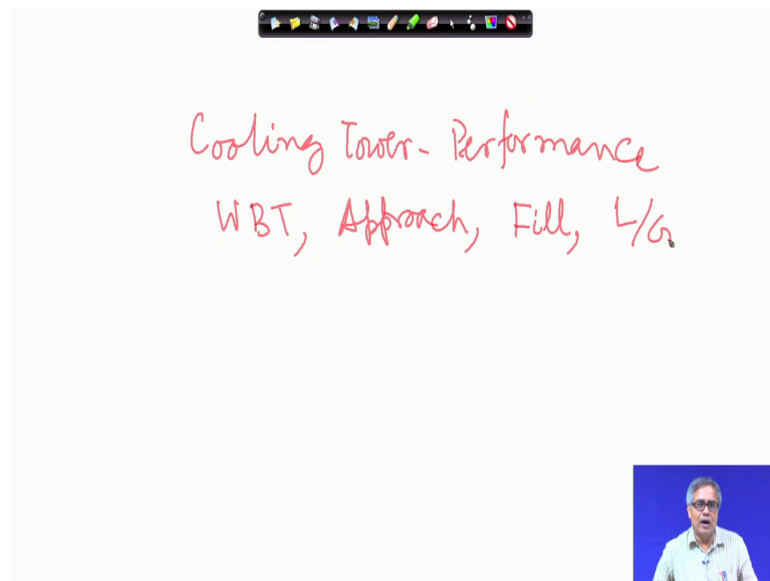
So, with that we can go to the next step and ultimately we can get an equation like this and in this equation dy that is the infinitesimal or small incremental height of the tower is coming in and then if we do this integration we can get, the, tower height.

But for getting the tower height we have to know this quantity. This equation gives the required height of the tower. The left hand side of the equation is called Markel Integral I_M and the expression on the right hand side is called the packing fraction I_p . Now, I_M and I_p they are same, all right or there is a matching. So, but this is how it has to be determined the crux of the problem lies that this is the mass transfer co-efficient, this is the area which is Interfacial area per unit volume as we have defined earlier and these things cannot be determined separately. See when there is a Fill or there are large number- number of droplet us are formed.

It is very difficult to know what is the area, so, what we do? Up to this with some sort of a simplification we can derive the equation this is called Markels. Cooling Tower equation we can derive it. Basically, I would, like you to follow any kind of book on, refrigeration air conditioning, where you will find this kind of derivation even in many, many chemical engineering mass transfer book you will get this derivation, but what you can we do with this? See, this quantity it is very difficult to design or sorry determined from the fundamental physics though nowadays people are even telling that with cfd it can be determined.

But it is not easy and not whatever we determine that is not very reliable. So, the cooling tower manufacturers have got their tested cooling towers. The heat transfer characteristics, comes from the field characteristics. So, they have got testing of different field geometry and they have got the field characteristics. Now, cooling tower, performance that will depend on many things what are the things it will depend on cooling tower performance let us say.

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Cooling Tower Performance let us say. It will depend on WBT, it will depend on Approach, it will depend on the type of Fill we are using it will depend on L by G ratio mainly ok. So, basically, the cooling tower manufacturer, they have to perform a large number of experiments. Let us say, they are using a typical kind of a Fill. So, they have to perform and Fill. It is not only having a geometry. It is also having some sort of a dimension like; this is a 3 dimensional volume. So, how much is the width, breadth and height. So, all these things it is having. So, Fill and Fill geometry then.

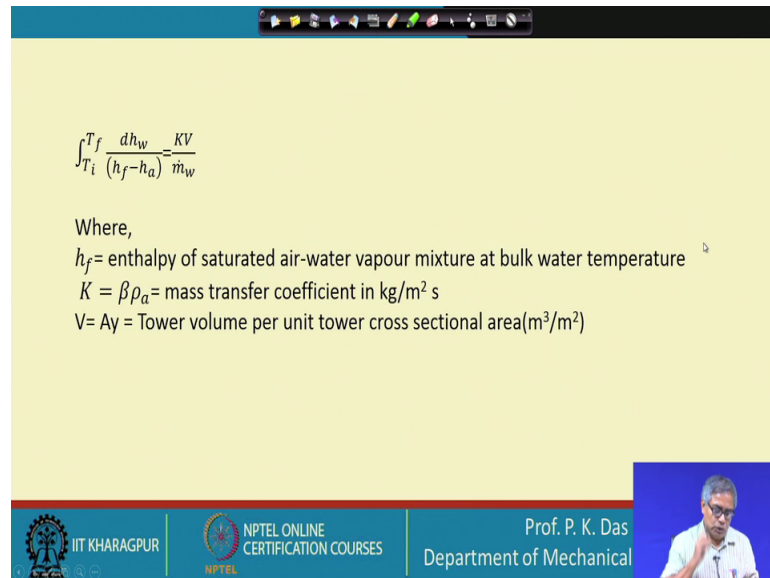
There are variables like WBT, Approach and L by G. So, with this variable they produced some sort of a curve Ok; and those curves are used for Heat Exchanger Design or based on those curve they do the design of Heat Exchanger, in this case the Heat Exchanger is nothing, but you are cooling tower. So, let us proceed little bit and then let us see, How it is done? So, we have stopped over here and I told the right hand side, is very difficult to determine left hand side is some sort of if you know the psychometric

property etcetera you can determine it by some sort of numerical integration, but the right hand side is very difficult to determine.

Because, we do not have, idea regarding beta rho a and a sorry rho a is the density beta and a we do not have the idea. So, then what is done overall Cooling Tower Performance? or overall Fill performance? Fill is the main part of the Cooling Tower that is the Heat Exchanger, element Heat Exchanger element. So, that is determined by the Cooling Tower manufacturers for a given Approach for a giving WBT for a given L by G ratio. So, that is there and then, they are doing actually over a range of WBT over a range of Approach and over a range of L by G. So, if that is known then it has to be matched the left hand side and the right hand side is to be can be matched.

And then, we can have the value of y; that means, what could be the tower height? It is not the physical height of the tower. It is the height of the Fill or packing or the Heat Exchange element when the, breadth and width of the Fill element are some sort of a given quantity. This is how Heat Exchanger is designed. Now, as a, thermal engineer or as an engineer of the plant, you never have to design a Heat Exchanger you cannot design it and you cannot make it unless you are working in a Heat Exchanger company you are really not very much bothered regarding the design of the Heat Exchanger, but how the design is done to have some idea, I mean it is important to have some idea and that is why, I have given this explanation and with this explanation we can go to the next slide.

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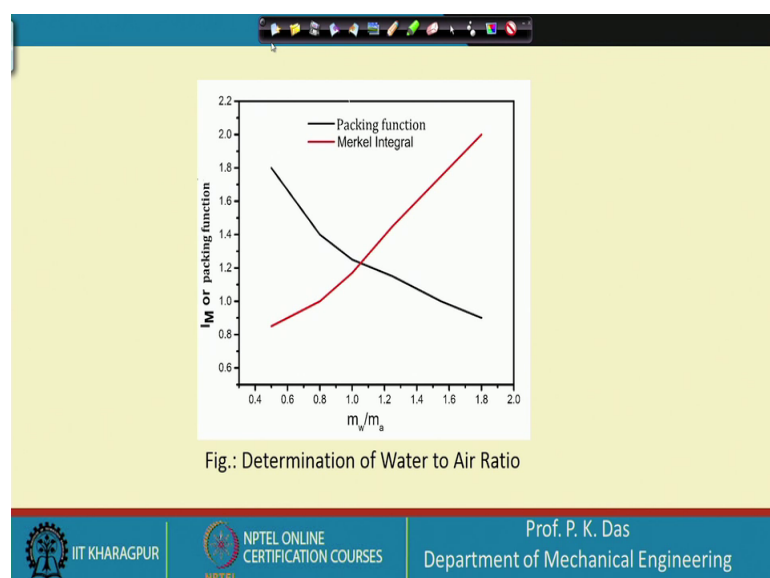
$$\int_{T_i}^{T_f} \frac{dh_w}{(h_f - h_a)} = \frac{KV}{\dot{m}_w}$$

Where,
 h_f = enthalpy of saturated air-water vapour mixture at bulk water temperature
 $K = \beta \rho_a$ = mass transfer coefficient in $\text{kg}/\text{m}^2 \text{ s}$
 $V = Ay$ = Tower volume per unit tower cross sectional area (m^3/m^2)

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So, here you see, the previous equation has been written in a different form, K is the mass transfer coefficient here the unit has been given h f is the enthalpy of saturated water vapour mixture at the bulk water temperature and h a is the h a is the what? h a is the enthalpy of air which is unsaturated., and V is the Ay that is, Tower volume per unit tower cross sectional area metre cube per metre square. Actually, in different books it is given in different way, as I have told still I tell the same thing left hand side can be determined easily, but the right hand side we cannot determine and we have to determine depend on the, the supplier of the Heat Exchanger.

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So, basically there is they, for a given WBT and given Approach given kind of Fill design what we can do, the both the left hand side and the right hand side will be a function of L by G which is nothing, but $m \cdot w$ by $m \cdot a$ ok. So, this is nothing, but L by G ., let me write it. So, that you can understand this is nothing, but L by G . So, basically What you can do? Suppose you have got a Cooling Tower; that means, Fill volume etcetera are given you are, design condition is a given w by the Wet Bulb Temperature and.

And, Approach. So, what you get this is your Markles the redline is Markles integral what you get, that is your left hand side of the equation which I was showing and how the Cooling Tower volume or mass transfer coefficient plus volume sorry multiplied by the volume that changes, that is done experimentally and again let us say for a wet bulb temperature and Approach the Cooling Tower manufacturer have done this. So, Cooling Tower, manufacturer will get some sort of a packing function left hand side is called a packing function or Fill element function of the Fill element. So, he will get the black curve. So, where both of these things are, intersecting you will get your Cooling Tower operating point.

That means to, I mean if you operate that L by G ratio. So, we will get the performance of the Cooling Tower ok. So, this is how the Cooling Tower analysis can be done as I have told that, most of the design engineer cannot get much benefit out of this analysis, but it is good to know it is important to know how the Cooling Tower works and how this thing is done and, with this almost we are towards the end of the, thing which we wanted to design for indirect for direct contact, Heat Exchanger certain thing you can um, estimate, by the analysis which we have I which I have given.

One thing let us say that, you want to determine how much water will evaporate. So, what we can do? We can assume, we can we know the ambient air condition and one can assume that, the water vapour will go out at almost at the saturated condition that is the best utilization of the Cooling Tower, out of the Cooling Tower saturated condition of what saturated condition at the temperature of the incoming hot water. So, if you do that then from there, huh.

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So, let us say this is a Cooling Tower, let me get something. So, let us say. So, very thin lines are coming, but let us, let us say this is the Cooling Tower, let me try to make it thicker.

So, this will do, let us say, this is our Cooling Tower ok. So, this is hot water temperature, hot water is coming and this is your cold water temperature cold water is going out. So, this is Ambient air and this is Air out. Now, this is your psychrometric chart. So, Ambient air is somewhere at this condition.

And, let us rub it, Ambient air is somewhere at this condition this has become red no problem. So, what will happen you see, we have shown let us show the air and water path hot water is coming from the hot water is coming in this direction and air Ambient air, is going in this direction. So, initially you see, the air will come in contact with the cold water because hot water as it is falling towards the downward direction it is cold water, and let us say this is your ambient air temperature.

And hot water temperature is more than that. So, hot water temperature is somewhere here. So, the air path will be something like this. This is very interesting. Initially, air may get cooled air may get cooled air will take energy from the, water, but even then it may get cooled why? Because, some amount of moisture is coming latent heat is coming, but there is no sensible heating. So, air may get cooled, but ultimately air will come out

almost close to the hot water temperature and in the saturated condition. So, you see air enthalpy will change this much this is the air enthalpy change.

So, this is actually higher enthalpy. So, let us say if it is $h_{a\ out}$ and this is $h_{a\ in}$. So, $h_{a\ out}$ is greater than $h_{a\ in}$, and by that process, the water temperature will also reduce because the water temperature cold water temperature will be somewhere over here and the hot water temperature is somewhere over here. So, this much of change in temperature, we will get. So, this is a complex process please try to imagine how it happens on a psychrometric plane. This things cannot be explained, to a very great detail in a course of Heat Exchanger, but I, expected to be I attempted to give you some glimpse of how a cooling, tower works. I have not given much, attention for the, very, methodical derivation of the Cooling Tower equation which is there in many textbook you can pick it up.

But, you have to understand one thing that how the energy exchanges is taking place. In this kind of direct contact Heat Exchanger which is Cooling Tower, both sensible and latent heat transfer is taking place and, the latent heat transfer actually plays a very important role sensible heat transfer is not that important and by this latent heat transfer we are having the maximum amount of heat exchange. That is why, in a Cooling Tower there is only little bit of evaporation loss less than 3 percent or around 3 percent of the circulating water, but we can cool the water to a very great extent. So, Cooling Tower is not only a Heat Exchanger it is also a water conservation device.

And in big power plant we only sacrifice 3 percent of the circulating water and get a great amount of cooling. Ultimately Cooling Tower is the device where the heat due to second law or according to the second law it has to be dumped in the atmosphere and that is how it is being dumped and ultimately it is being dumped in the Cooling Tower with the help of the Cooling Tower with this few words I end my lecture. And

Thank you for your kind attention.