

Heat Exchangers: Fundamentals and Design Analysis
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Lecture - 55
Fixed Bed Regenerator (Numerical)

(Refer Slide Time: 00:43)

The slide is titled "Packed Sphere: Correlations". It contains a text box with the following problem statement: "Regenerator matrix consists of lead shot with $D_s = 1.6$ mm diameter. Porosity, $e_v = 0.38$. The lead shots are housed inside a pipe of diameter $D = 154$ mm and $L = 915$ mm. Nitrogen gas at an av. pressure $p = 2$ atm and av. temperature $T = 200$ K flow through the regenerator with $\dot{m} = 0.080$ kg/s. Determine the heat transfer coefficient and the pressure drop in the regenerator." The slide footer includes the IIT Kharagpur logo, NPTEL Online Certification Courses logo, and the name "Indranil Ghosh, Cryogenic Engineering Centre".

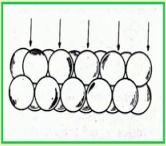
Welcome to this lecture. We are talking about the Fix Bed Regenerator. Today we will try to solve some numerical problem, we have already stated about that problem in the last class. And this is about the heat transfer and pressure drop estimation for packed bed I mean packed bed regenerator with the spheres spherical shots or it was something like this. It has been made of some lead shot and its diameter is specified it is specified I mean the porosity has already been specified, and it was inside a pipe of diameter 154 millimetre and length 915 mm.

So, the gas which is used in this I mean process is nitrogen at a pressure of 2 atmospheric pressure and at an average temperature of 200 Kelvin, and then flow rate of the nitrogen is also specified. So, what you need to find out is the heat transfer coefficient and the pressure drop, which is necessary for any regenerator design. When we talk about the regenerator simulation, not design, we will find that these two parameters will be essential to or it will be necessary to estimate the design or it will be necessary to simulate the regenerator. So, if we now first concentrate on this packed bed with the

sphere and with this particular problem, so we have been told about certain correlations that we have to use for this packed spheres.

(Refer Slide Time: 02:06)

Packed Sphere: Correlations



$$j_H = 0.23Re^{0.3}$$

<p>Re < 1000</p> $f = \left(\frac{172.6}{Re}\right)(1 + 0.0288Re^{0.86})$	$D_e = \left(\frac{4A_{ff}}{A_w/L}\right) = \frac{2e_v D_s}{3(1 - e_v)}$ $G = \left(\frac{\dot{m}}{A_{ff}}\right) = \left(\frac{\dot{m}}{e_v A_{fr}}\right)$
<p>Re > 1000</p> $f = 5.375Re^{-0.14}$	

$$A_w = \left[\frac{6(1 - e_v)V_0}{D_s}\right] \quad \Delta P = \left[\frac{fLG^2}{2\rho D_e}\right]$$

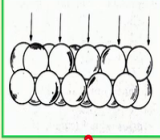
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And it is the heat transfer is expressed in terms of the Coleman j factor. And then we have also the friction factor given for different Reynolds number region for Re less than 1000, and for Re greater than 1000. So, we can understand that we have to first evaluate the Reynolds number, and then only we will be able to find out the heat transfer coefficient and the pressure drop. And associated parameters for you know evaluating the Reynolds number, one of them is the equivalent diameter based on the spherical this spherical factor of a spherical spheres.

So, then you have to find out what is the mass velocity, and that will be based on the free flow area or the frontal area and the porosity. So, then we have the heat transfer surface area that will also be necessary a particularly, and it may be different for packed sphere or if it is made out of the wire screen. So, here this is the finally, this is the expression to be used for the pressure drop.

(Refer Slide Time: 03:33)

Packed Sphere: Correlations



$$D_e = \left(\frac{4A_{ff}}{A_w/L} \right) = \left(\frac{2e_v D_s}{3(1-e_v)} \right) = 6.54 \times 10^{-4} \text{ m}$$

$$G = \left(\frac{\dot{m}}{A_{ff}} \right) = \left(\frac{\dot{m}}{e_v A_{fr}} \right) = \frac{0.80 \text{ kg/s}}{0.38 \times \frac{\pi D^2}{4}}$$

$$A_{fr} = \frac{\pi D^2}{4}$$

$$Re = \frac{GD_e}{\mu} = 11.308 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$

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So, in particular for this problem, if we now look into, we will find that we have the Re value, the D_e or the equivalent diameter, and then G the mass velocity. Now, we would like to calculate first the D_e or the equivalent diameter. So, the parameters are known like the e_v is known, D_s is known. So, if we put these values, we will find that this will come out to be we have the value of e_v we have the value of D_s , and then if you put, this will come out to be 6.54 into 10 to the power minus 4 metre. So, this is the equivalent diameter.

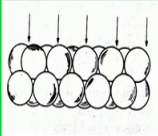
So once we know the equivalent diameter, we will come to the mass velocity part, where we know the mass velocity, this is 0.80 kg per second, then we have this porosity that has been given as 0.38. And the frontal area obviously, you can understand that a frontal will be $4 \pi D^2$ by 4. So, this is what is the and this is of course, this is like a tube, which is filled with this kind of spheres, and the frontal area is obviously this surface area, which is nothing but πD^2 by 4. So, if we now calculate the free flow area, which will be this frontal area multiplied by this porosity. So, this is πD^2 by 4, we have been given this internal diameter. And, if we put this value, you will find G will become eleven point 11.308 kg per metre square into second. So, this is about the G value.

So we have the value of this equivalent diameter, we have the value of G. And what we need to know is the viscosity part of the fluid, so this viscosity we have already been told

about the fluid. The fluid is nitrogen at an average pressure of 200 sorry 200 Kelvin, and its temperature is pressure is already known, so and that temperature and pressure we have to evaluate the velocity I mean properties of nitrogen, and mu will come out to be we will put its value it is 12.95.

(Refer Slide Time: 06:37)

Packed Sphere: Correlations



$$D_e = \left(\frac{4A_{ff}}{A_w/L} \right) = \frac{2e_v D_s}{3(1 - e_v)}$$

$$G = \left(\frac{\dot{m}}{A_{ff}} \right) = \left(\frac{\dot{m}}{e_v A_{fr}} \right)$$

$$Re = \frac{GD_e}{\mu} = \frac{11.308 \times 6.54 \times 10^{-4}}{12.947 \times 10^{-6}} = 571$$

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So, if we now put this G is equals to 11.308 multiplied by equivalent diameter was 6.54 into 10 to the power minus 4, and then you have the mu equals to 12.947 into 10 to the power of minus 6, and then we get the Reynolds number to be 571. So, now we know the Reynolds number. And once we know the Reynolds number, we would be able to decide, whether we will use which correlation we have to use for the friction factor, and we already know that the correlation for the Coleman J-factor.

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Packed Sphere: Correlations

$$j_H = 0.23 Re^{0.3} = 0.23 (571)^{0.3} = 0.034256$$

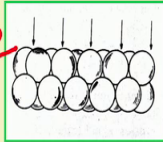
$$j_H = \left(\frac{h}{G c_p} \right) Pr^{\frac{2}{3}} \Rightarrow 0.034256 = \frac{h}{11.308 \times 1.047 \times 10^3} \Rightarrow h = 502.7 \text{ W/m}^2\text{K}$$

$$\Delta P = \frac{f L G^2}{2 \rho D_e} = \frac{0.72567 \times 11.308^2 \times 10^6}{2 \times 1000 \times 0.01} = 45.7 \text{ Pa}$$

$$Pr = 0.72567$$

$$Re < 1000$$

$$f = \left(\frac{172.6}{Re} \right) (1 + 0.0288 Re^{0.86})$$



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So, now if we go back to our discussion now, we will come back to the heat transfer coefficient in the form of Coleman J-factor, and this Coleman J-factor is basically nothing but 0.23 into Re to the power 0.3 0.23 multiplied by Re to the power 0.23. So, here we will also put the other parameters, and then we will see how we will go about it sorry previous ok.

(Refer Slide Time: 08:01)

Regenerator Simulation

Regenerator matrix is made of 1.882 kg of metal spheres ($c_s = 0.840 \text{ kJ/kg.K}$, $k_s = 155 \text{ W/m.K}$). Porosity, $e_v = 0.38$. Heating and cooling periods are equal and the total time period is 4s. Fluid flowing is helium ($c_p = 5.2 \text{ kJ/kg.K}$). Hot helium flow rate is 40 g/s and that of cold helium is 36 g/s. The heat transfer coefficients $h_c = h_h = 8320 \text{ W/m}^2\text{K}$ and heat transfer areas are $A_c = A_h = 0.95 \text{ m}^2$. Determine the regenerator effectiveness.

Barron R.F. Cryogenic Heat Transfer 1st Ed.

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So, here it is if we put in this correlation, we will find that this is 0.23, and the Re is 571 to the power 0.3 to the power 0.3, so this will come out to be 0.034256. So, this would be

the value of the Coleman J-factor. And the Coleman J-factor is nothing but the Stanton number and Prandtl number to the power two-third the Stanton number is h by $G C_p$. So, we already have an idea about the G , we already we obtained, now we also know from the fluid properties of nitrogen the value of C_p , so we can calculate the heat transfer coefficient that is what we are looking for from this relation.

So, this is already it is known 0.034256, and then that is equals to h by G is eleven point G is 11.308, and then you have the C_p value. C_p is already known, so because we know the temperature and pressure of the fluid and C_p will become h by $G C_p$. C_p value is 1.04 into 47 into 10 to the power 3 joule per kg Kelvin, so this is the value of C_p . And once we put this value, and we have also the value of Prandtl number Pr is equals to 0.72567.

So, this is the value of the Prandtl number. So, we have to put that Prandtl number here 0.72567 to the power two-third, and accordingly we will find that the heat transfer coefficient will come out to be 502.7 watt per metre square Kelvin. So, this is what is needed for this heat transfer that is what we are looking for.

(Refer Slide Time: 10:39)

Packed Sphere: Correlations

$$j_H = 0.23Re^{0.3}$$

$$j_H = \left(\frac{h}{G C_p} \right) Pr^{\frac{2}{3}}$$

$Re < 1000$

$$f = \left(\frac{172.6}{Re} \right) (1 + 0.0288 Re^{0.86}) = 2.346$$

Handwritten calculations:

$$\Delta P = \left[\frac{f L G^2}{2 \rho D_e} \right] = 61.2 \text{ kPa}$$

Intermediate values: $\rho = 3.428 \frac{\text{kg}}{\text{m}^3}$

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Then we will go for the friction factor, because we have seen that the Reynolds number is less than 1000. So, we will put it in this relation, and from there we will be able to estimate the friction factor. So, the friction factor is coming to be 2.346, if you put the Re is equals to 571 that is what we have estimated for the this problem. And once we know

the friction factor, we can put all these values over here, the friction factor already we have known is it has already been obtained.

The length of the heat exchanger is sorry the regenerator has already been told, so its length is 0.914 metres, and then G value is known. The equivalent diameter, we have calculated the density. Since, the nitrogen is flowing at a temperature of at an average temperature of 200 Kelvin and 2 atmospheric pressure, so the density of the gas is also known, and this density is equals to 3.428 and kg per metre cube.

So, if we put all these values in this relation, we find that this will come out to be 61173 KPa sorry P_a , and that will be equivalent to 61.1 or 2 K P a. So, we can estimate the pressure drop for this packed bed for a flow rate of the given flow rate of 0.8 point sorry the flow rate was given 0.08 kg per second, we find that the estimated pressure drop is 61.2 K P a. So, like that we have to depending on the regenerator matrix material, we have to calculate the pressure drop and I mean the heat transfer coefficient as it is necessary for the design and the simulation.

So, now we will go back to our discussion on the simulation of the regenerator. Here comes problem, we have taken it from the Barron's cryogenic heat transfer book. And it is a bulk type regenerator fixed bed regenerator. It is made of say 1.182 kg metal spheres, and its material properties unknown. So, it is both the specific heat and the thermal conductivity of this material has already been given. The porosity of this is also known, and it is made of packed bed spheres.

And heating cooling periods are equal and the total time period is 4 second. So, the hot period is 2 second, and the cold period is also of 2 second. The fluid flowing through this system is helium. So, its C_p is also given. And the hot helium flow rate is flow rate is 40 gram per second, whereas the cold helium flow is slightly less than that and it is 36 gram per second. The heat transfer coefficients we need not calculate, it has already been specified. You can understand that depending on the situation or like here already the flow rates have already been given.

And we need to calculate you know as we have specified just, we have to use similar expressions to calculate the heat transfer coefficient on this. And the heat transfer coefficient on the cold side and the for the hot stream and the cold stream. And its a heat transfer surface area is also known. So, as you can understand that it is the quite a similar

to the heat exchanger design process where or design of the simulation process. Particularly this is we are talking about the simulation, where we find that the simulation is about I mean a situation, where we try to find out the performance of something, where all the parameters related to its configuration or the geometry is already known.

Here as we can understand that the materials property are known, I mean the heat transfer surface area is known, its other properties are known, so that we can calculate the heat transfer coefficient. So, this is also known. And what we find is that is what we need to know is the effectiveness, so that heat exchanger effect regenerator effectiveness we need to calculate, and we have been told about the other parameters. So, now as you can understand that some of the parameters are given and what we to have to find out is the regenerator effectiveness, and what would be the process to do that. So, we will discuss about it in the next slide.

(Refer Slide Time: 16:16)

Regenerator Simulation

$$NTU = \frac{1}{C_{min}} \left[\frac{1}{h_h A_h} + \frac{1}{h_c A_c} \right]^{-1} \quad C_m = \frac{m_s C_s}{C_{min} P_o}$$

$$NTU_e = \frac{2 C_R NTU}{1 + C_R} \quad C_{m,e} = \frac{2 C_m C_R}{1 + C_R}$$

$$X = \frac{(1 - C_R^2) \left(\frac{\epsilon_1}{1 - \epsilon_1} \right)}{2 C_R} \quad \epsilon = \frac{1 - e^{-X}}{1 - C_R e^{-X}}$$

Barron R.F. & Nellis G. F. 2015 Cryogenic Heat Transfer 2nd Ed.

NTU	Matrix Capacity Rate Ratio, C_c							
	0.8	1.0	1.25	1.50	2.0	3.0	5.0	∞
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.330	0.332	0.333	0.333
1.0	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5000
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6000
2.0	0.566	0.611	0.623	0.626	0.649	0.659	0.664	0.6667
2.5	0.599	0.639	0.664	0.679	0.694	0.705	0.711	0.7143
3.0	0.622	0.667	0.696	0.712	0.728	0.740	0.746	0.7500
3.5	0.642	0.690	0.721	0.738	0.755	0.767	0.774	0.7778
4.0	0.659	0.709	0.741	0.759	0.776	0.789	0.796	0.8000
4.5	0.673	0.724	0.758	0.776	0.794	0.807	0.814	0.8182
5.0	0.685	0.738	0.772	0.791	0.809	0.822	0.829	0.8333
5.5	0.696	0.749	0.785	0.803	0.821	0.834	0.842	0.8462
6.0	0.705	0.759	0.796	0.814	0.832	0.845	0.853	0.8571
6.5	0.713	0.768	0.805	0.824	0.842	0.855	0.862	0.8667
7.0	0.721	0.776	0.814	0.833	0.850	0.863	0.870	0.8750
20	0.796	0.865	0.906	0.922	0.935	0.943	0.948	0.9524
30	0.816	0.889	0.928	0.943	0.947	0.961	0.964	0.9677
40	0.826	0.903	0.942	0.955	0.963	0.971	0.974	0.9756
50	0.843	0.914	0.951	0.962	0.970	0.975	0.978	0.9804
60	0.844	0.920	0.957	0.968	0.974	0.980	0.982	0.9836
80	0.857	0.931	0.966	0.975	0.980	0.984	0.986	0.9877
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.9890
100	0.867	0.939	0.971	0.979	0.984	0.987	0.989	0.9901
500	0.891	0.974	0.993	0.995	0.996	0.997	0.998	0.9980

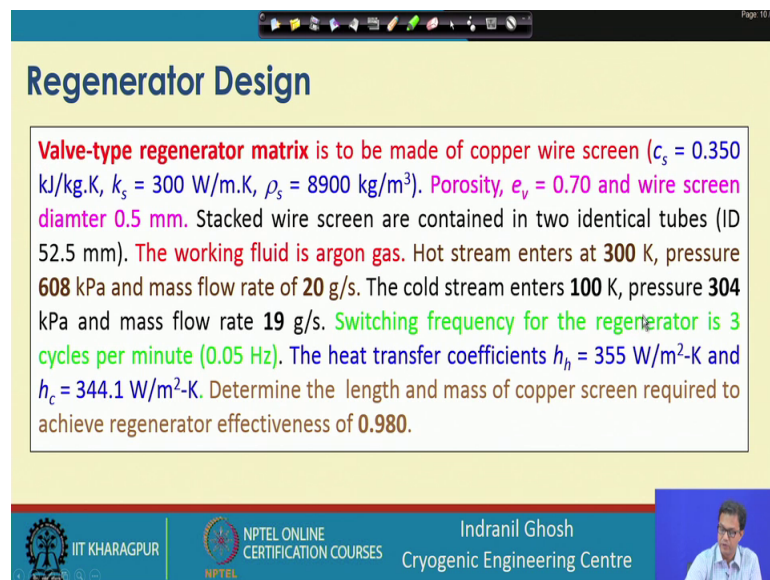
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So, first of all we need to find out what is the minimum capacity fluid flow. And for that we of course need to find out the C_p value of the gas flow. And already it has already been told the C_p of helium has already been given. So, we can understand that we will be able to find out the C_{min} . So, the heat transfer coefficient and the heat transfer surface area is also given. So, the NTU can also be calculated. So, once we know the NTU, we will be able to calculate the ratio C_m that is the capacity rate ratio.

So, here once we know the C_m and NTU, and also we need to find out the depending on the value of the C_R , if it is not one we have to find out an equivalent NTU and equivalent C_m , or the capacity rate ratio. So, depending on that we will be using this chart for the given as a function of a NTU and C_m , and that you know on the basis of NTU effective values and C_m , effective.

So, we will try to find out the epsilon. And once we find out that epsilon we know that we have to calculate a factor X , and depending on that X we will be able to find out the correlation. We will be using this correlation to calculate this regenerator effectiveness. So, this is the procedure that is to be followed. So, we will go one by one in this case.

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Regenerator Design

Valve-type regenerator matrix is to be made of copper wire screen ($c_s = 0.350$ kJ/kg.K, $k_s = 300$ W/m.K, $\rho_s = 8900$ kg/m³). Porosity, $e_v = 0.70$ and wire screen diameter 0.5 mm. Stacked wire screen are contained in two identical tubes (ID 52.5 mm). The working fluid is argon gas. Hot stream enters at 300 K, pressure 608 kPa and mass flow rate of 20 g/s. The cold stream enters 100 K, pressure 304 kPa and mass flow rate 19 g/s. Switching frequency for the regenerator is 3 cycles per minute (0.05 Hz). The heat transfer coefficients $h_h = 355$ W/m²-K and $h_c = 344.1$ W/m²-K. Determine the length and mass of copper screen required to achieve regenerator effectiveness of 0.980.

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So, here what we do is first of all sorry we will go back to our earlier slide. So, here we would go to this calculation. First of all we know that what are the property values, that is given. So, here we find that this regenerator is made of C P is already given.

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Handwritten calculations on a whiteboard:

$$C_h = \dot{m}_h C_p = 0.04 \frac{\text{kg}}{\text{s}} \times 5.2 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.2080 \frac{\text{kW}}{\text{K}} \rightarrow C_{\text{max}}$$

$$C_c = \dot{m}_c C_p = 0.036 \frac{\text{kg}}{\text{s}} \times 5.2 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.1872 \frac{\text{kW}}{\text{K}}$$

Additional notes:

- $C_{\text{min}} = 0.9$
- $CR = \frac{C_{\text{min}}}{C_{\text{max}}}$
- $CR \neq 1$
- Labels: C_{min} , C_{max} , C_{me} , $N_{\text{tu,e}}$

So, first of all we will find out the C_h , so that is equals to \dot{m}_h and C_p , and that comes out to be 0.04 is the mass flow. And then we have the corresponding 5.2 the specific heat that is this many kilo joule per kg Kelvin, and we have this is kg per this is kg per second. So, if we multiply this two parameters, this comes out to be 0.2080 then kilo watt per this is kilo watt per Kelvin. And we can also calculate the C_c . So, this will become \dot{m}_c and then C_c or then what is C_c , \dot{m}_c dot is slightly less, and it is 0.36, so 0.036 multiplied by 5.2 kilo joule per kg Kelvin. And then you have for this one this many kg per second. So, this comes out to be 0.1872 kilo watt per Kelvin.

So, you can understand that this is the C_{min} and the cold fluid stream is the C_{min} fluid. And this is the C_{max} fluid. So, we can now try to find out the CR that is equals to C_{min} by C_{max} , and that will come out to be 0.90, so this is not equals to 1. So, we know that that capacity rate ratio C_{min} equivalent we have to find out we have to also find out the N_{tu} equivalent. So, we will find out the appropriate relation to calculate the C_{me} and N_{tu} effective. And then we will proceed to the calculation of the values of the effectiveness.

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Handwritten derivation on a whiteboard:

$$Ntu = \frac{1}{\frac{1}{UA C_{min}} + \frac{1}{h_c A_c + h_h A_h}}$$

Given values:

$$h_c = h_h = 8320$$

$$A_c = A_h = 0.950$$

Substituting into the Ntu formula:

$$Ntu = \frac{1}{187.2} \left[\frac{2}{8320 \times 0.95} \right]$$

$$= 21.11$$

Final result circled in red:

$$Ntu,e = 20.0$$

So, now if we try to calculate the Ntu, we know that Ntu is already it is Ntu value is given by 1 by heat transfer surface area 1 by $h A_c$ plus 1 by $h A_h$ this basically this is nothing but $U A$ by C_{min} , so this is the value of C_{min} and this is nothing but this is $U A$ is 1 by $h_c A_c + h_h A_h$ to the power this 1 . So, this is in this case we have seen that the both the $h_c A_c$ and A_h are the same. Here the h_c is equals to h_h is equals to 8320 , and A_c is equals to A_h is equals to 0.950 . So, basically these two terms are getting added and the Ntu becomes say 1 by what is the; that $1.187.2$ this kilo watt per Kelvin we have converted into watt per Kelvin. And then we have this 2 by this 8320 multiplied by 0.95 .

So, this whole to the power minus 1 will give you the value of Ntu equals to 21.11 . So, once we know the Ntu value we would be able to calculate the Ntu effective value and that Ntu effective value will become we will use that expression that we have already talked about. So, the Ntu effective value comes out to be 20 ok.

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$$C_m = \frac{1.882 \times 0.840 \times 4}{0.1872}$$
$$= 2.111$$

$C_m(e) = 2.0$
 $N_{tu}(e) = 20.0$
 $(\epsilon) = 0.935$

$CR = 1$
 $\epsilon =$

And then we have also, now we need to as we have said the matrix capacity rate ratio, that C_m that is equals to it is 1.882 multiplied by 0.840, and that is the total time 4 and then you have to divide it by 0.1872. So, then we have this will come out to be 2.11.

Now, we have the equivalent C_m then you will find that it is coming nearly to be 2.0 and then if we now have the value of this effective N_{tu} , and the effective value of C_m that is equals to 20 and 2. And the corresponding value for the epsilon that would become from there we will get an epsilon is equals to 0.935. So, we now have an estimate of the rough estimate of the effectiveness, because we have used that chart, which corresponds to CR equals to 1, but since the CR is not equals to 1 for us we have to find out this effective value for corresponding to this effective C_m and this N_{tu} effective value, and that there we are getting the epsilon 1.

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$$X = 1.5184$$

$$\epsilon = \frac{1 - e^{-X}}{1 - CR e^{-X}}$$

$$= 0.9727$$

So, as we have said earlier that based on this value, we will be calculating the parameter x , and that parameter X comes out to be 1.5184. And you already you have told it about the expression of X . And from there if we now put the value of that 1 and the corresponding epsilon would be 1 minus e to the power minus X divided by 1 minus $C R$ into e to the power minus x . So, this is nothing but if we put the value of X in this expression, you will find that this is 9727. So, the regenerator is 97.27 percent effective or in its ineffectiveness is basically 0.0273 or 2 percent 2.73 percent. So, this is how we estimate the effectiveness of the regenerator or simulated the performance of any regenerator in a nutshell.

Thank you for your attention.