Heat Exchangers: Fundamentals and Design Analysis

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Lecture - 54
Design and Simulation of Regenerator (Fixed Bed)

You are welcome to this lecture. Today, we are going to talk about the Design and Simulation of Regenerator. You may remember that we have talked about different type of regenerator, and it is a fixed bed type, it is a rotary type. And out of that we have specifically looked into the fixed bed type regenerator, you may remember that we have solved some governing equation for those regenerator fixed bed type, counter current regenerator.

And to add to that just we there are regenerator of counter current and parallel current heat regenerator similar to the heat exchanger, but it is not possible to have a cross flow type a regenerator. So, generally we have counter current and parallel current regenerator. And here we are going to talk about the fixed bed regenerator.

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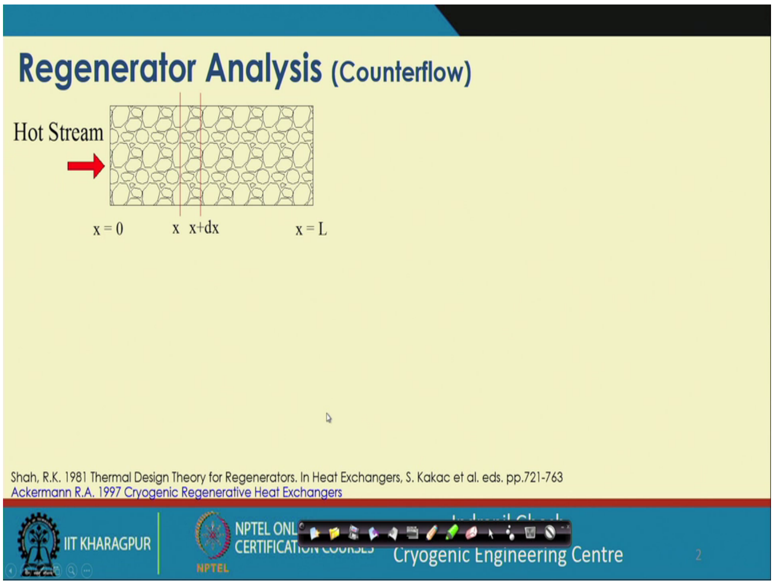

Regenerator Analysis (Counterflow)

Shah, R.K. 1981 Thermal Design Theory for Regenerators. In Heat Exchangers, S. Kakac et al. eds. pp.721-763
Ackermann R.A. 1997 Cryogenic Regenerative Heat Exchangers

So, now in case of fixed bed regenerator, you may remember that we have a solved, this kind of solved. This kind of regenerator problem, where we have seen that in one cycle, the hot stream is flowing.

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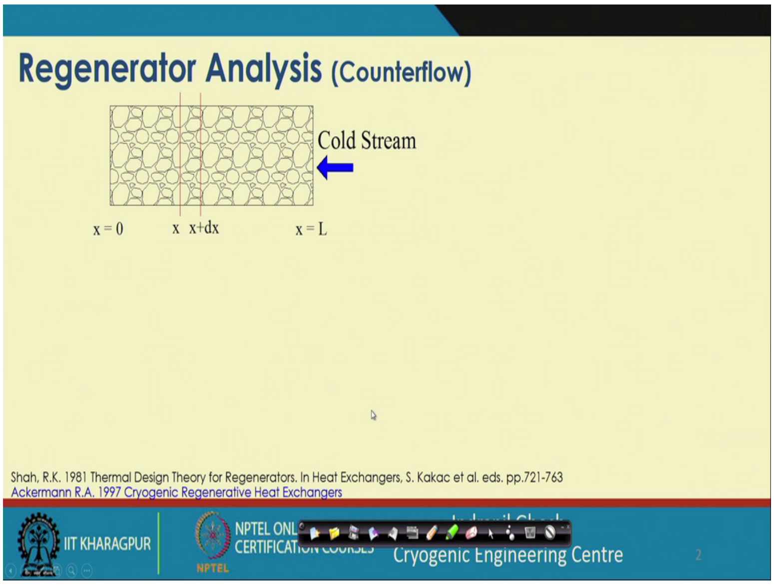

Regenerator Analysis (Counterflow)

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In other cycle, the cold stream is flowing from the opposite direction, so that is how it is a forming a counter flow regenerator.

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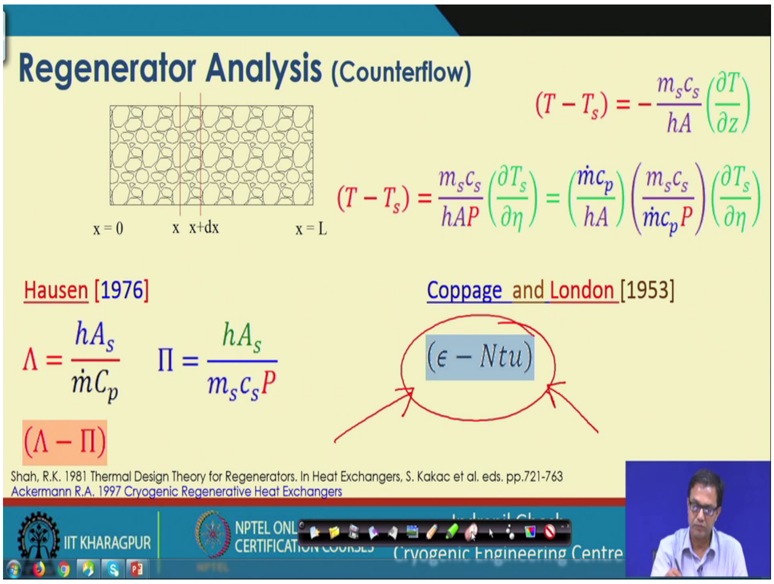

Regenerator Analysis (Counterflow)

$$(T - T_s) = -\frac{m_s c_s}{hA}\left(\frac{\partial T}{\partial z}\right)$$

$$(T - T_s) = \frac{m_s c_s}{hAP}\left(\frac{\partial T_s}{\partial \eta}\right) = \left(\frac{\dot{m}c_p}{hA}\right)\left(\frac{m_s c_s}{\dot{m}c_p P}\right)\left(\frac{\partial T_s}{\partial \eta}\right)$$

Hausen [1976]

$$\Lambda = \frac{hA_s}{\dot{m}C_p} \qquad \Pi = \frac{hA_s}{m_s c_s P}$$

$(\Lambda - \Pi)$

Coppage and London [1953]

$(\epsilon - Ntu)$

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And while analysing it, we have seen that these are the governing equations. And in that governing equations, we have also talked about the boundary conditions, but we have said that we will not solve it. It is beyond the scope of this class. And if anyone is interested specifically he or she can, I mean look into the R.K Shah's book or this particular reference or he can also or he can also look into the Ackermann's book for the solution of these equations, and then we will try to later on utilise the solutions of these equations.

And while solving this equation, we will find that there are different kind of dimensionless parameters. And here we will find that there are particularly two type of a dimensionless parameter. One is the reduced length, another is the reduced time. Here this is called the reduced length parameter, and this is the reduced time parameter. And this you can I mean correlated with the Ntu, but basically this is called the lambda and pi, and this has been done by Hausen. And this analysis particularly is known as what is called the lambda pi method.

And the other method is proposed by London and Coppage, Coppage and London. And this is pretty similar to the heat exchanger analysis that we have we are familiar with and this is called the epsilon Ntu the approach of solving the counter current regenerator, which is fixed bed type. So, we have chosen in this analysis in this slide or class to look into this particular epsilon Ntu method, where we are familiar with the terms the epsilon the heat regenerators effectiveness and the Ntu is regenerator Ntu.

So, we will look into the details of this definition of the epsilon and Ntu. And since we are bit familiar with these terminologies, we will go into the analysis by epsilon Ntu. And we will try to later on simulate and design the fixed bed counter flow type regenerator by taking this approach called the epsilon Ntu.

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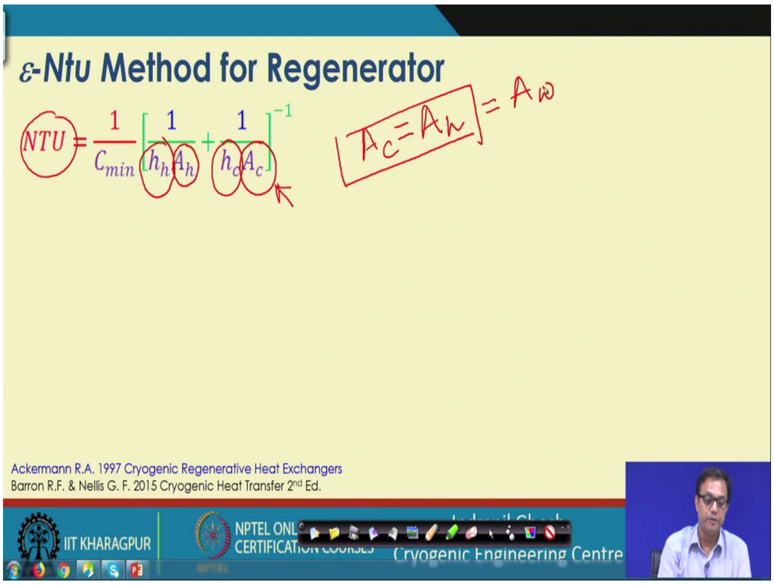

ε-Ntu Method for Regenerator

$$NTU = \frac{1}{C_{min}}\left[\frac{1}{h_h A_h} + \frac{1}{h_c A_c}\right]^{-1}$$

$$\boxed{A_c = A_h} = A_w$$

Ackermann R.A. 1997 Cryogenic Regenerative Heat Exchangers

Barron R.F. & Nellis G. F. 2015 Cryogenic Heat Transfer 2nd Ed.

So, now if you remember the definition of Ntu, where we have NTU is defined by u a by C min, where the u a is the overall conductance and C min is the minimum capacity fluid. In case of a regenerator we have seen that in one cycle, the hot fluid flowing through the regenerator bed. In the other cycle, the cold fluid is flowing through the bed. And you know the one of the fluid, whether it is the cold or the hot fluid becomes a minimum capacity fluid. And it may be also possible that both of them are the C min or C max are equal in that case we will have C R equals to 1.

So, otherwise if I look into this definition of the NTU, it comprises of the hot side heat transfer area, the hot side heat transfer coefficient, and this is the cold side heat transfer coefficient and the cold side heat transferred its surface area. And often you will find for the fixed bed regenerator that this A c and A h are mostly equal in case of a fixed bed type regenerator, but which is not the case in case of a rotary type heat exchangerregenerator, but that rotary type regenerator is not the subject that we are going to deal in this class. So, will be mostly assuming that A c equals to A h and that is equals to A w, and so that is how the NTU of this regenerator is defined.

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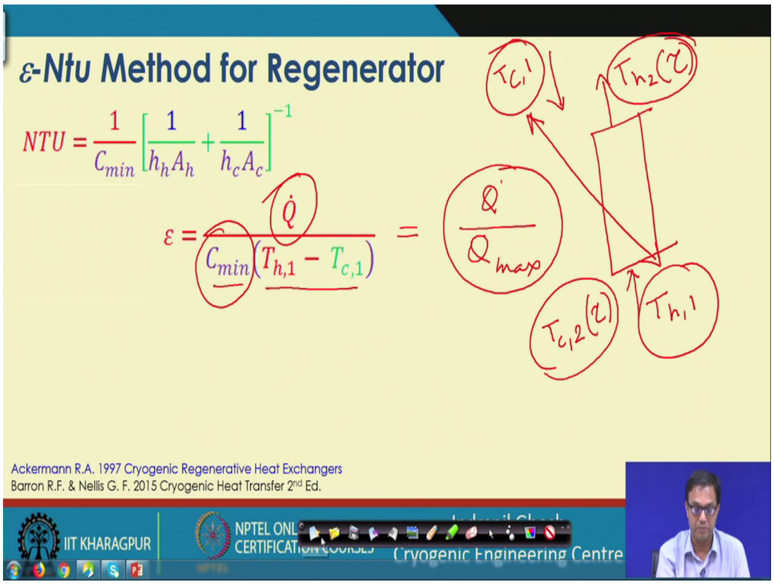

ε-Ntu Method for Regenerator

$$NTU = \frac{1}{C_{min}}\left[\frac{1}{h_h A_h} + \frac{1}{h_c A_c}\right]^{-1}$$

$$\varepsilon = \frac{\dot{Q}}{C_{min}(T_{h,1} - T_{c,1})} = \frac{\dot{Q}}{\dot{Q}_{max}}$$

Ackermann R.A. 1997 Cryogenic Regenerative Heat Exchangers
Barron R.F. & Nellis G. F. 2015 Cryogenic Heat Transfer 2nd Ed.

And we also know that the other definition. It is quite similar to the definition of the heat exchanger effectiveness, where we find that it is epsilon is equals to Q dot by Q dot max. So, the maximum heat transfer that is possible. And here in this case this will be C minimum multiplied by the maximum difference in temperature. So, this can be I mean correlated like this that in the regenerator, when the hot fluid stream entering, this is entering at temperature of T h, 1. And as time passes this T h, 2, this is dependent on the time.

And in the later part of the cycle, you will find that the cold fluid is coming at a temperature of T c, I and it is coming out at T c, 2 and it is dependent on time as time passes this is changing, as time passes this hot fluid will be changing. So, what is fixed is T h, 1 and this is fixed at T c, 1. So, T c, 1 and T c, 2 are the maximum difference in temperature multiplied by the minimum capacity fluid.

And this is Q dot is the amount of exact or actual heat that is getting transferred in the regenerator. So that is how you know we define the regenerator effectiveness and it is pretty similar to the definition of the heat exchanger effectiveness.

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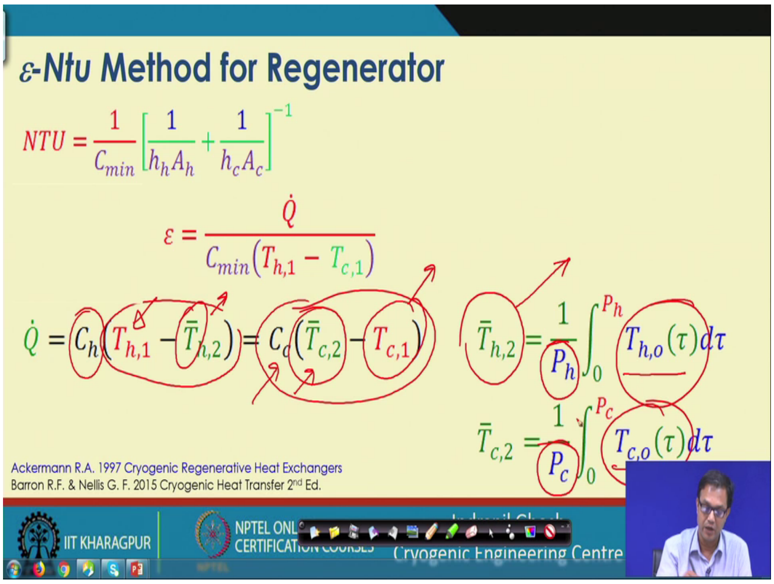

ε-Ntu Method for Regenerator

$$NTU = \frac{1}{C_{min}}\left[\frac{1}{h_h A_h}+\frac{1}{h_c A_c}\right]^{-1}$$

$$\varepsilon = \frac{\dot{Q}}{C_{min}\left(T_{h,1}-T_{c,1}\right)}$$

$$\dot{Q} = C_h\left(T_{h,1}-\bar{T}_{h,2}\right) = C_c\left(\bar{T}_{c,2}-T_{c,1}\right)$$

$$\bar{T}_{h,2} = \frac{1}{P_h}\int_0^{P_h} T_{h,o}(\tau)\,d\tau$$

$$\bar{T}_{c,2} = \frac{1}{P_c}\int_0^{P_c} T_{c,o}(\tau)\,d\tau$$

Ackermann R.A. 1997 Cryogenic Regenerative Heat Exchangers
Barron R.F. & Nellis G. F. 2015 Cryogenic Heat Transfer 2nd Ed.

Here what we find is the Q dot and Q dot is defined as the heat capacity of the hot fluid multiplied by this cold sorry hot fluid inlet and hot fluid outlet, but this is the average outlet temperature of the hot fluid. Similarly, in case of a cold fluid, if it is minimum capacity fluid will take here; you know the C c as the minimum capacity fluid. And in that case we have to define; this is the average exit temperature of the cold fluid, whereas T c, 1 is the entry temperature of the cold fluid.

So, this is how we define the hot and cold average temperature, here is the formula for the hot and cold fluid exit temperatures. Here you can see that T h, 2 is average to over the hot cycle duration and T c, 2 is average to over the cold cycle duration. And this is how the T h, o and T c, o are changing. But we are not going into the details of this temperature profile as we have said, but we will be using the final outcome of these expressions. And we will see, how we best we can utilise this one.

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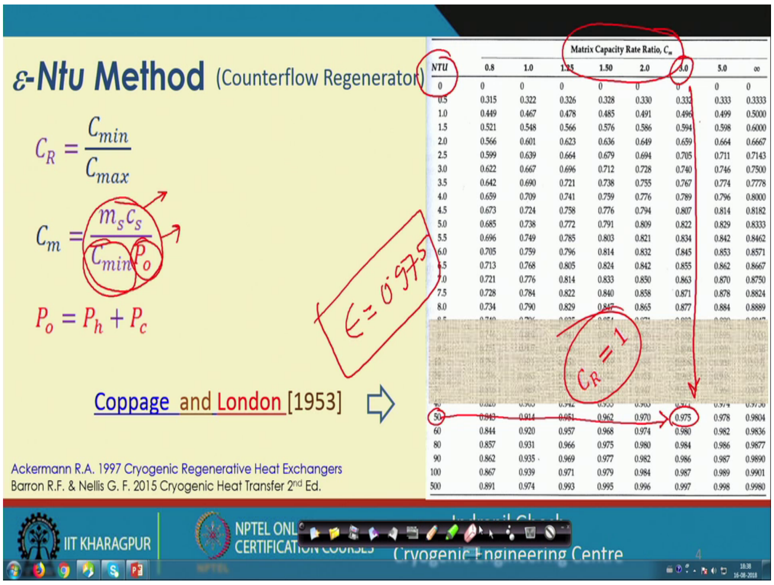

ε-Ntu Method (Counterflow Regenerator)

$$C_R = \frac{C_{min}}{C_{max}}$$

$$C_m = \frac{m_s c_s}{C_{min} P_o}$$

$$P_o = P_h + P_c$$

$\varepsilon = 0.975$

Coppage and London [1953]

NTU	Matrix Capacity Rate Ratio, C_m 0.8	1.0	1.25	1.50	2.0	3.0	5.0	∞
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.330	0.332	0.333	0.3333
1.0	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5000
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6000
2.0	0.566	0.601	0.623	0.636	0.649	0.659	0.664	0.6667
2.5	0.599	0.639	0.664	0.679	0.694	0.705	0.711	0.7143
3.0	0.622	0.667	0.696	0.712	0.728	0.740	0.746	0.7500
3.5	0.642	0.690	0.721	0.738	0.755	0.767	0.774	0.7778
4.0	0.659	0.709	0.741	0.759	0.776	0.789	0.796	0.8000
4.5	0.673	0.724	0.758	0.776	0.794	0.807	0.814	0.8182
5.0	0.685	0.738	0.772	0.791	0.809	0.822	0.829	0.8333
5.5	0.696	0.749	0.785	0.803	0.821	0.834	0.842	0.8462
6.0	0.705	0.759	0.796	0.814	0.832	0.845	0.853	0.8571
6.5	0.713	0.768	0.805	0.824	0.842	0.855	0.862	0.8667
7.0	0.721	0.776	0.814	0.833	0.850	0.863	0.870	0.8750
7.5	0.728	0.784	0.822	0.840	0.858	0.871	0.878	0.8824
8.0	0.734	0.790	0.829	0.847	0.865	0.877	0.884	0.8889
50	0.843	0.914	0.951	0.962	0.970	0.975	0.978	0.9804
60	0.844	0.920	0.957	0.968	0.974	0.980	0.982	0.9836
80	0.857	0.931	0.966	0.975	0.980	0.984	0.986	0.9877
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.9890
100	0.867	0.939	0.971	0.979	0.984	0.987	0.989	0.9901
500	0.891	0.974	0.993	0.995	0.996	0.997	0.998	0.9980

$C_R = 1$

Ackermann R.A. 1997 Cryogenic Regenerative Heat Exchangers
Barron R.F. & Nellis G. F. 2015 Cryogenic Heat Transfer 2nd Ed.

So, now we if we look into this epsilon Ntu method, we have other parameters like C R as we have said that it is ratio between the C min and C max. And in addition to that we have another parameter in this case, which is called the matrix heat capacity ratio. So, this is the ratio, you can understand that m s C s basically that m s C s is a product of the mass of the regenerator matrix multiplied by the specific heat of the regenerator matrix. And you have the C min, the minimum capacity fluid multiplied by P 0. P 0 is the total time period, it comprising of both the hot and cold fluid time period. So, this is the P h plus P c that is equals to P 0.

Now, with this number of I mean different type of I mean dimensionless and dimensional parameters, if we look into the solution, it has been proposed by the Coppage and London. And they have obtained you know solution in terms of the number of heat transfer units, and the matrix capacity rate ratio that is C m. Here we find that this has been given as a in terms of NTU and matrix capacity rate ratio. So, these are the different epsilon values corresponding to different NTU and different matrix capacity rate ratio.

So, if we know the C m, and if we know the NTU, we would be able to find out the epsilon or the effectiveness of the regenerator. Now, say imagine that if the matrix capacity rate ratio is 3 and the NTU is somewhere at 50, then we will have something like 0.975 as the regenerator effectiveness. So, this corresponds to this one, and this corresponds to you know this value.

So, this is what is the effectiveness or the regeneral effectiveness is 0.975 corresponding to a value of NTU equals to 50, and C m equals to 3. Now, this particular solution has been obtained for C R equals to 1, when the regenerator is in balanced condition or C min by C max is equals to 1. So, we will try to understand, how best we can utilise this chart for designing and simulating the regenerators. So, in the next slide, we are going to see, the we are going to talk about the regenerator simulation.

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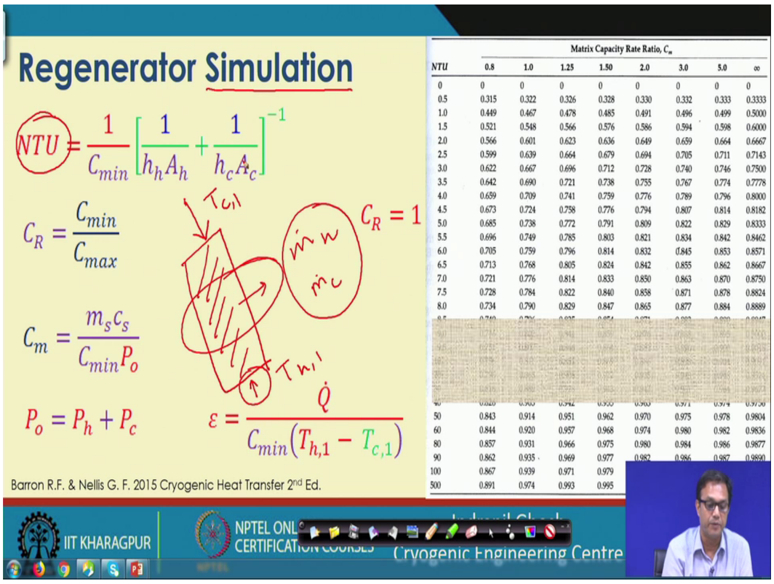

Regenerator Simulation

$$\boxed{NTU} = \frac{1}{C_{min}}\left[\frac{1}{h_h A_h} + \frac{1}{h_c A_c}\right]^{-1}$$

$$C_R = \frac{C_{min}}{C_{max}}$$

$$C_m = \frac{m_s c_s}{C_{min} P_o}$$

$$P_o = P_h + P_c$$

$$\varepsilon = \frac{\dot{Q}}{C_{min}(T_{h,1} - T_{c,1})}$$

$C_R = 1$

	Matrix Capacity Rate Ratio, C_m							
NTU	0.8	1.0	1.25	1.50	2.0	3.0	5.0	∞
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.330	0.332	0.333	0.3333
1.0	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5000
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6000
2.0	0.566	0.601	0.623	0.636	0.649	0.659	0.664	0.6667
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4.5	0.673	0.724	0.758	0.776	0.794	0.807	0.814	0.8182
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7.5	0.728	0.784	0.822	0.840	0.858	0.871	0.878	0.8824
8.0	0.734	0.790	0.829	0.847	0.865	0.877	0.884	0.8889
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
50	0.843	0.914	0.951	0.962	0.970	0.975	0.978	0.9804
60	0.844	0.920	0.957	0.968	0.974	0.980	0.982	0.9836
80	0.857	0.931	0.966	0.975	0.980	0.984	0.986	0.9877
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.9890
100	0.867	0.939	0.971	0.979	[illegible]	[illegible]	[illegible]	[illegible]
500	0.891	0.974	0.993	0.995	[illegible]	[illegible]	[illegible]	[illegible]

Barron R.F. & Nellis G. F. 2015 Cryogenic Heat Transfer 2nd Ed.

Here we will find that we have what are the parameters known, NTU is known. You may remember that when we talk about the simulation that means, the regenerator matrix, I mean all the internal things are known, its geometry is known and its inlet pressure is known, inlet temperature of the hot fluid is known, inlet temperature of the cold fluid is also known. So, T c, 1, T h, 1 these are known, geometry and all other details like you know heat transfer surface area, then you know the heat transfer coefficient can also be obtained, because we know the hot fluid, fluid flow rate and cold fluid flow rate.

So, from there we would be able to calculate the heat transfer coefficient. And since the geometry is known, we also know the heat transfer surface area. So, accordingly we would be able to calculate the NTU.

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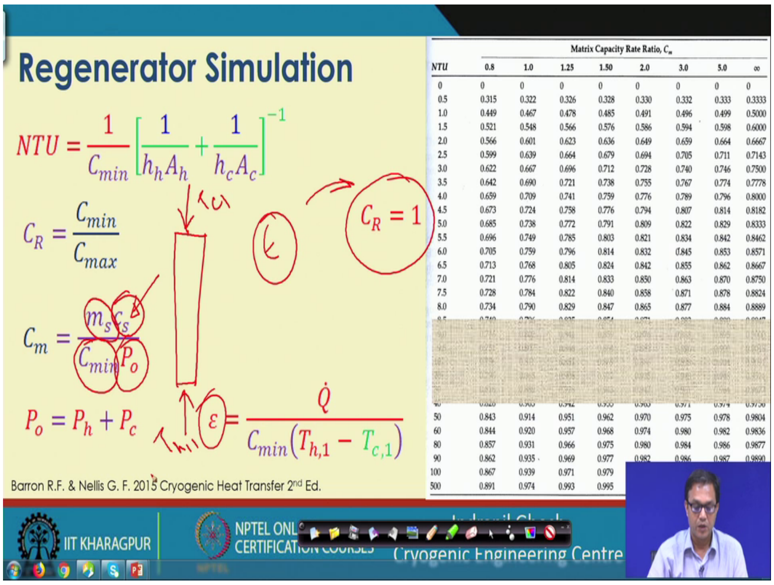

Regenerator Simulation

$$NTU = \frac{1}{C_{min}}\left[\frac{1}{h_h A_h} + \frac{1}{h_c A_c}\right]^{-1}$$

$$C_R = \frac{C_{min}}{C_{max}}$$

$$C_m = \frac{m_s c_s}{C_{min} P_o}$$

$$P_o = P_h + P_c$$

$$\varepsilon = \frac{\dot{Q}}{C_{min}(T_{h,1} - T_{c,1})}$$

$$C_R = 1$$

Barron R.F. & Nellis G. F. 2016 Cryogenic Heat Transfer 2nd Ed.

NTU	0.8	1.0	1.25	1.50	2.0	3.0	5.0	∞
	Matrix Capacity Rate Ratio, C_m							
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.330	0.332	0.333	0.3333
1.0	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5000
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6000
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3.5	0.642	0.690	0.721	0.738	0.755	0.767	0.774	0.7778
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80	0.857	0.931	0.966	0.975	0.980	0.984	0.986	0.9877
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.9890
100	0.867	0.939	0.971	0.979	[illegible]	[illegible]	[illegible]	[illegible]
500	0.891	0.974	0.993	0.995	[illegible]	[illegible]	[illegible]	[illegible]

Please mind that we are looking for a similar, I mean we are trying to find out the epsilon or the simulation problem for a balanced condition, when C R equals to 1. Now, here the geometry being known we know the mass of the matrix that is present and C s is the specific heat, because we know what is the temperature of or tentatively what is the exit temperature of the hot fluid T h, 1 and T c, 1 being known.

We know the to estimate is, so we would be able to find out, what is the operating temperature of the C s average. And C min is already known, P 0 is the period total period the duration of the hot and cold fluid flow rate is known, and the time is also known. Here what we are now trying to find out is the effectiveness of that particular regenerator.

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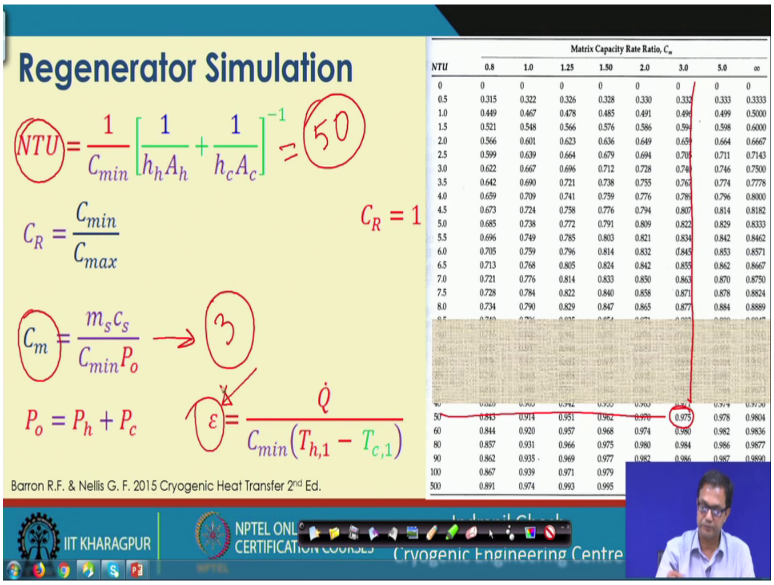

Regenerator Simulation

$$NTU = \frac{1}{C_{min}}\left[\frac{1}{h_hA_h}+\frac{1}{h_cA_c}\right]^{-1} \simeq 50$$

$$C_R = \frac{C_{min}}{C_{max}}$$

$$C_R = 1$$

$$C_m = \frac{m_s c_s}{C_{min}P_o} \rightarrow 3$$

$$P_o = P_h + P_c$$

$$\varepsilon = \frac{\dot{Q}}{C_{min}\left(T_{h,1} - T_{c,1}\right)}$$

	Matrix Capacity Rate Ratio, C_m							
NTU	0.8	1.0	1.25	1.50	2.0	3.0	5.0	∞
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.330	0.332	0.333	0.3333
1.0	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5000
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6000
2.0	0.566	0.601	0.623	0.636	0.649	0.659	0.664	0.6667
2.5	0.599	0.639	0.664	0.679	0.694	0.705	0.711	0.7143
3.0	0.622	0.667	0.696	0.712	0.728	0.740	0.746	0.7500
3.5	0.642	0.690	0.721	0.738	0.755	0.767	0.774	0.7778
4.0	0.659	0.709	0.741	0.759	0.776	0.789	0.796	0.8000
4.5	0.673	0.724	0.758	0.776	0.794	0.807	0.814	0.8182
5.0	0.685	0.738	0.772	0.791	0.809	0.822	0.829	0.8333
5.5	0.696	0.749	0.785	0.803	0.821	0.834	0.842	0.8462
6.0	0.705	0.759	0.796	0.814	0.832	0.845	0.853	0.8571
6.5	0.713	0.768	0.805	0.824	0.842	0.855	0.862	0.8667
7.0	0.721	0.776	0.814	0.833	0.850	0.863	0.870	0.8750
7.5	0.728	0.784	0.822	0.840	0.858	0.871	0.878	0.8824
8.0	0.734	0.790	0.829	0.847	0.865	0.877	0.884	0.8889
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
50	0.843	0.914	0.951	0.962	0.970	0.975	0.978	0.9804
60	0.844	0.920	0.957	0.968	0.974	0.980	0.982	0.9836
80	0.857	0.931	0.966	0.975	0.980	0.984	0.986	0.9877
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.9890
100	0.867	0.939	0.971	0.979	[illegible]	[illegible]	[illegible]	[illegible]
500	0.891	0.974	0.993	0.995	[illegible]	[illegible]	[illegible]	[illegible]

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So, how do you do that? Here once we know the NTU, and once we know the C m, we have to look you know in the chart. What is the corresponding value, corresponding to this NTU, so in imagine as we have said earlier that this comes out to be 50, and this comes out to be 3, and or it may be some intermediate value also in that case, we have to interpolate from this chart. So, if that is so, we will find that directly, we get the value as 0.975 corresponding to this one, and this one ok.

So, this is how we calculate the effectiveness or simulate the performance of the regenerator, when we know its geometry. And we know some of the parameters of fluid flow parameters. And this is pretty similar to our heat exchanger design where we know, you know the inlet conditions and the heat exchanger geometry. And we try to find out what is the exit temperature. So, this is how, we know the performance or the overall performance of the regenerator.

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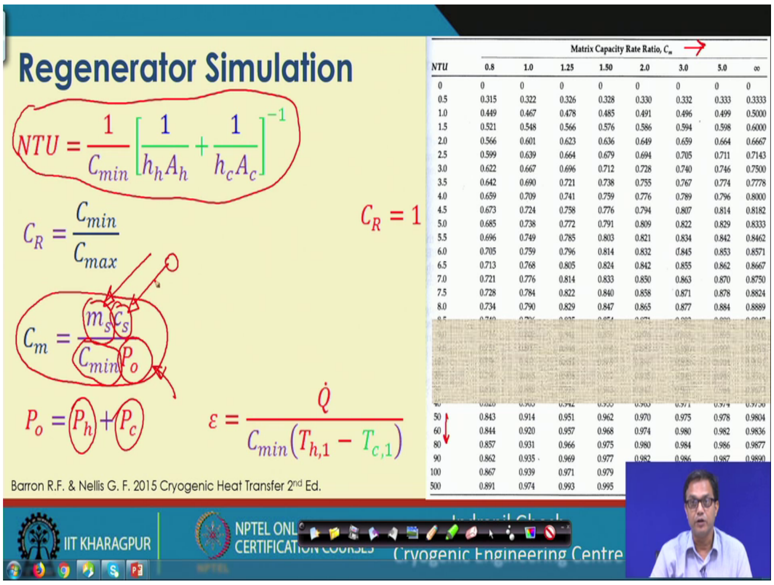

Regenerator Simulation

$$NTU = \frac{1}{C_{min}}\left[\frac{1}{h_h A_h} + \frac{1}{h_c A_c}\right]^{-1}$$

$$C_R = \frac{C_{min}}{C_{max}}$$

$$C_m = \frac{m_s C_s}{C_{min} P_o}$$

$$P_o = P_h + P_c$$

$$\varepsilon = \frac{\dot{Q}}{C_{min}\left(T_{h,1} - T_{c,1}\right)}$$

$C_R = 1$

	Matrix Capacity Rate Ratio, C_m →							
NTU	0.8	1.0	1.25	1.50	2.0	3.0	5.0	∞
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.330	0.332	0.333	0.3333
1.0	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5000
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6000
2.0	0.566	0.601	0.623	0.636	0.649	0.659	0.664	0.6667
2.5	0.599	0.639	0.664	0.679	0.694	0.705	0.711	0.7143
3.0	0.622	0.667	0.696	0.712	0.728	0.740	0.746	0.7500
3.5	0.642	0.690	0.721	0.738	0.755	0.767	0.774	0.7778
4.0	0.659	0.709	0.741	0.759	0.776	0.789	0.796	0.8000
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5.0	0.685	0.738	0.772	0.791	0.809	0.822	0.829	0.8333
5.5	0.696	0.749	0.785	0.803	0.821	0.834	0.842	0.8462
6.0	0.705	0.759	0.796	0.814	0.832	0.845	0.853	0.8571
6.5	0.713	0.768	0.805	0.824	0.842	0.855	0.862	0.8667
7.0	0.721	0.776	0.814	0.833	0.850	0.863	0.870	0.8750
7.5	0.728	0.784	0.822	0.840	0.858	0.871	0.878	0.8824
8.0	0.734	0.790	0.829	0.847	0.865	0.877	0.884	0.8889
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
40	0.826	0.903	0.942	0.955	0.963	0.971	0.974	0.9756
50	0.843	0.914	0.951	0.962	0.970	0.975	0.978	0.9804
60	0.844	0.920	0.957	0.968	0.974	0.980	0.982	0.9836
80	0.857	0.931	0.966	0.975	0.980	0.984	0.986	0.9877
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.9890
100	0.867	0.939	0.971	0.979	[illegible]	[illegible]	[illegible]	[illegible]
500	0.891	0.974	0.993	0.995	[illegible]	[illegible]	[illegible]	[illegible]

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Now, in this context, I would like to mention that if we are looking for a high regenerator value and effectiveness value, we not only need a high NTU value, we also need to have a high value of the matrix capacity rate ratio, so that means, we need a high value of this C m, and we also need a high value of these NTU. Now, in order to get a high value of the C m or the matrix capacity rate ratio, you can understand that we should have a large mass of the matrix, we should also have a large value of the C s. C min is basically you know the process dependent, we do not have much control on that. And P 0 is again the time period, it comprises of the hot and cold fluid time period. So, here this P 0 should ideally be smaller ok.

Now, if we you know if it is there in our control the mass of the regenerator matrix, obviously this larger mass will give a better value. We can put lot of mass, but obviously lot of mass will also ensure, I mean lot of pressure drop. But, this C s is something you know, where we do not have much control. C s is the specific heat of the solid and that is temperature dependent. So, when it is temperature dependent, particularly if we have to design a regenerative for cryogenic application or low temperature application, you will find that the specific heat of the regenerator is changing you know very fast with the temperature.

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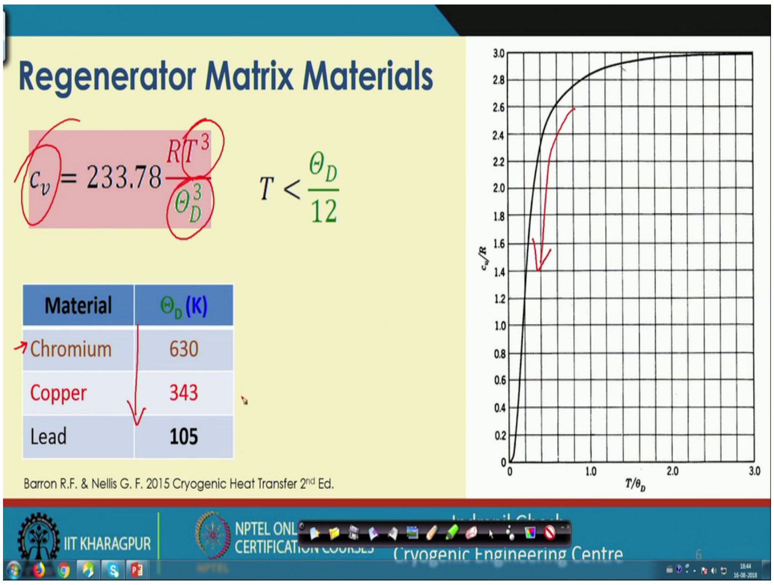

Regenerator Matrix Materials

$$c_v = 233.78\frac{RT^3}{\Theta_D^3} \qquad T < \frac{\Theta_D}{12}$$

Material	Θ_D (K)
Chromium	630
Copper	343
Lead	105

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When the temperature is less than theta D by 12, we will find that the C v or the specific heat at constant volume or constant volume and constant pressure its equivalent in case of solid. So, its varying with a T cube, I mean as it is as the temperature is falling, you will find that the specific heat is falling rapidly. And it depends also on the divide temperature theta D and some of the common material for the theta D or the I mean common material which are used for the regenerator of chromium, copper, lead, shots or I mean their products material. So, they had having you know typical this kind of theta D.

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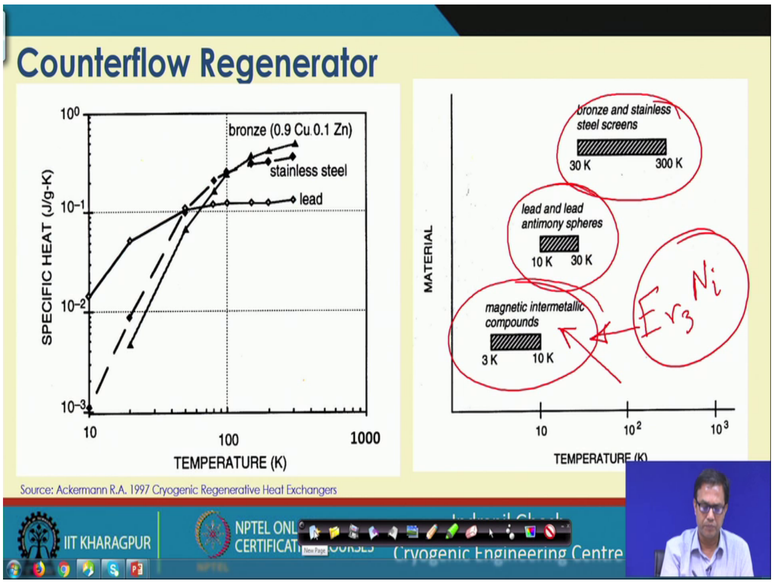

Counterflow Regenerator

Source: Ackermann R.A. 1997 Cryogenic Regenerative Heat Exchangers

So, if you look into the specific heat variation of such materials, we will find that this is the typical temperature variation of the specific heat of the bronze, stainless steel and lead. And this is more or less you know, when it is above the room temperature or near room temperature is becomes 3 (Refer Time: 20:07) that is the Dulong-Petit law. But, at low temperature that is the T cube law, it follows and the you know it falls quite rapidly, so that means, the specific heat of the material is very small, and in that case, m s C s becomes you know very small.

So, whatever you know whether you put a large mass or not, it becomes I mean bits it is a very small amount you are multiplying with a small value of the C s. And we are losing or decreasing that particular value. So, we have to be very careful, particularly when we are designing the regenerator in this range. So, here this is lead is the appropriate material in the range 10 to 30 K, because it is having a divide temperature. And low divide temperature will ensure a higher value of the C p of the C p and specific heat of the material.

But, when it is a design for you know higher temperature, so the stainless steel or bronze material is suitable. Particularly when it is going below 10 K on this lead or you know, I mean so called ordinary this materials and you know not appropriate, and in that range we will look for a special property of the magnetic material like erbium, nickel, E E r 3 nickel. So, these are the kind of material, which is suitable for very low temperature regenerator.

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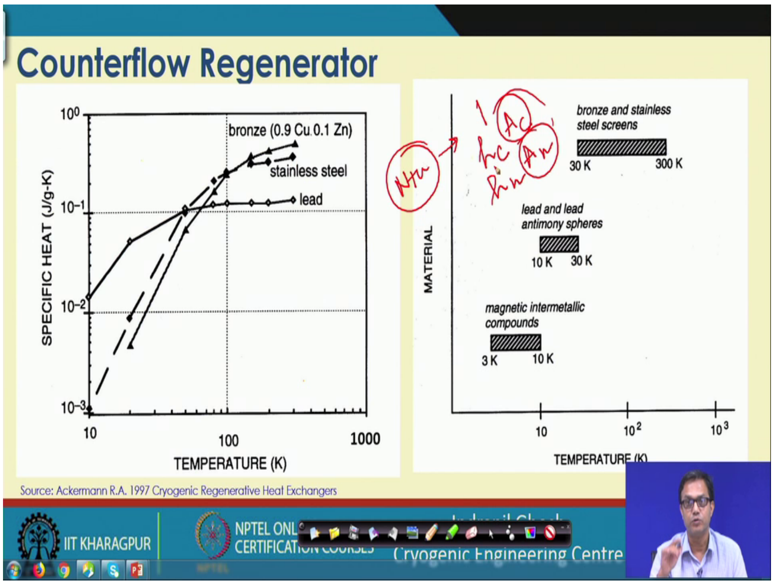

Counterflow Regenerator

Source: Ackermann R.A. 1997 Cryogenic Regenerative Heat Exchangers

So, this is what we have you know and other than that if we look into the Ntu part, if we have to make the Ntu high, it is obvious that we should have a high heat transfer coefficient for the cold side and the hot side. And then you know obviously have to put a more amount of surface area, associated both with the cold and the hot fluid. So, with this one we will move to the design part once again.

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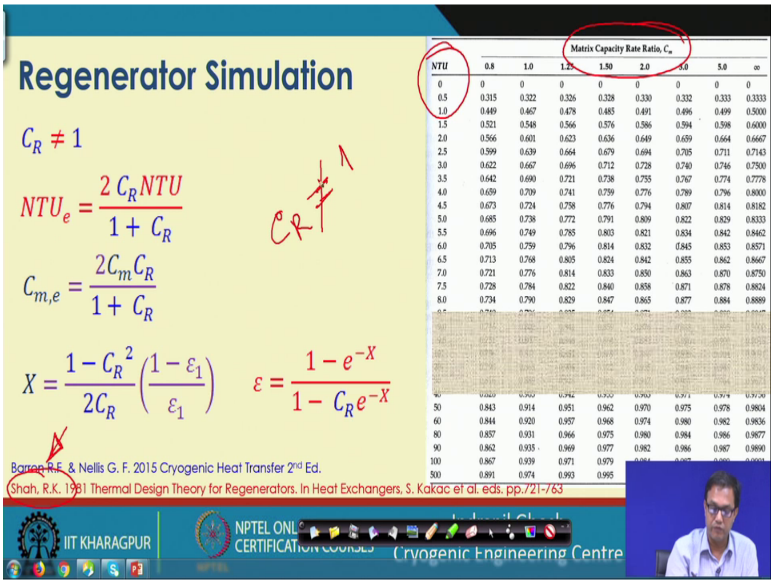

Regenerator Simulation

$C_R \neq 1$

$$NTU_e = \frac{2\,C_R NTU}{1 + C_R}$$

$$C_{m,e} = \frac{2 C_m C_R}{1 + C_R}$$

$$X = \frac{1 - C_R^{\,2}}{2 C_R}\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) \qquad \varepsilon = \frac{1 - e^{-X}}{1 - C_R e^{-X}}$$

$C_R \neq 1$

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Shah, R.K. 1981 Thermal Design Theory for Regenerators. In Heat Exchangers, S. Kakac et al. eds. pp.721-763

	Matrix Capacity Rate Ratio, C_m							
NTU	0.8	1.0	1.25	1.50	2.0	3.0	5.0	∞
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.330	0.332	0.333	0.3333
1.0	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5000
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6000
2.0	0.566	0.601	0.623	0.636	0.649	0.659	0.664	0.6667
2.5	0.599	0.639	0.664	0.679	0.694	0.705	0.711	0.7143
3.0	0.622	0.667	0.696	0.712	0.728	0.740	0.746	0.7500
3.5	0.642	0.690	0.721	0.738	0.755	0.767	0.774	0.7778
4.0	0.659	0.709	0.741	0.759	0.776	0.789	0.796	0.8000
4.5	0.673	0.724	0.758	0.776	0.794	0.807	0.814	0.8182
5.0	0.685	0.738	0.772	0.791	0.809	0.822	0.829	0.8333
5.5	0.696	0.749	0.785	0.803	0.821	0.834	0.842	0.8462
6.0	0.705	0.759	0.796	0.814	0.832	0.845	0.853	0.8571
6.5	0.713	0.768	0.805	0.824	0.842	0.855	0.862	0.8667
7.0	0.721	0.776	0.814	0.833	0.850	0.863	0.870	0.8750
7.5	0.728	0.784	0.822	0.840	0.858	0.871	0.878	0.8824
8.0	0.734	0.790	0.829	0.847	0.865	0.877	0.884	0.8889
8.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	0.942	[illegible]	[illegible]	0.971	0.974	0.9756
50	0.843	0.914	0.951	0.962	0.970	0.975	0.978	0.9804
60	0.844	0.920	0.957	0.968	0.974	0.980	0.982	0.9836
80	0.857	0.931	0.966	0.975	0.980	0.984	0.986	0.9877
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.9890
100	0.867	0.939	0.971	0.979				
500	0.891	0.974	0.993	0.995				

Here it is not always expected that we will have or we will encounter with the situation that C R equals to always small. This C R earlier we have talked about this that we have this matrix capacity rate ratio and this NTU, this chart whole chart is meant for C R equals to 1, but we may encounter situation where C R equals to not equals to 1.

In that case, how do we make an effective use of this chart that has been given or suggested by R.K Shah in this particular reference you can have it. And this is also I mean just to remind you that whether it is a pi lambda method or it is epsilon Ntu method. R.K shah has also shown that these two methods are nearly equal, and I mean they give equal results. So, it is whether you follow the epsilon Ntu method or the pi lambda method ultimate results are going to be the same.

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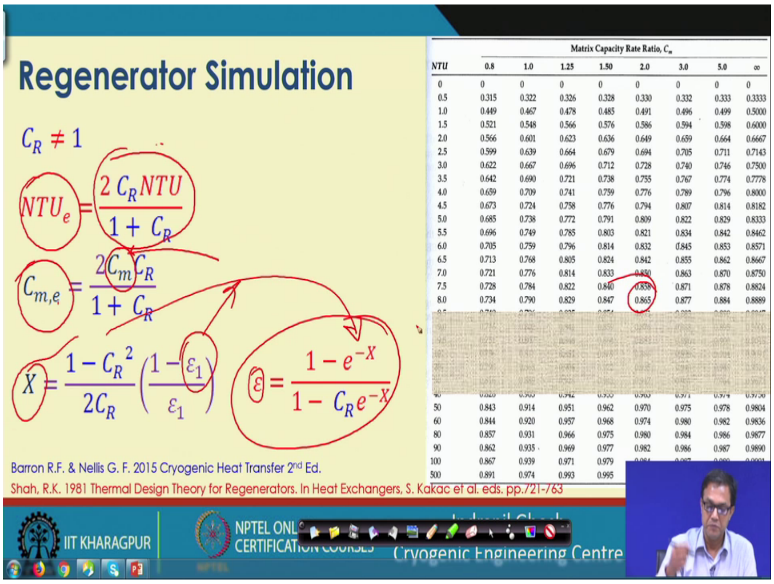

Regenerator Simulation

$C_R \neq 1$

$NTU_e = \frac{2\,C_R NTU}{1+C_R}$

$C_{m,e} = \frac{2C_m C_R}{1+C_R}$

$X = \frac{1-C_R^2}{2C_R}\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)$

$\varepsilon = \frac{1-e^{-X}}{1-C_R e^{-X}}$

	Matrix Capacity Rate Ratio, C_m							
NTU	0.8	1.0	1.25	1.50	2.0	3.0	5.0	∞
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.330	0.332	0.333	0.3333
1.0	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5000
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6000
2.0	0.566	0.601	0.623	0.636	0.649	0.659	0.664	0.6667
2.5	0.599	0.639	0.664	0.679	0.694	0.705	0.711	0.7143
3.0	0.622	0.667	0.696	0.712	0.728	0.740	0.746	0.7500
3.5	0.642	0.690	0.721	0.738	0.755	0.767	0.774	0.7778
4.0	0.659	0.709	0.741	0.759	0.776	0.789	0.796	0.8000
4.5	0.673	0.724	0.758	0.776	0.794	0.807	0.814	0.8182
5.0	0.685	0.738	0.772	0.791	0.809	0.822	0.829	0.8333
5.5	0.696	0.749	0.785	0.803	0.821	0.834	0.842	0.8462
6.0	0.705	0.759	0.796	0.814	0.832	0.845	0.853	0.8571
6.5	0.713	0.768	0.805	0.824	0.842	0.855	0.862	0.8667
7.0	0.721	0.776	0.814	0.833	0.850	0.863	0.870	0.8750
7.5	0.728	0.784	0.822	0.840	0.858	0.871	0.878	0.8824
8.0	0.734	0.790	0.829	0.847	0.865	0.877	0.884	0.8889
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
50	0.843	0.914	0.951	0.962	0.970	0.975	0.978	0.9804
60	0.844	0.920	0.957	0.968	0.974	0.980	0.982	0.9836
80	0.857	0.931	0.966	0.975	0.980	0.984	0.986	0.9877
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.9890
100	0.867	0.939	0.971	0.979				
500	0.891	0.974	0.993	0.995				

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Shah, R.K. 1981 Thermal Design Theory for Regenerators. In Heat Exchangers, S. Kakac et al. eds. pp.721-763

Now, in this situation when C R equals to not equals to 1, in that case if we have to make use of this particular chart, then we have to define some new parameters. The one is the effective NTU, and effective C m, e that is the matrix capacity rate ratio we have to redefine. And this will contain this has been suggested by R.K Shah that C m equivalent becomes C m, e. And it comprises of C m multiplied by C R divided by 1 plus C R.

And effective NTU is you know modified by this relation. So, now you have the effective NTU and effective C m, e and that is a matrix capacity rate ratio. So, now we can again use this chart and we get the value say like 0.865 or 0.85. So, these are the values. And these values are the epsilon 1, we will get this kind of regenerator effectiveness, but we know that that regenerator effectiveness is not appropriate for us, because this chart is meant for C R equals to 1.

Now, we are trying to use it for C R not equals to 1. And we have defined the NTU effective and Ntu effective C m effective. So, with that value whatever C m the effectiveness, we are getting that is known as epsilon 1. So, based on that epsilon 1 we need to find out another parameter X. And that x parameter when we put in this equation, we get the actual regenerator effectiveness. So, this is how we can make use of this chart suggested by Coppage and London, and suggested I mean modified subsequently by R.K Shah to find out the effectiveness of this regenerator, when the C R not equals to 1.

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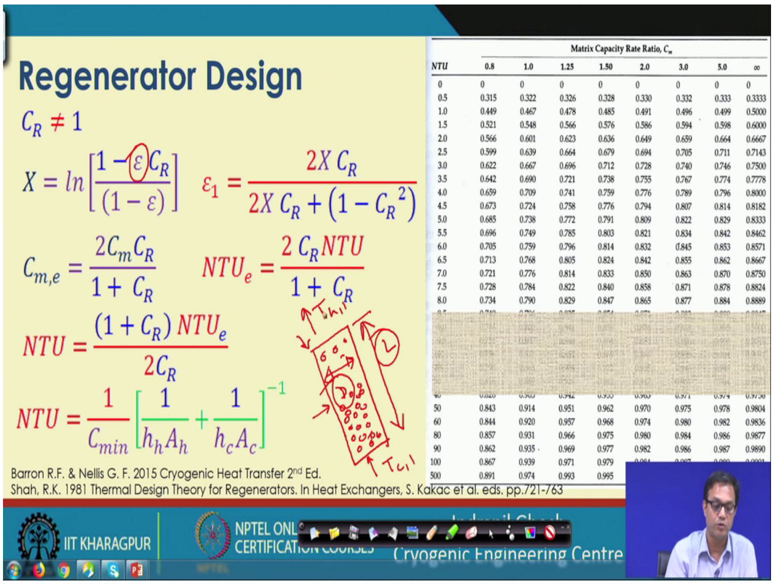

Regenerator Design

$C_R \neq 1$

$$X = ln\left[\frac{1-\varepsilon C_R}{(1-\varepsilon)}\right] \qquad \varepsilon_1 = \frac{2X\,C_R}{2X\,C_R + \left(1 - {C_R}^2\right)}$$

$$C_{m,e} = \frac{2C_m C_R}{1+\ C_R} \qquad NTU_e = \frac{2\,C_R NTU}{1+\ C_R}$$

$$NTU = \frac{(1+C_R)\,NTU_e}{2C_R}$$

$$NTU = \frac{1}{C_{min}}\left[\frac{1}{h_h A_h} + \frac{1}{h_c A_c}\right]^{-1}$$

	Matrix Capacity Rate Ratio, C_m							
NTU	0.8	1.0	1.25	1.50	2.0	3.0	5.0	∞
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.330	0.332	0.333	0.3333
1.0	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5000
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6000
2.0	0.566	0.601	0.623	0.636	0.649	0.659	0.664	0.6667
2.5	0.599	0.639	0.664	0.679	0.694	0.705	0.711	0.7143
3.0	0.622	0.667	0.696	0.712	0.728	0.740	0.746	0.7500
3.5	0.642	0.690	0.721	0.738	0.755	0.767	0.774	0.7778
4.0	0.659	0.709	0.741	0.759	0.776	0.789	0.796	0.8000
4.5	0.673	0.724	0.758	0.776	0.794	0.807	0.814	0.8182
5.0	0.685	0.738	0.772	0.791	0.809	0.822	0.829	0.8333
5.5	0.696	0.749	0.785	0.803	0.821	0.834	0.842	0.8462
6.0	0.705	0.759	0.796	0.814	0.832	0.845	0.853	0.8571
6.5	0.713	0.768	0.805	0.824	0.842	0.855	0.862	0.8667
7.0	0.721	0.776	0.814	0.833	0.850	0.863	0.870	0.8750
7.5	0.728	0.784	0.822	0.840	0.858	0.871	0.878	0.8824
8.0	0.734	0.790	0.829	0.847	0.865	0.877	0.884	0.8889
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
50	0.843	0.914	0.951	0.962	0.970	0.975	0.978	0.9804
60	0.844	0.920	0.957	0.968	0.974	0.980	0.982	0.9836
80	0.857	0.931	0.966	0.975	0.980	0.984	0.986	0.9877
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.9890
100	0.867	0.939	0.971	0.979	[illegible]	[illegible]	[illegible]	[illegible]
500	0.891	0.974	0.993	0.995	[illegible]	[illegible]	[illegible]	[illegible]

Barron R.F. & Nellis G. F. 2015 Cryogenic Heat Transfer 2nd Ed.
Shah, R.K. 1981 Thermal Design Theory for Regenerators. In Heat Exchangers, S. Kakac et al. eds. pp.721-763

Now, how do we design in case of a design problem, as you understand that. Situation will be slightly a difference. In this case, just we have to I mean follow the reverse path here what is known is the effectiveness, because in this case we are trying to design the regenerator. Maybe you know, we know the diameter of it, you know we have to find out the length of it or sometime it may be such that you know we have to find out both length, diameter, and etcetera.

And even the what are the particular type of regenerator matrix that may also be you know that may also be decided, we have to decide about that one and there are different options. So, one by one we have to finalize, and there would be different type of constants. So, as we understand that the design becomes always a multi objective or multivariable process. So, in this case imagine that we have certain parameters known, what is known is the overall effectiveness, what is known as the inlet temperature of the hot and the cold fluid, and what is the exit fluid temperature that is also known average exit of both hot and cold fluid are known.

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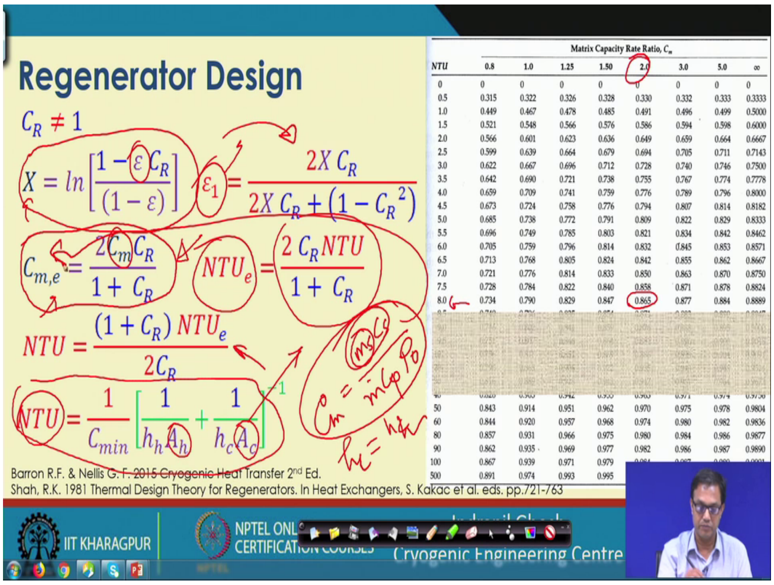

Regenerator Design

$C_R \neq 1$

$$X = ln\left[\frac{1-\varepsilon C_R}{(1-\varepsilon)}\right] \qquad \varepsilon_1 = \frac{2X\,C_R}{2X\,C_R + (1 - {C_R}^2)}$$

$$C_{m,e} = \frac{2C_m C_R}{1+C_R} \qquad NTU_e = \frac{2\,C_R NTU}{1+C_R}$$

$$NTU = \frac{(1+C_R)\,NTU_e}{2C_R}$$

$$NTU = \frac{1}{C_{min}}\left[\frac{1}{h_h A_h} + \frac{1}{h_c A_c}\right]^{-1}$$

$m_s C_s$

$C_m = \dot{m} C_p P_0$

$h_c = h_h$

NTU	Matrix Capacity Rate Ratio, C_m 0.8	1.0	1.25	1.50	2.0	3.0	5.0	∞
0	0	0	0	0	0	0	0	0
0.5	0.315	0.322	0.326	0.328	0.330	0.332	0.333	0.3333
1.0	0.449	0.467	0.478	0.485	0.491	0.496	0.499	0.5000
1.5	0.521	0.548	0.566	0.576	0.586	0.594	0.598	0.6000
2.0	0.566	0.601	0.623	0.636	0.649	0.659	0.664	0.6667
2.5	0.599	0.639	0.664	0.679	0.694	0.705	0.711	0.7143
3.0	0.622	0.667	0.696	0.712	0.728	0.740	0.746	0.7500
3.5	0.642	0.690	0.721	0.738	0.755	0.767	0.774	0.7778
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4.5	0.673	0.724	0.758	0.776	0.794	0.807	0.814	0.8182
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8.0	0.734	0.790	0.829	0.847	0.865	0.877	0.884	0.8889
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
50	0.843	0.914	0.951	0.962	0.970	0.975	0.978	0.9804
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80	0.857	0.931	0.966	0.975	0.980	0.984	0.986	0.9877
90	0.862	0.935	0.969	0.977	0.982	0.986	0.987	0.9890
100	0.867	0.939	0.971	0.979	[illegible]	[illegible]	[illegible]	[illegible]
500	0.891	0.974	0.993	0.995	[illegible]	[illegible]	[illegible]	[illegible]

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So, in that case we have since epsilon is known. We can find out just as we have defined, if you remember that earlier expression. So, from there we can find out the X, once we know the X we can find out the epsilon 1. This is the reduced value, but we do not have any idea about say the C m, e, because C m, e contains C m and that C m contains m s C s by m dot C P P 0. So, this m s C s we do not have any idea about the amount of mass that is going to be there inside the regenerator.

So, here at this point we have to make an assumption of this regenerator a matrix C m effectiveness. So, based on that now we have epsilon 1 and C m, e, if it is a 0.861 and C m if it is 2 in that case you know, we know what is the value of the corresponding NTU. So, with that NTU we that is that becomes you know the effective NTU, and from there we have to calculate the actual NTU by this process.

And once we know the actual NTU, this NTU can also be related by this relation, where now we have to this NTU is now known, and this we can find out h w A c and h c and h h can be obtained. And finally, we will be able to find out what is the area. So, once this area is known, we can also find out what is the amount of material involved with that you know regenerator. And when we know the material, we get a value of the same. If it is a good guess, if you have made you know, there would be that number of iterations will be small. Otherwise, with that value we have to go back to the step, and we have to make an iterative solution. So, now with this value, we will go to the next slide.

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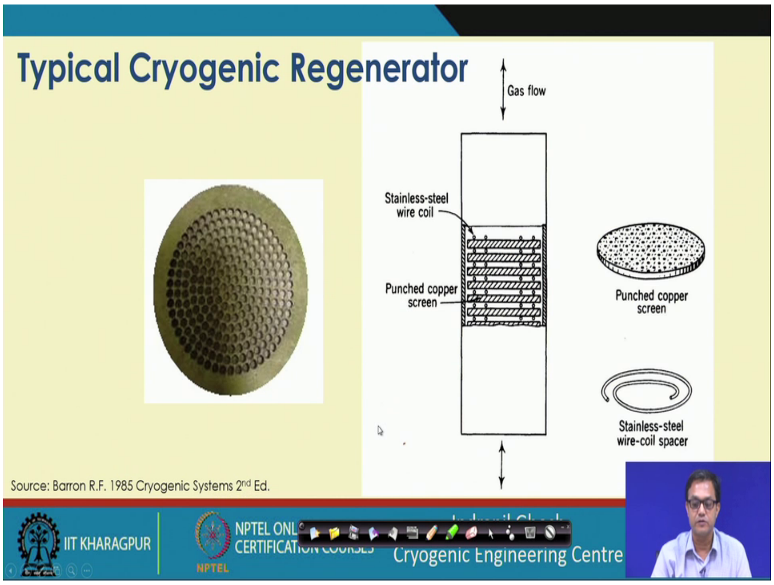

Typical Cryogenic Regenerator

Source: Barron R.F. 1985 Cryogenic Systems 2nd Ed.

And this is a typical you know cryogenic regenerator, where we find that alternative layers of such this is a perforated plate, and alternative such perforated plate and stainless steel you know spacers. This spacers are given to adjust or reduce the actual conduction of the heat from coming from top to the bottom. So, this is a typical look of a cryogenic regenerator. This has been taken from the Barron's book of Cryogenic Systems.

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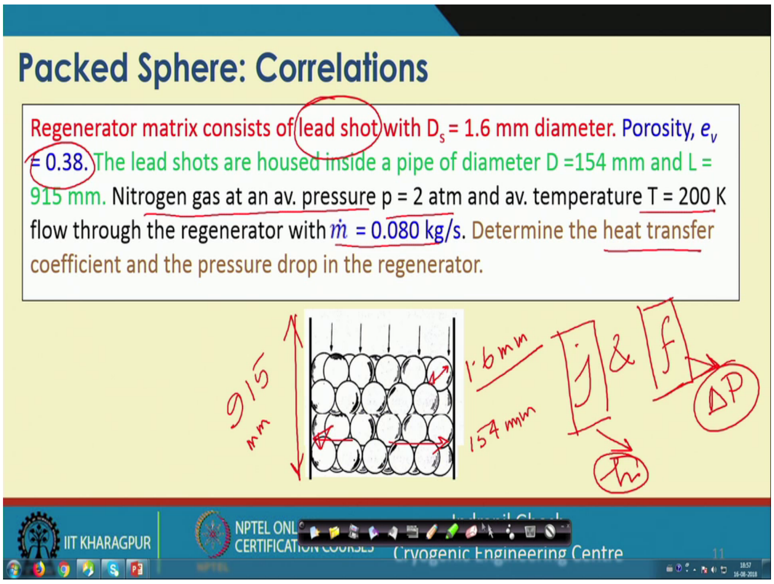

Packed Sphere: Correlations

Regenerator matrix consists of lead shot with D_s = 1.6 mm diameter. Porosity, e_v = 0.38. The lead shots are housed inside a pipe of diameter D =154 mm and L = 915 mm. Nitrogen gas at an av. pressure p = 2 atm and av. temperature T = 200 K flow through the regenerator with $\dot{m}$ = 0.080 kg/s. Determine the heat transfer coefficient and the pressure drop in the regenerator.

So, with this knowledge, we will now move to a small problem, where we will try to calculate the heat transfer coefficient and the pressure drop in regenerator, where it is packed with you know the lead shot. And this you know is diameter of this lead shot is 1.6 millimetre. And their packed inside a bed of diameter it is given as 154 mm, and the length of this regenerator is 915 mm; the porosity is also given as 0.38. And the mass flow rate is basically m dot is 0.080 kg per second, and the gas is nitrogen gas flowing at a pressure of 2 atmosphere and at average temperature of 200 Kelvin.

So, with this information we need to find out what is the heat transfer coefficient, and what is the friction factor. So, you know that by this time that this j and f are you know the value from which we can calculate the overall heat, I mean the heat transfer individual, heat transfer coefficient. And this will be link to the pressure drop associated with this flow of gas through this packed bed. So, we will try to solve this example in the next class.

And thank you for your attention.