

**Heat Exchangers: Fundamentals and Design Analysis**  
**Prof. Prasanta Kumar Das**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 46**  
**Heat pipes and Heat exchangers (Contd.)**

Welcome everyone. If we recall, we were halfway in between the modelling effort of heat pipe. We have taken conventional heat pipe that is heat pipe of constant cross sectional area constant constructional area with a circular cross section, and where the liquid returned from the condenser to the evaporator is supported by capillary structure or tweak that kind of heat pipe was developed at the beginning. And so far most of the heat pipe we use are of that kind though over the years different variations have come.

And if you recall that we have adopted network analysis for the design of this kind of heat pipe, which is a very innovative and intelligent kind of approach. And for such heat pipe, there are two networks; one is thermal network, and another is fluidic network. The thermal network I have explained at a very basic level, though there could be more complicated thermal network. And the fluidic level, we have just started describing. And last lecture I have told that before learning the fluidic network, the fluid flow inside the heat pipe has to be understood.

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**Fluidic Network:**

- To develop the fluidic network, first the fluid flow (liquid and vapour) within the pipe needs to be appreciated.

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Prof. P. K. Das  
Department of Mechanical Engineering

So, again we are continuing with this Heat pipes and heat exchanger. What I will do? I will go back to our previous last slide of the previous lecture, so this slide. So, fluidic network to develop the fluidic network, first the fluid flow liquid and vapour within the pipe needs to be appreciated. First, we have to understand this.

Let us recapitulate once again. So, what we are having that we are having a heat pipe, where there is the working fluid and the working fluid is in the wick region. And as we as we start transferring heat in the evaporator section, the evaporation of the fluid in the wick structure will start. And then this vapor will flow and it will reach the condenser section. So, here heat is being extracted. So, here it will get into liquid state, and that liquid state will return through the wick structure to the evaporator section. So, this will continue. This is our fluidic network vapor liquid, liquid vapor. So, this is our fluidic network. So, again we can see that evaporator to condenser, condenser to evaporator. So, this is our fluidic network.

Now, what is driving the flow? There is no pump, there is no fan, but there is a flow and circulatory flow, and not only that two fluids are involved, two types of fluids are involved; one is a gas or a vapor, and another is a liquid. So, you see this is a very intelligent kind of arrangement. So, this I have told, again I will repeat that here we can see the meniscus is formed curve meniscus is formed, so there is a pressure difference between liquid phase vapor phase. And this meniscus is not uniform all through, so there is a change in the meniscus shape in the axial direction. So, this will give rise to some sort of a capillary pressure, and this capillary pressure will give rise to the driving action. So, let us see the next diagram.

So, in the next diagram, we have shown distance along the heat pipe, evaporator length, then the adiabatic length, and then the condenser length. And then this side, we have got so, this length is the independent dimension. And on the other side, we have seen how the pressure varies along the length; and how the pressure varies along the length that we want to see both for the liquid and for the vapor.

So, for the vapor, what I have shown that the vapor pressure it starts with a high value at the evaporator end or at the beginning of the evaporator and then it falls. It falls, and it comes to some sort of a minimum value at the end of the adiabatic section. So, how it can be appreciated in a minute that vapor will generate more and more as we move along

the length of the evaporator. And again, it is travelling a distance, so the mass flow rate is increasing, and there is some sort of a resistance, which is it has to overcome.

So, initially if there is a high pressure, then there will be fall in pressure. As it overcomes the evaporator length, then there will not be any further vapor generation, mass flow rate will not increase, but the flow resistance that will still continue in the adiabatic section, so there will be still a fall in the pressure of the vapor. So, it will come to the lowest pressure at the end of the evaporator sorry adiabatic section, and then it enters the condenser section.

So, in the condenser section what will happened? In the condenser section gradual condenser section of vapor will take place, mass flow rate will reduce, so we will have some sort of a pressure rise, many people call it pressure recovery. So, there will be some sort of a pressure recovery. And at the end of the evaporate sorry condenser, the pressure will be somewhere over here. So, at the beginning of the evaporator, the pressure was here; and at the end of the condenser, the pressure is here, and this difference is the vapor pressure drop ok. And you see this is kind of positive this is kind of positive.

Then we will have liquid. So, liquid is getting form from the vapor or transform from the vapor in the condenser section. So, in the condenser section, liquid is formed, condensate is being formed. And then the condensate flows through the through the wick structure. And then we have to understand that there is not much change of the liquid volume, liquid being in compressible, vapor is compressible, vapor density is also less, but the liquid density is not that less. So, liquid mass flow rate there is slight change, but after that, the volume flow rate does not change much.

So, for the liquid, this blue line if we see that there will be continuous fall in pressure, as it moves from the condenser end to the evaporator end, because there is flow resistance. All these reduction or increase whatever we have seen, these are non-linear. But, initially when the analysis started, I have told that we will assume that there is no effect of gravity. So, this is without any effect of gravity.

If the effect of gravity if the effect of gravity is included, then there will be further pressure drop and we will get the rate curve. So, what we can see that there is some sort of a net pressure head for the vapor phase, and there is pressure drop for the liquid phase. And if the circulation has to be there, then the these two things should balance, and then

only we will have circulation. So, this is what we get from our fluid flow. Configuration of the fluid flow, we can understand what is taking place.

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**Fluidic Model:**

To develop the fluidic model the following assumptions are made:

1. Fluid path is one-dimensional.
2. Liquid is incompressible.
3. Vapour is represented by the ideal gas equations.
4. Vapour compression and expansion are polytropic processes.
5. Flow is laminar for both vapour and liquid.
6. Evaporation and condensation phenomenon are confined respectively in the evaporator and in the condenser zones.
7. Liquid mass flow rate is constant in the wick adiabatic section.
8. Liquid temperature is equal to the wick temperature in each HP section.

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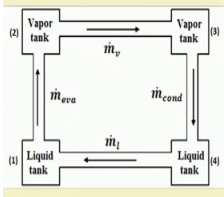
Now, if we move to the next slide, next slide means we have to go here fluidic model. To develop the fluidic model, the following assumptions are made. Fluid path is one-dimensional. Liquid is incompressible. Vapour is represented by the ideal gas equation. And then vapour compression and expansion are polytropic process ok; it could be adiabatic polytropic process or some sort of polytropic process. Flow is laminar for both vapor and liquid.

And then evaporation and condensation phenomena are confined respectively in the evaporator and in the condenser zone. Liquid mass flow rate is constant in the wick adiabatic section. In the wick adiabatic section, we have constant liquid pass flow rate, because this we adiabatic section take a very major role in the pressure drop of the liquid phase. And liquid temperature is equal to the wick temperature in each heat pipe section. This is what we have also assumed earlier that the wick structure and the liquid, they are at thermal equilibrium at any axial position. So, with this, let us with this assumption, let us move forward for the modelling.

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**Fluidic Model**

The model conceptualized as a combination of four reservoirs;



- The liquid tank in the evaporator zone (1) is heated up; a mass of vapour  $\dot{m}_{eva}$  is produced and flows into the vapour tank (2)
- Pressure increases inside the vapour tank (2) and a mass of vapour  $\dot{m}_v$  is pushed in the condenser vapour tank (3) through the vapour path.
- The condenser zone is cooled down; a mass of fluid  $\dot{m}_{cond}$  is condensed and flows into condenser liquid tank (4).
- The wick capillary pressure pushes a mass of liquid  $\dot{m}_l$  from the condenser liquid tank (4) back to the evaporator liquid tank (1) through the liquid path.

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For the fluid model, the fluid model as I have told that it is a circulatory kind of a model. Vapor is generated in the condenser sorry vapor is generated in evaporator, it goes to the condenser section, it transformed into liquid, liquid comes back in the evaporator, and it gets transformed into vapor. So, we have assumed that there are two vapor tank, one vapor tank is there in the evaporator section, one vapor tank is there in the condenser section.

So, vapor goes from one tank to another. Then in the condenser section what vapor tank is there, so there the vapor is getting transformed into liquid, and that goes to a liquid tank in the condenser section imaginarily liquid tank. Then from the liquid tank situated in the condenser, liquid flows to another liquid tank, which is situated in the evaporator. And there, the liquid goes to the vapor tank through a transformation process from liquid to vapor. So, this is kind of a circulatory model, people could people could imagine.

See the model conceptualized as a combination of four reservoirs. The liquid tank in the evaporator zone 1 is heated up. A mass of  $\dot{m}_{eva}$  is produced and the and flows into the evaporator tank 2. Pressure increases inside the vapor tank 2 and a mass of  $\dot{m}_v$  is pushed in the condenser pressure tank 3, though the vapor path. The condenser zone is cool down. A mass of fluid  $\dot{m}_{cond}$  is condensed and flows into the condenser liquid tank 4. The wick capillary pressure pushes the mass of liquid  $\dot{m}_l$  from the condenser liquid tank 4, back to the evaporator liquid 1 through the liquid

path. So, four tanks four conceptualized tanks, and four pipelines again conceptualized pipelines constitute some sort of a circulatory system, and represents the circulation inside a heat pipe good.

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**Governing Equations of the fluidic elements:**

- Vapour inside the tank is modelled as an ideal gas.
- The continuity equation gives,

evaporator:  $\frac{V_{ve}}{\gamma R T_{ve}} \frac{dp_{ve}}{dt} = \dot{m}_{eva} - \dot{m}_v \rightarrow C_{ve} \frac{dp_{ve}}{dt} = \dot{m}_{eva} - \dot{m}_v$

condenser:  $\frac{V_{vc}}{\gamma R T_{vc}} \frac{dp_{vc}}{dt} = \dot{m}_v - \dot{m}_{cond} \rightarrow C_{vc} \frac{dp_{vc}}{dt} = \dot{m}_v - \dot{m}_{cond}$

- Considering adiabatic polytropic process, relation between pressure and temperature is obtained.

$$\frac{dT_{ve}}{dt} = \frac{\gamma - 1}{\gamma} \frac{T_{ve}}{p_{ve}} \frac{dp_{ve}}{dt}$$

$$\frac{dT_{vc}}{dt} = \frac{\gamma - 1}{\gamma} \frac{T_{vc}}{p_{vc}} \frac{dp_{vc}}{dt}$$

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The vapor inside the tank is modelled as an ideal gas. This is what we have told earlier, and we will follow that. The continuity equation that gives, we will get this kind of a continuity equation. So, actually you see the sub scripts are not very difficult to follow that is vapor in the evaporator. So,  $p_{ve}$  that is the pressure of the vapor in the evaporator section, and  $V_{ve}$  is the volume of the vapor in the evaporator section,  $R$  is the gas constant, and  $\gamma$  is  $C_p$  by that is the ratio of the specific heats, so  $C_p$  by  $C_v$ .

And this quantity this is equal to our  $\dot{m}_{eva}$  minus  $\dot{m}_v$ , so which is going out just mean that means, let us if we go to the previous slide, so what we can see that we are doing some sort of a transient analysis. If we do not do the transient analysis, then whatever  $\dot{m}_{eva}$  is coming that will be  $\dot{m}_v$ . But, if we are considering transient analysis, then some sort of a vapor accommodation can be done in this vapor tank, which is tank number 2. So, this is what we have written in the next slide. In the next slide, if we see that  $\dot{m}_{va}$  that is entering the tank and  $\dot{m}_v$  that is going out, and then we can write some sort of an equation like this. And then we can simplify the equation something like this.  $C_{ve}$ , it is kind of a capacitance we can write.

For the condenser also similar equation, I do not like to explain it as details, as I have explained the earlier equation. So, for the condenser also, we can get some sort of an equation. So, what we can get, so again some sort of a capacitance for the condenser, we can get. Considering adiabatic polytropic process, relation between pressure and temperature is obtained. So, we assume that adiabatic polytropic process is taking place, when there is an expansion or there is a change in temperature. So, the change in temperature with time, we can get with this kind of equation. With this, if we go to the next level, then how we get that how much mass is coming to the evaporator tank.

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**Governing Equations of the fluidic elements:**

- Mass flow rates for evaporator and condenser are obtained as,
 
$$\dot{m}_{eva} = \frac{\dot{Q}_{eva}}{h_{fg}} \quad \dot{m}_{cond} = \frac{\dot{Q}_{cond}}{h_{fg}}$$
- For the vapour duct one gets,  $L_{va} \frac{dm_v}{dt} = R_v m_v + p_v - p_{vc}$
- For the liquid tanks
 
$$\dot{m}_l - \dot{m}_{eva} = \frac{dM_{li}}{dt} \quad \dot{m}_{cond} - \dot{m}_c = \frac{dM_{li}}{dt}$$

Where,  $M_{li}$  is the liquid mass in the wick volume.

$$M_{li} = \rho_l \epsilon \pi (r_w^2 - r_c^2) L_w \begin{cases} i = e, \text{ evaporator} \\ i = c, \text{ condenser} \end{cases}$$

So,  $\dot{m}_{eva}$  that is  $\dot{Q}_{eva}$  divided by the latent heat. Similarly,  $\dot{m}_{cond}$  that is whatever is the heat divided by the latent heat. And then there are ducts, which are the there is a duct we have conceptualized a duct, which is carrying the vapor. For that, some sort of a pipe line equation with resistance etcetera, we will get like this. For liquid in tank, we will get this kind of equation. Two liquid tank, one for evaporator, and one for condenser, we will get these two equation.

$M_{li}$  is the liquid mass in the wick volume. So, in the wick volume, whatever liquid mass is there that we can get, how? This is the rho density, this is the porosity, and this is the cross sectional area, and this is  $L_w$  that means, the cross sectional area into length that will give us volume of the wick structure multiplied by the porosity that will give is give the pore volume multiplied by rho that will give us the mass, so that is what we are

getting. And then this porous structure is there, both in this evaporator and in the condenser. So, we have to take care of, whether it is in the condenser or in the evaporator.

So, more or less, the important equations I have give, but there are I have done separately the analysis of quite a few things. First, the thermal analysis I have done with thermal network, then the fluidic analysis I have done with fluid network. And then in the fluid network, I have separately considered the vapor path, I have separately considered the liquid path. So, there should be some sort of a coupling between liquid and vapor, and then ultimately there should be a coupling between the fluidic network and the thermal network. So, these two couplings are very important ok. So, let us now see, how these two couplings can be done.

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• For the liquid line one gets integrating over the adiabatic zone length  $L_a$ .

$$\frac{\mu_a L_a}{K \rho_a} \frac{1}{c \pi (r_{ev}^2 - r_{iw}^2)} m_l = R_{la} m_l = p_c - p_e$$

• K is an empirical constant given by, 
$$K = \frac{4r_g^2 \epsilon^2}{150(1-\epsilon)}$$

• Fluidic circuit is represented here.

The diagram shows a fluidic circuit with two parallel paths. The top path consists of a capacitance  $C_{ve}$  on the left, a resistance  $R_{va}$  in the middle, and a capacitance  $C_{vc}$  on the right. The bottom path consists of a resistance  $R_{la}$  in the middle. The left side of the circuit is connected to a source with temperature  $T_{ve}, p_{ve}$  and the right side to a sink with temperature  $T_{vc}, p_{vc}$ . The bottom path is connected to a source with temperature  $T_{we}, p_{we}$  and a sink with temperature  $T_{wc}, p_{wc}$ . A source credit is given at the bottom: Source: <http://hdl.handle.net/10589/51066>

So, now for liquid line, we have to get the equation for the liquid line, as we have got earlier for the other lines. So, here we bring some sort of a resistance K. So, K is an empirical constant for actually liquid line, you have to see it is through the porous structure. So, this is how we can bring it. Then the fluid circuit, whatever we have got. So, this fluidic circuit can be represented like this. So, fluidic circuit we can see that there are different resistances, and then there are other kind of elements also. So, with that, we can write down the fluidic circuit for our conceptual understanding, but already we have written the equation. Now, whatever I was telling let us move to that.



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The coupling between liquid and vapour:

Mass flow rates for evaporator

$$\Delta P_{v-l} = \frac{2\sigma}{r_c} \quad \text{Y-L Eqn}$$
$$r_c = 0.41r_g$$
$$P_w - P_v = f \frac{2\sigma}{r_c}$$

So, first we are seeing that coupling between liquid and vapour. So, coupling between liquid and vapour if we think of the evaporator section, so this is a very exaggerated way of drawing the evaporator section. So, this is evaporator wall, and this is the porous structure, and this is the curve interface, this is your liquid, and this is your vapour. So, here we see that we get the curve interface. And curve interface, so some sort of radius of curvature one can get, which is  $r_i$ . Let us say this is your  $r_i$ . Let us say this is your radius of curvature and  $r_i$ .

So, the vapour and liquid pressure difference that is given by Young-Laplace equation, this is nothing but your Young-Laplace equation. And one can derive the situation, so we will get this relationship between vapour pressure and the liquid pressure; they are related by the surface tension of the liquid and the radius of curvature. How do you get the radius of curvature, one cannot measure it within a within a heat pipe, when it is under operation.

So, there is some sort of a relationship here you can see that if it is made of different grains the porous structure and the grain radius is  $r_g$ , then  $r_c$  is related to  $r_g$ . And then, again we get finally one equation between as a difference between the liquid phase and the vapour phase. What we have written the previous equation itself and with a factor  $F$ , some sort of a correction factor one has to take. So, with this, we will get a coupling between the liquid phase and the vapour phase. So, with thus with this, let us move.

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Solid-Fluid thermal coupling :

$$\dot{Q}_{eva} = \frac{T_{we} - T_{ve}}{R_{2we}} \rightarrow \dot{m}_{eva} = \frac{T_{we} - T_{ve}}{R_{2we} h_{lv}}$$
$$\dot{Q}_{cond} = \frac{T_{vc} - T_{wc}}{R_{2wc}} \rightarrow \dot{m}_{cond} = \frac{T_{vc} - T_{wc}}{R_{2wc} h_{lv}}$$
$$T_{we} = T_{le}$$
$$T_{wc} = T_{lc}$$

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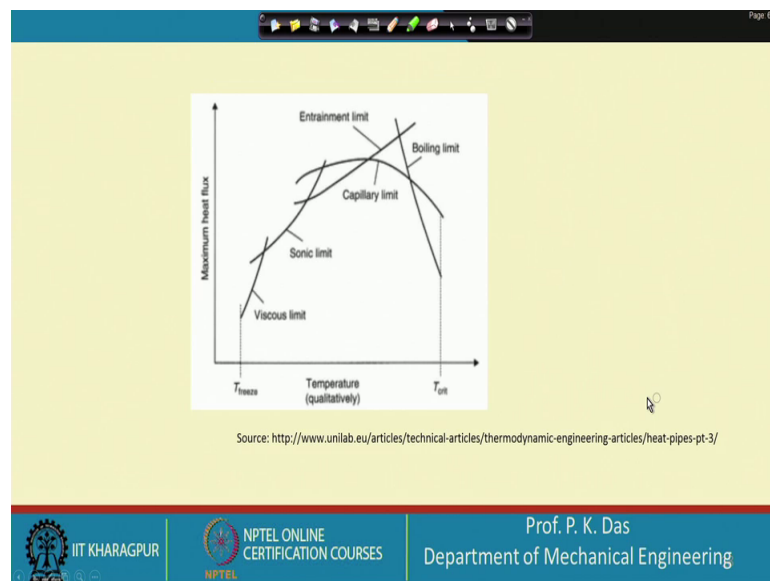
Solid and fluid thermal coupling. So, now what we can do that  $\dot{Q}$  evaporator, evaporator side (Refer Time: 23:25) so that we can determine knowing that what is the wick temperature, temperature within the wick, and temperature of the vapor in the evaporator and then the resistance, so that will give you some sort of mass flow rate, and that is equal to no. From this equation, we will get the mass flow rate by this particular formula, where  $h_{lv}$  is the latent heat of vaporization. So, this resistance, we have to calculate.

These two actually wick temperature can be related to liquid temperature in the evaporator, liquid and vapor temperature can be related by the Young-Laplace equation. So, and the resistance calculation, we have mentioned earlier. So, with all these things, we can be able to couple the solid and fluid thermal solid and fluid, and then this thermal coupling between fluidic and thermal network and the fluidic network can be coupled. So, similar set of equation, one can get for the condenser also.

So, with all these things, we will be able to close our model, close our network model. So, we need property values, we need of course, some small correlation coefficients small number of correlation coefficient we have used, those thing we need, we need the dimension of the heat pipe, we need the some details regarding the wick structure. So, with all these things, we will be able to calculate the performance or we will be able to assess the performance of the heat pipe or we will be able to predict the transient performance transient characteristics of the heat pipe ok.

So, this is what I like I wanted to convey in the design of the heat pipe or analysis of the heat pipe. Details can be obtained by from different books. In the next class, I like to give you some I like to give you mention some books on heat pipe. And those books can be referred for getting details. If possible, I will also give some details of the papers or reference of the papers. From there also one can see, what is the network analysis. So, with this, we move to a important topic, but unfortunately time is short.

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So, today I will not be able to tell this, only let me once I have introduce the slide, let me tell you. Here what we want to do, we want to see what is the working range of a heat pipe. So, there are different limiting phenomenon. So, with this limiting phenomena, what is the working range of the heat pipe that we want to see, and that we will do in the next class. For the time being, I like to thank all of you for your attention.

Thank you.