Heat Exchangers: Fundamentals and Design Analysis
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Lecture – 34
Plate fin heat exchanger: Multistream Analysis

Welcome to this lecture. We are going to talk about the Plate fin type heat exchangers and we were discussing about the analysis of Plate fin type heat exchangers where we have analyzed the fin equation and we tried to differentiate between the different type of flow resins not resins rather the temperature domains or the temperature configurations that is possible in a flood plate fin type multi stream exchangers.

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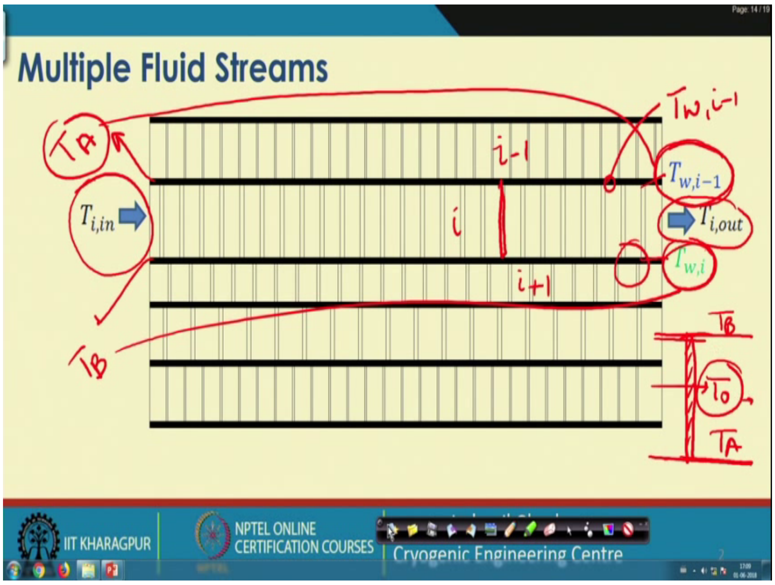

And in that context we have seen that the plate fin type heat exchangers where there may be multiple fluid streams and we have to analyze this kind of heat exchanger where we were attempting basically a type of what is called the simulation problem, we are trying to solve.

And here we have seen that we have the fluid stream going in at T in and it is coming out at T i out whereas, this is for the i th fluid streams. And this is T w i minus 1 is the wall temperature and this is the T w, i this plate is at temperature T w, i and we have this plate temperature at T w, i minus 1. So, this is the generalized configuration where we may have say this is the i th fluid stream and we have this i minus 1 th fluid stream. This is i plus 1 th fluid stream and this wall temperature is designated at T w, i minus 1 and this wall temperature is designated as T w, i.

So, in this generalized configuration, we now want to find out how this temperature of the plate and the fluid streams are correlated. And in that context we have solved the basic fin equation connecting between the two walls may be we have solved it as considering it to be T A and T B in the generalized form. But here it is in the more generalized form of T w, i minus 1 and T w, i. So, here basically T w, i is T b and T w a is T w, i minus 1. So, this is this correspondence is there.

So, now we have seen that if we are solving that basic fin equation where the walls are at T B and T A and we find out the heat transferred equation for this fin joining these two parallel plates or the separating plates. And the fluid flowing on top of it is T 0 at a temperature T 0 and it has already been assumed though we have not explicitly it is said that the fluid is not undergoing any phase change.

So, now in this situation what we have seen that the, if the fluid temperature and the plate temperatures are like that we can have the.

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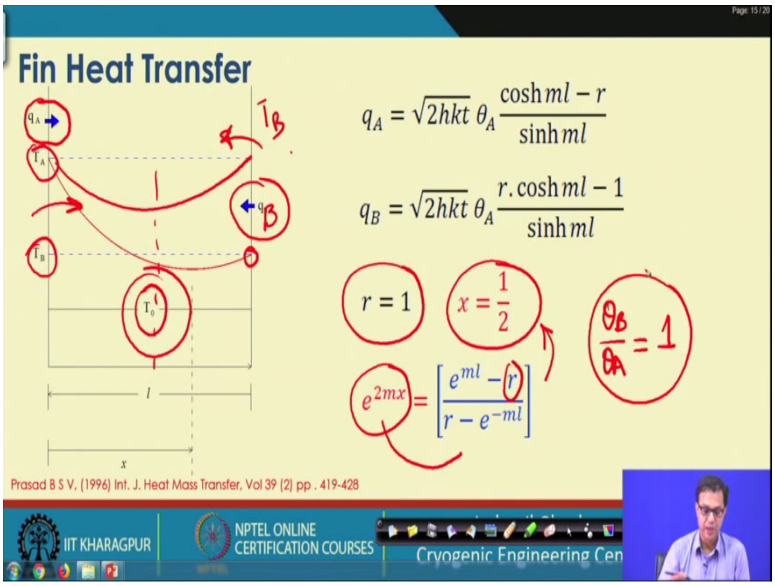

We will come back to this again that generalized configuration, before that we just want to recapitulate what we have learnt in the earlier class; where we have seen that this q A the amount of heat getting transferred from the wall to the fluid where the fluid temperature is T 0.

And the wall the temperature is T A and this is basically q B; it is not looking properly ok. So, this is q A and this is q B getting transferred from the wall to the fluid and here this is from this wall T A to the fluid where the fluid temperature is T 0. And this wall is at a temperature T B this is where it is its temperature is at this point is T B.

So, now in this context we have seen if r equals to 0 or theta B by theta A that is equals to r and that is equals to 1, we have seen that this x becomes half where do we get this we get this from this relation that if there has to be a minima then this minima will give a criteria like e to the power 2mx is equals to this co relation. And here in this context, if we put r equals to 1 or theta B by theta A equals to 1, we find that x becomes half. So that means, when this two temperatures are the same, we have a minima at the middle or at the center of this plate.

Now, if this is not equals to 1 or if it is having some other value and we were talking about the different possibilities, in that context we have seen that sometime it may be possible that the location of this extremum may go beyond the physical boundary.

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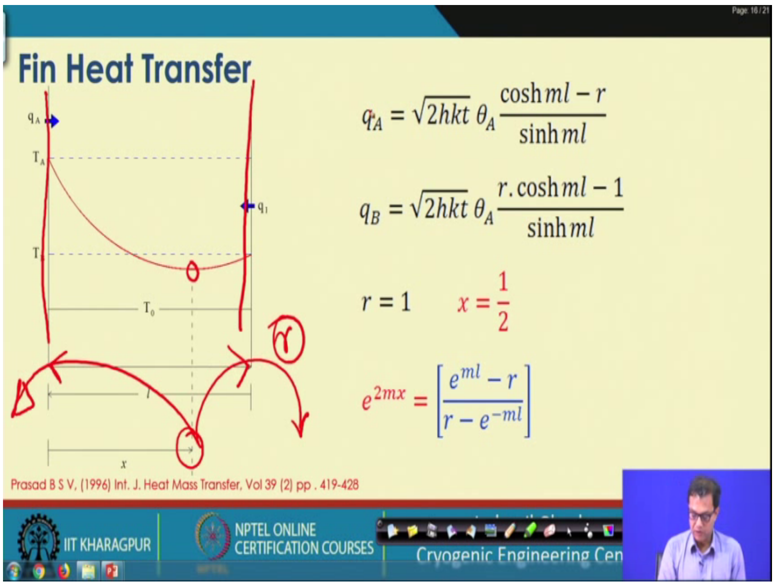

Physical boundary mean that this is the fin, these are the two separating plates and this minima may occur beyond the physical boundary of this one either on this side or on this side depending on the value or path. And we tried to analyze the different situations.

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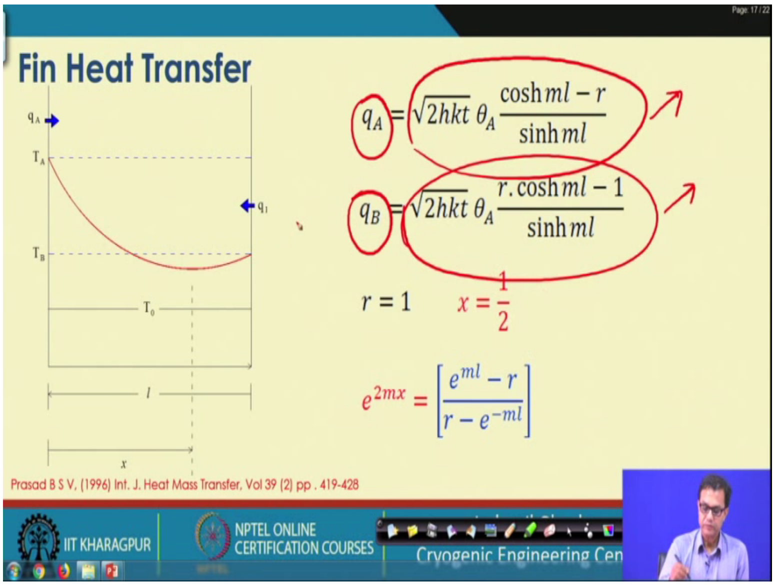

So, in that context we have seen that we have first of all we have evaluated this q A and q B and we have seen these are the expressions for q A and q B which is important and we will look into this again when we talk about the other I mean configurations or possibilities or the of the temperature of the plate and the fluid.

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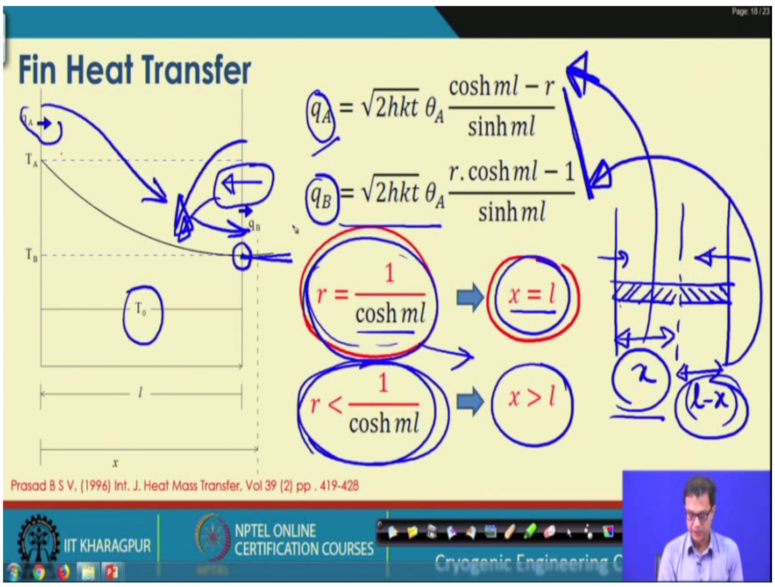

So, now, what is the other possibility that we have already talked about in the last class? It was such that in case this r equals to exactly 1 by cos hyperbolic m l. So, in that situation we find that the extremum is occurring at the fin base of the q B. And now if we find that if this r equals to one by cos hyperbolic m l, it is not only giving a the extremum at the fin base of the other of the other plate; we are also finding that this q B will change its I mean direction.

So, now, this heat is no longer I mean earlier we have seen that heat was coming from the plate to the fluid. I mean since that that is expected because the plate temperature is higher than the fluid temperature. So, this fluid is supposed to give this plate is supposed to give heat to the fluid. But here we are finding that if it is such that the situation is such that 1 by I mean r becomes so, an r becomes 1 by cos hyperbolic m l it may. So, happen that r is 1 by cos hyperbolic m l, you will find that that x is not only l, but this fluid is this plate is no longer giving heat to the fluid rather you know this heat q A has already been you know which is having origin from this fin base it is coming to the fin base b. So, this is nothing, but what we have said that transverse conduction of heat and sometimes we have to I mean, face this situation depending on the different fluid temperatures temperature and the plate temperature.

So, now here also we find that if we analyze this q A and q B I mean how did we do that ? You may remember that q A has been obtained you know by interrogating or calculating the heat transfer within this length x and this l minus x and we have done the interrogation over this part and this part. Separately we have calculated and that is q A is from here and q B was from here. And according to for this even if it is r equals to 1 by cos hyperbolic m l, we will find that for this length x, we have q A amount of heat equals to this much and l minus x over this length l minus x. So, we will have this heat transfer q B.

Now, what happens if r is still smaller? This, cos hyperbolic m l is more than 1 and we find that this becomes you know it is exactly I mean less than 1. And this is in this situation what happens this x becomes x goes beyond the physical boundary of this physical I mean plate.

So, in that case also you will find that these two equations are taking care of that even if there is actual conduction I mean sorry transverse conduction of heat it is automatically being taken care. Now, there may be another situation what we have talked in the last class.

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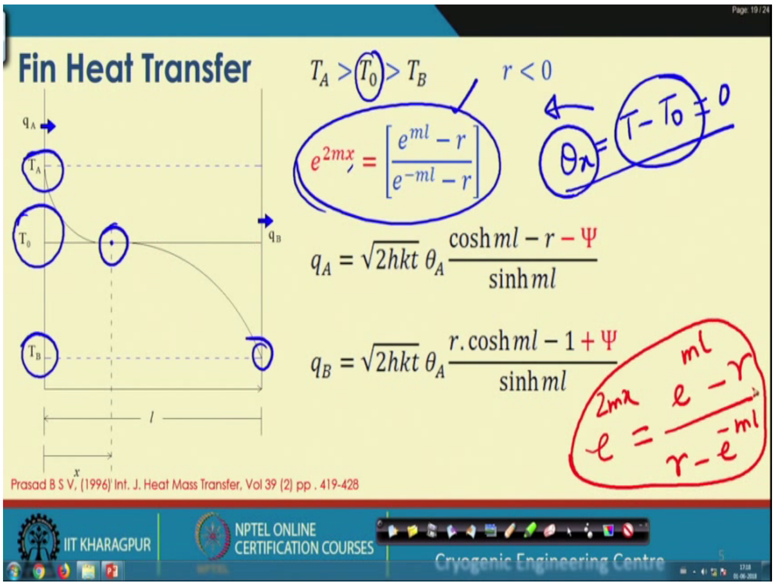

So, here this is T A this plate is at a temperature T B are corresponding to this one and the fluid temperature is intermediate to T A and T B.

So, if we have a situation where this T 0 is at the middle of T A and T B. So, we will find some position where this temperature difference of this theta x which was T minus T 0. So, this will vanish at some point intermediate and that will be equal to say 0 or as it is as we have said it is vanishing.

So, if we put this condition and this is theta x already we have obtained the fin equation and if we equate it to 0, we will have a correlation I mean relation for the x and the I mean length at which it is having theta x equals to 0 is given by this relation. So, if it is; so, then you will find this is not exactly what we have obtained earlier. The earlier expression if you remember it will be something like this. It was e to the power 2mx is equals to e to the power m l minus r divided by r minus e to the power minus m l. This was the expression when the minima was lying in between or beyond the physical boundary of the fin.

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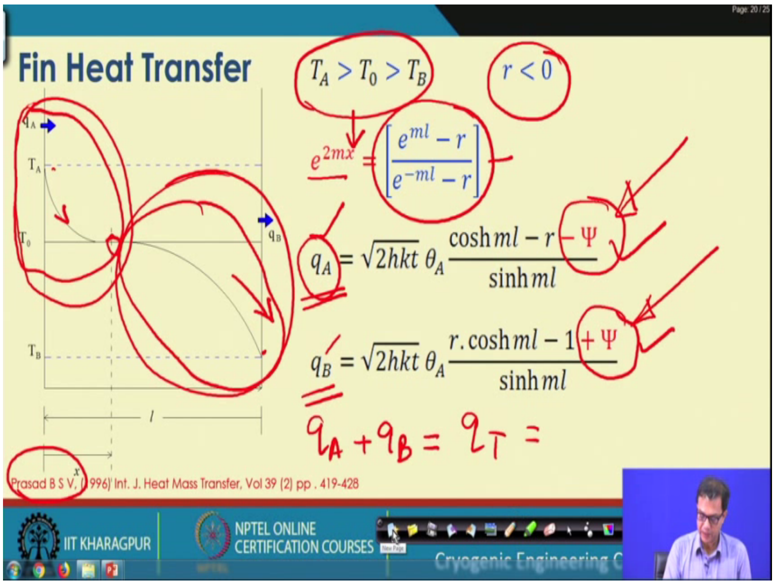

But in contrast to that here what we have is some relation this is like this. So, here the corresponding the x at which you know this fin I mean, from here to here the heat is coming from this wall to the fluid and between this section the fluid to this wall the heat is coming.

So, we have two distinct region to calculate this q A and q B and here the x is given by this relation. So, this is not exactly similar to the earlier situation and its reflection will be there in the amount of q A that is you know there will be a term like minus psi and plus psi. So, this terms will be added to this. So, that you find that this q A plus q B that is remains the same as earlier because these two terms will cancel each other when we combine together that or q T that remains same but these individual heat transfer q A the plate A and from the plate B. So, that becomes slightly different and. In fact, we need both q A and q B separately to estimate the plate temperature. So, it is important.

Now, it has become case dependent that if the r equals to less than 0 or T 0 is intermediate to T A and T B, we find that we have to keep it in mind that to take care of the transverse heat conduction from one plate to the other plate. There has to be some kind of additional parameter psi that is to be added or subtracted with the appropriate one with the appropriate q A or q B. So, this is I mean an additional task the, but have to overcome that one has also been suggested by B S V Prasad and we will find that there is an alternative way of calculating this q A and q B.

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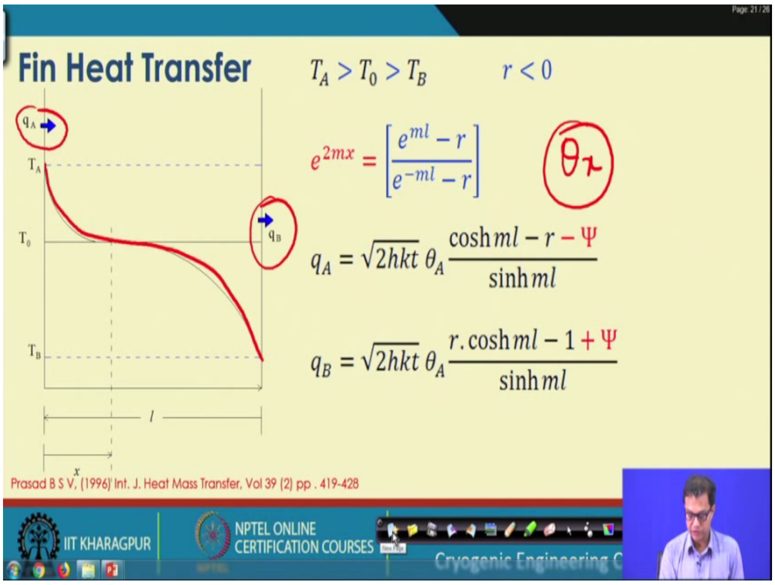

So, we will try that one and how is that I mean, since we know the temperature profile of this fin I mean fin temperature profile will already we have calculated that is the theta x we have solved. So, we know the temperature profile. So, another way of calculating the q A or q B is you know from the fin equation and we will look into that in the next slide.

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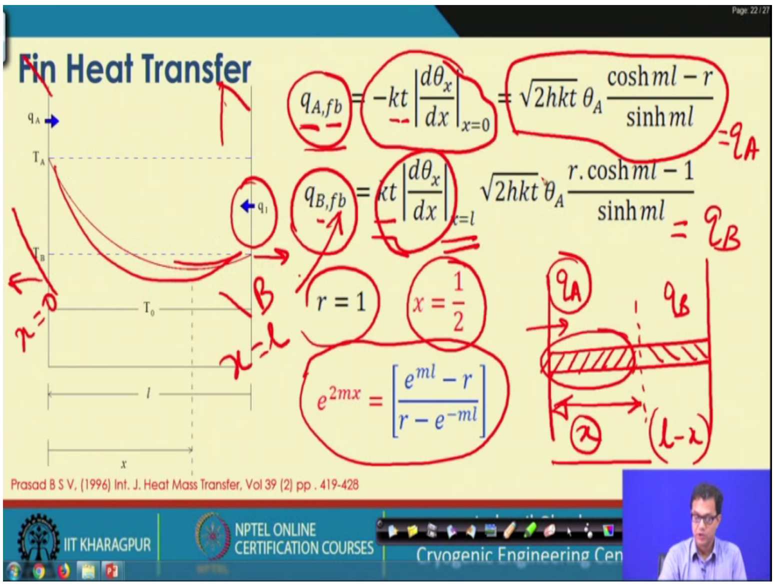

So, we find that we can find out I mean all the situations we will see one by one and we will find that this heat transferred from the q A earlier we were telling it as q A only. Now we are telling it as q A fin base so; that means, earlier we were trying to estimate the heat transfer and where we have I mean tried to locate where is the minima. So, that is the distance x, if it is located at a distance x, then we intricate and I mean find out q A over this length.

Now, this q B we have calculated over this length l minus x. But here, how we are we estimating this q A and how we are differentiating it? It is differentiated as q A fin ways and that is equals to minus kt dt dx please mind that this kt, it is you know only we have not considered the other dimension or the this length dimension of the fin. This is basically heat transfer per unit length.

So, this is kt into d theta x dx at a location x equals to 0 and we can try to find out this q B at the fin base of b and we designate it as q B fin base that is why f b and that is similar to kt d theta dx at x equals to l. So, this is x equals to l. So, here exactly we will find that this expression is nothing, but the expression q A and this expression is nothing, but q B. But why did we do that? I mean is there any special significance that q A f b and q A are equal. We it is expected that the fin base heat transfer and the heat transfer taking place through this fin is supposed to be the same.

But we will find the difference later on. So, here in this case as such when r is equals to 1 and x is half or for other relations where we know, the fin extrema is located within this region or it is slightly on this side or on this side this expression is remaining valid.

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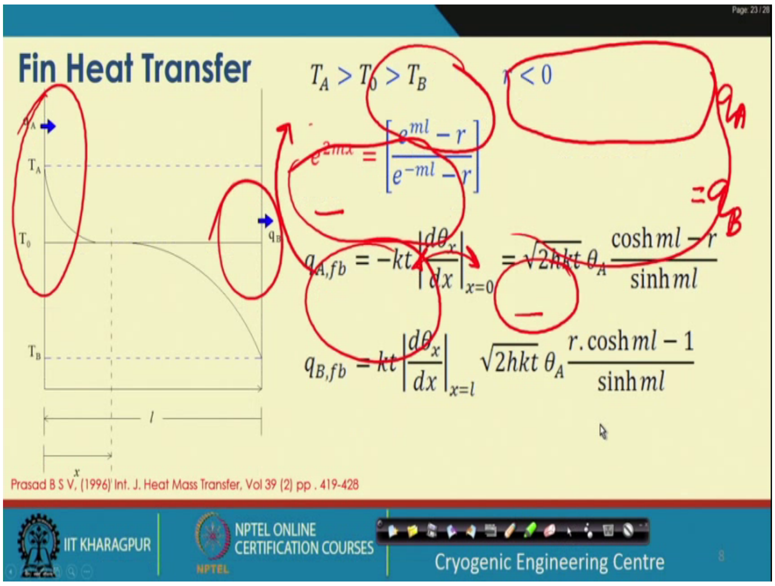

So, now, let us go look into the other situation where as we are saying that it is beyond the physical boundary of the fin. So, corresponding to that situation we have r equals to 1 by cos hyperbolic ml and this also we have analyzed earlier. Now in this situation we know that at x equals to l corresponding to r equals to 1 by cos hyperbolic ml that gives the minima at x equals to l. But if we find the heat transfer at the fin base expression like you know minus kt dt dx at x equals to 0 that is at x equals to 0 on this side. And fin base at d theta x dx at x equals to l on this side that will be this one. And you see again here this q A is equals to q A fin base and q B is equal to q B fin base.

So, here also we do not find any difference between the fin base heat transfer or the heat transfer from the fin length calculation. But what is about the situation when we go to the other I mean, when we go to the next case where we find that this temperature is a intermediate to this plate temperature T A and T B.

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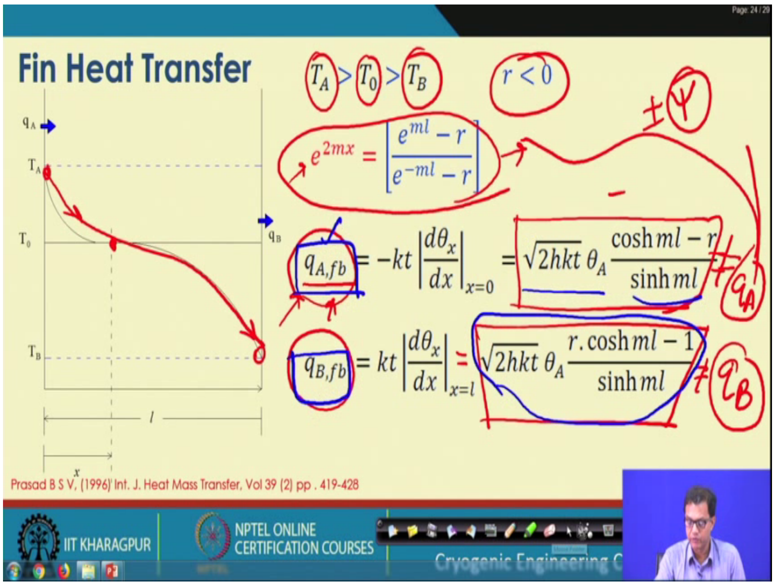

And in this situation the r value is less than 0 and this is corresponding to the situation where x is at some intermediate point and giving I mean co related to this temperature ratio I mean ratio r by this relation.

Here you see this fin base q A and fin base q B. They are not similar to they are not similar to the q A and the q B expression that we have obtained based on the length. Because this length was different you know the x that we have estimated for this q A was taken from this x and that is why our q A and q B expression was either having a plus minus psi term. But here in this case since this is completely based on the fin base heat transfer and that is automatically, this expression is automatically taking care of the actual I mean sorry transverse conduction of heat taking place from one plate to the other plate.

So that means, while analyzing the plate fin type heat exchangers I mean, when there would be number of multiple number of multiple fluid streams or multiple adding in multiple layers, what we need to look into I mean, we cannot separately keep notice whether you know there is a kind of transverse conduction or not. Because it is not possible at some intermediate location what is happening in between the plate and the fluid temperature.

So, automatically the equation should be framed such a way that that effect is taken care. So, for that we find that if we are taking this q A fin base expression, then it is always taking care of that heat transfer I mean that is this is equals to this. And that is taking care of the transverse conduction from one fin to the other fin without bypassing the fluid or we call it bypass heat transfer. This is very frequently I mean it is a common phenomena in multi stream plate fin heat exchanger. So, that will be taken care automatically by this expression.

So, we understand that this particular expression, we will be using for doing the multi stream plate fin heat exchanger analysis. So, but expression wise it looks this for the q fin q fin base and this is the expression what we get for the q A.

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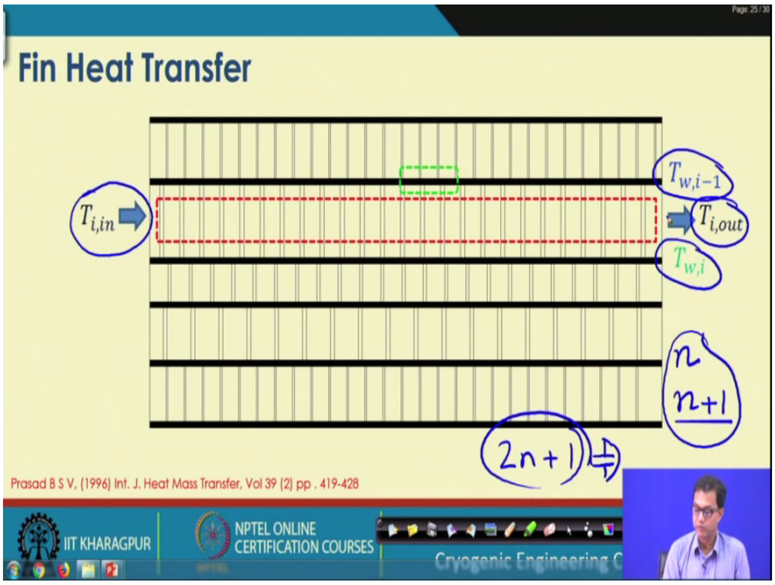

So, now, based on this information we will again go back to our that where will have started with where in a in a in a generalized I mean heat exchanger configuration where there may be I mean n number of fluid streams and n plus 1 number of wall temperatures. Then we have these in a generalized condition we have this is the inlet temperature and this is the outlet temperature of fluid stream surrounded by this two walls T w, i and T w, i minus 1

Now, here in this context we will write that we have already told that if there are n number of fluid streams, we have n number one number of wall and we have altogether two n plus 1 number of equations and unknowns are there. So, we have to frame those two n plus 1 number of equations.

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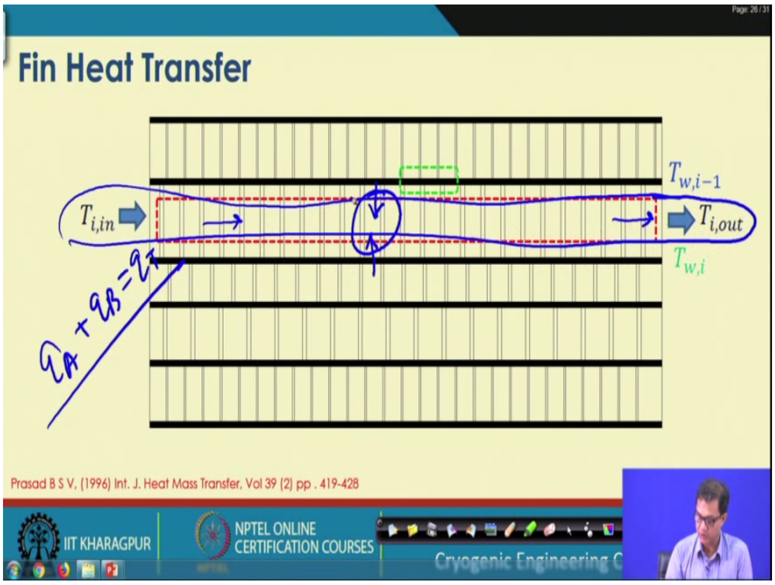

So, first of all what we will like to do is that we will take a energy balance over this i'th number of channel where this is entered in at T in and moving out at T out. And what are the heat transfer that is going to take place? So, some amount of heat will be coming from here, some amount of the heat will be coming from this and the fluid will be receiving that amount of heat and it will come out.

So, this together combined heat q A plus q B that becomes q T and that we have seen as constant. And we will take care of I mean this if we take an energy balance we will look into that in details.

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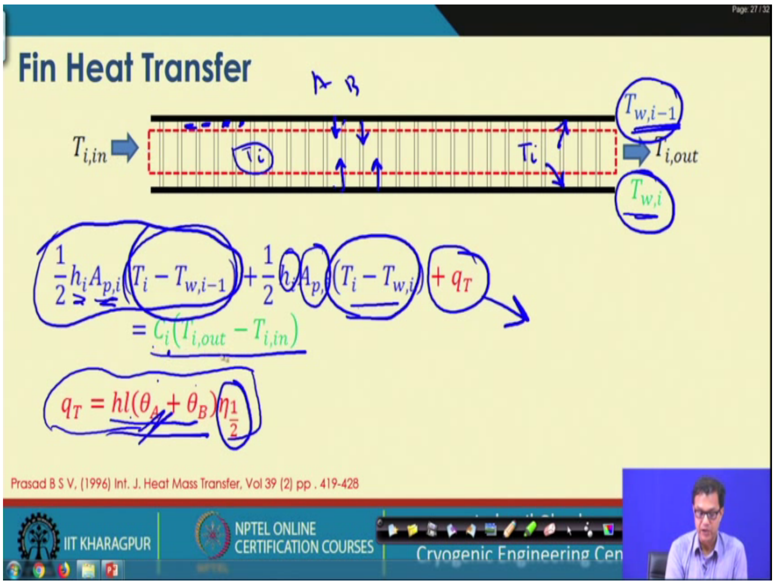

So, in the next slide we will now go to this that when we do the heat transfer or I mean energy balance this for the i th fluid stream we find that the heat is getting transferred from the primary surface as well as heat is coming from the fin. So, both for this type of fin and both I mean for this fin also either it is A and B and the separate I mean this primary surface area. So, this is the primary heat transfer surface area contribution from the primary heat transfer surface area and of the i th plate. This is h i A primary i and this is the difference in temperature. This is say, if we consider it to be T i which is an average between T in and T out and then it is T i minus T w, i and then we have this is for the T w, i.

So, this is this temperature difference between this fluid and this is this one the other part T w, i minus T w sorry T w, i and T w T i and the corresponding surface area and the heat transfer co efficient. And then we have q T as we have said that q T is nothing, but q A plus q B and this is eta half. And that is just you know in all the cases whether it is having transverse heat conduction or not, we can write this total heat transfer to be h into fin length and then theta A and theta B and eta half.

So, this theta A is basically T w, i minus 1 and T i and theta B will be T w, i minus T i. So, that is the energy balance we will have for the fluid stream and that will be equal to the heat capacity the mass flow rate multiplied by the C T and T out minus T in. So, that will write or give the total energy balanced equation for this i th fluid stream.

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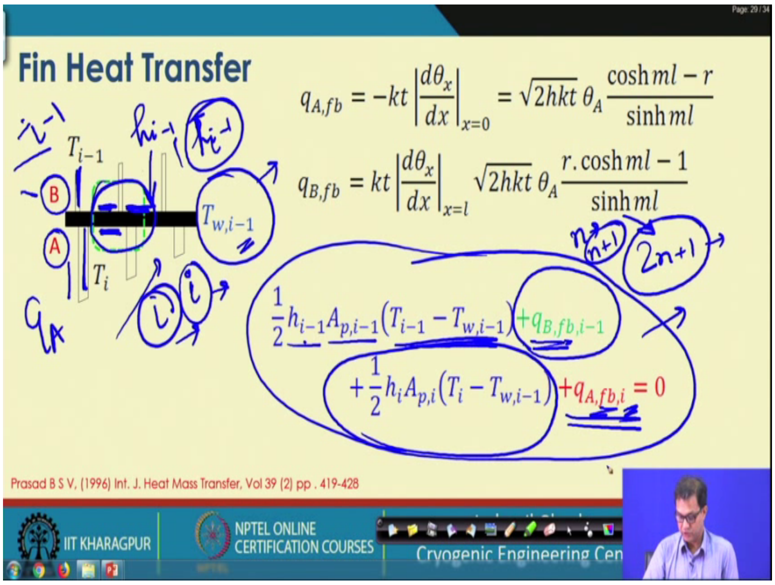

Now we need to look into the heat balance on the wall T w, i minus 1. So, now we have the contribution from the primary surface and this primary surface on the B side fin and primary surface on the A side fin. We designate this fin as a and this is as B. So, now we have this overall energy balance if we take around this plate, we find that half of h i minus 1. I mean that is h i minus this is i minus T i minus 1 th fluid and its temperature is T i minus 1. This side is fluid is i minus 1 and this is i i'th fluid.

So, we have the heat transferred between this surface, I mean this primary surface area on the i minus 1 th side. Heat transfer surface area in the heat transfer coefficient temperature difference between the fluid and the wall and then the primary heat transferred you know contribution on the wall side. So, and then we have this contribution from the B side fin and the heat transfer from the fin base of the i th minus I mean i minus 1 th side fluid. And then we have the heat transfer q A for the A side and the fin base as we here decided on the i th side fluid. So, that is equals to 0.

So, like that we have to this is the I mean equation for the wall of i minus 1 th 1 and this is the heat transfer we have also looked for the I mean already we have figured out or estimated the heat transfer for the i th fluid. So, like that we will have then 2 n plus 1 number of fluid streams and I mean sorry n number of fluid streams and n plus 1 number of walls.

So, all together 2 n plus n number of such equations we would be able to frame and those becomes you know algebraic equation, then we need to solve those equations to find out the wall temperature as well as the fluid exit temperature. And that is exactly what we are we were looking for to simulate the plate fin type heat exchangers.

So, once we know the exit temperatures of the fluid streams and we would be able to calculate what is the heat duty of this bridged aluminum plate fin type heat exchangers.

So, thank you for your attention.