Heat Exchangers: Fundamentals and Design Analysis
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Lecture – 33
Plate fin heat exchanger : Multistream (Contd.)

Your welcome to this lecture, this is in continuation to our earlier discussion, where we are trying to analyze the plate fin heat exchanger are multiple where you know multi stream are handled and we talked about the fin temperature profile connecting between the 2 surfaces are connecting between the plates at temperature TA and TB and we said that the temperature profile between the plates when known to us.

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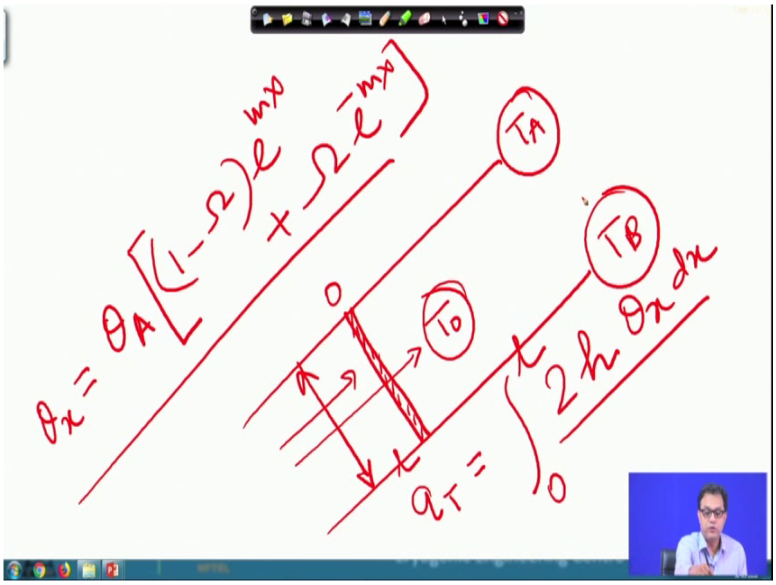

And we have derived some kind of we have obtained some kind of temperature difference or temperature profile theta x is equals to theta A multiplied by omega into to the power x, it was of this nature 1 minus omega into e to the power m x plus omega into e to the power minus m x, where omega is having a relation between r and sign hyperbolic m l.

And then you also trying to find out the amount of heat getting transferred. So, this is a T A and T B, we have the temperature profile for this fin connecting between T A and T B, if the fluid flowing on top of this, I mean fin is of temperature T 0 then we have will able to find out the total amount of heat getting transferred as function, I mean like this 2 h multiplied by theta x into d x and this interrogation was done over the length 0 to l this is 0 and this is the length of the fin l.

So, from here we got the total amount of heat transfer, when the edges in fin stream are T A and T B.

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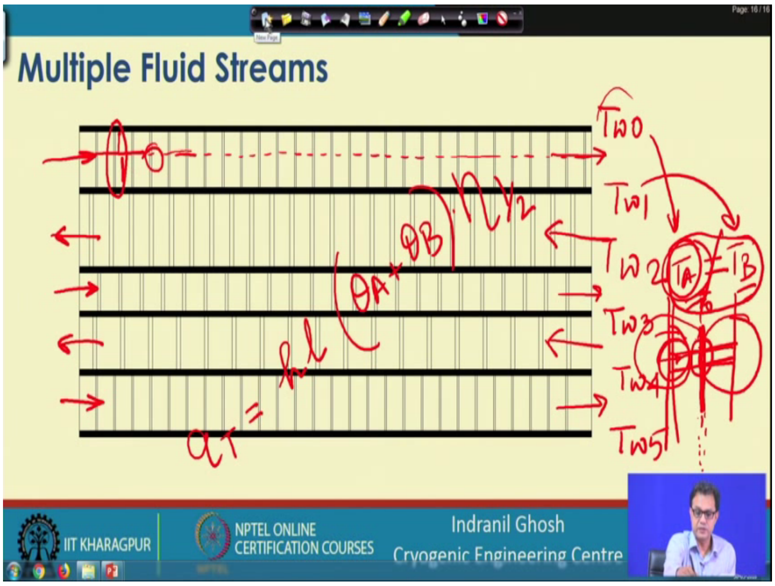

Now this is exactly what we are looking for in case of a multiple fluid stream if you look at that particular geometric. So, if it is the 5 fluid stream heat exchanger say, we have T w 0, then we have T w 1 then we T w 2 these are the wall temperatures separating wall temperatures, then we have T w 4 and T w 5 and we have for 5 fluid stream depending on their configurations may be you know there are encounter current to each other or it may be parallel counter combination.

So, that will be automatically taken into account, if we are able to write down fin equations and the plate equation separately. So, if we now try to find out the heat transfer are taking place to one single fin that is what exactly, we have told that this is the q T amount of heat getting transferred and that was given as you know as we have estimated it earlier that, it would come as h into l into theta A plus theta B and 10 hyperbolic, I am sorry eta of fin half.

So, as if half of the fin is at as do this plate, this plate and half of the fin is attached to this plate and as if you know there is a kind of adiabatic temperature profile, I mean condition prevailing at the fin middle. So, this is like this is T A T w T A and T B and these are actually nothing, but T w 0 and T w T w 1 this is in this now in place, they are T w in this now in place, we have talked about T A T B now if that is connected by 2 fins and we can show that when this T A equals to T B, we will have a minima at the or a maxima at the middle of this fin there by, I mean depending on the temperature of the fluid flowing on top of it, we will find that the heat either is flowing from this wall to the fluid or fluid to the wall, but there is a minima at the middle of this fin.

So; that means, some kind of adiabatic condition has been state at the middle of it. So, half of the fin is as if attached to this plate and half of the fin is attached as it to this plate, but it may not be the situation for every case, where T A and T B are not the same. So, depending on the T A T B, we may find various kind of configurations, where we will find that the extremum or this minima or the maxima is lying within the fin outside the fin and of course, it may be you know there is no minima at all.

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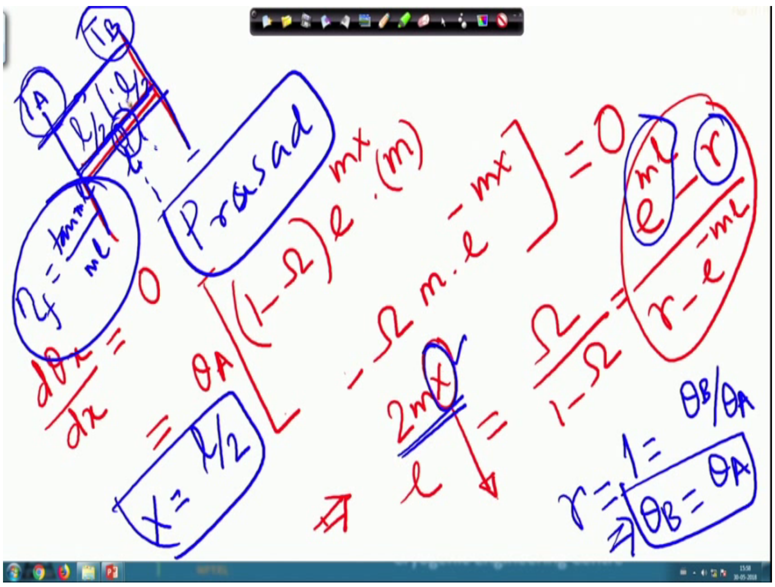

So, there may be situation. So, in those cases, how are you going to take account of those situations that we want to discuss in this class and we will start with the basically the fin equation again. So, now, if you look at we will find that the fin equation we have already talked about now, if we want to look at the maxima or the minima in that fin equation.

We have to take an, I mean the derivative of that one that equation fin equation and equated to 0 and then we can determine, what is the position of the maxima or the minima? So, we have theta x known to us, what we do is that, we differentiated with respect to x and then we equated to 0. So, this will lead to the situation, that theta A and 1 minus omega and e to the power m x. there would be another m here, then we will have minus omega into m into e to the power minus m x and that has to be equated to 0.

So, this will lead if you do is little algebra you will find that e to the power 2 m x is equals to omega by 1 minus omega and that will be you know little bit of algebra, you will find that this is coming nothing, but e to the power m l minus r by r minus e to the power minus m l. So, this will till the exact location of the extremum that is occurring in terms of this fin connected between the 2 plates.

As we have told that this has been taken from the paper by it is available with the Prasad and it has been taken from introductional journal of heat and must transferred volume 39 issued to of 1996 or otherwise you can also refer to the book of Sha and Sekulic, there also this analysis is available.

So, if we have this fin connecting between this two wall which are at say, T A T B and they are connected by a fin of length l and we are trying to find out what is the extremum. That is located depending on the as you can understanding that this r and e to the power m l and that ratio will determine, where exactly this x is getting located particularly, if we say that r is equals to 1, that is equals to e to the I mean theta B by theta A that is what is r and; that means, theta B is equals to theta A, you will find that, this x is coming exactly at l by 2 so; that means, we find that, when this temperature of this 2 temperatures are equal we have an extremum located at the middle and; that means, we have l by 2 fin attached to this plate and l by 2 fin attached to this plate. So, if that is. So, if there is the kind of adiabatic condition at the middle of that fin or anywhere at the fin.

We can assume that you know there exist a kind of adiabatic condition or at the fin tip and we can you know easily use that eta f is equals to 10 hyperbolic m l by m l that is was this the fin efficiency provided, we know the exact location of that extremum, but as we have said that this is the particular this corresponds to a particular situation, where this half of this fin is attached to this or half fin idealization as it is called.

Then it is possible only when this T A T B are equal, but in reality you may find that there are several situations, where T A not equals to T B and in those cases, we have to go for a different I mean, how do we estimate? One way of doing, I mean estimating the thing is that, we try to find out exactly what is the up to which that extremum is located, where it is exactly located.

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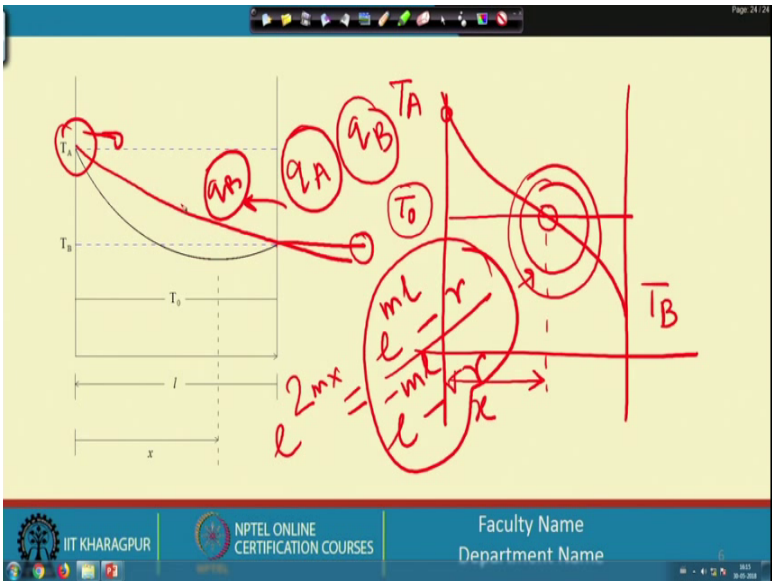

And if we look at to this diagram we may be it may be a situation. Where you will find that, this is like T A, this is T A and this end is that T B, this is this temperature is T B and the fluid temperature flowing on top of that fin is that T 0 and it is minima is somewhere at this point x, where the length is total length is l. So, now, in this case we have the extremum located at this point and we may try to find out the position x and that will be given by e to the power 2 m x is equals to e to the power m l minus r by r minus e to power minus m l. So, from there from depending on the temperature and this temperature ratio theta A by theta B and the value of m and value of l, we would be able to find out the position x, where the exits I mean extremism is located. So, we have x and then l minus x these are the 2 lengths over which we have to calculated the heat transfer. So, how do you do that? So, we will be able to find out to intrigues, where we should to able to calculate it over this 2 lengths, one is say if we take it tell it as q A, that is the heat transfer taking place from this fin.

And this is from this side, this plate to this fluid and from this plate to this fluid is this is q B, if we try to find out and this is nothing, but you know we have already try to find out that one earlier and this would become twice x into h into theta A and then eta A as, because we have assumed that or not assumed means, we have told that this, there is the extrema at this point so; that means, there is an adiabatic plane at this point.

So, we on we know that we have an adiabatic plane or at this point. So, fin tip which is the fin tip? Where is the fin tip? The at distance location x and what is the temperature difference here up to this point? Theta A then, what is the heat transfer coefficient is h and this eta A is taking care of that fin connected between this wall and this length, this is the adiabatic condition prevailing at this location. So, this is at T A and this fin temperature profile, we have already known to us and what is the length? This entire fin is not at T A. So, what is taking care of that one that we have you see you have not returned it as theta x or variable length.

So, here we have put it as theta A as if this entire fin is at the fin rest temperature T A. So, we are multiplying it with the fin effectiveness or fin I mean fin efficiency whether eta A and multiplied by the heat transfer coefficient and this is the length of the fin. So, by that way you would able to calculate q A, similarly we would be able to calculate q B and you will find q B would be twice l minus x into h into theta B and then you have eta B.

So, this is what is the amount of heat getting transferred from the wall B to the fluid so, this is 2 l minus x into h into theta B and eta B, here we are finding that this remaining length of the fin l minus x is having you know as if there is heat transfer coefficient h between the heat fin and the fluid and the fin is as if the entire fin is that fin based temperature of theta B, which is not actually the case, but this fin efficiency is taking care of that one. So, that you know this entire fin base is assume to be at theta B. So, this is q A and q B.

Now, if you put the values of q A and q B, you will find that there q A is coming as root over 2 h k t and theta a and then you have cos hyperbolic m l minus r divided by sin hyperbolic m l. Similarly q B would be, it would be root over 2 h by k t 2 h k t and then it have theta A then multiplied by r into cos hyperbolic m l minus 1 divided by sin hyperbolic 1 sin hyperbolic m l.

So, this is what is q B and this is q A. So, that gives you the temperature, I mean the heat getting transferred from this plate to the fluid and this plate to the fluid, this is q B and this is q A. So, once we know this q A and q B, we would be able to find out many things. Now as, we have told that if may not be always this situation that, this fluid temperature you cannot expect that it will be in this configuration then may be, other situations like this fluid temperature may be just the wall temperatures.

I mean, this is here, this is here and this may be at this point so; that means, this T 0 the fluid temperature may be hotter than the wall temperatures. So, in that case, what will happen? The heat will be getting transferred from the wall from the fluid to the wall. So, only thing that we need not worry about so, what we have to do is that we have to just you know this automatically, that will be taken into account.

Once we are able to I mean, once we are able to once we you know frame the equations, it will be automatically taken into account, I mean this q B or q A it will automatically take into account of this temperature, but the situation will really be different I mean all these things depend on that equation, where we have said that e to the power m x is equals to e to the power m l minus r divided by r minus e to the power m l and this r will determined, the position of the x and that r is basically nothing, but the theta B by theta A.

So, if r is such that you know theta A or theta B is such that this location of this x is going the physical boundary of this fin. It may lead to a situation that x is no longer the extremum is no longer existing within, this physical boundary of the fin. So, which means that at exactly, you know r is equals to if we put 1 by cos hyperbolic m l, you know exactly at 1 by cos hyperbolic m l, when this is the value of r then, we will find that this extremum is exactly at this location and when we have values are you know, when the value is r less than 1 by cos hyperbolic m l then it will indicate I mean let us look into that situation one by one.

So, here we have situation when we have say, exactly r is equals to 1 by cos hyperbolic m l. So, at that point the extremum is located at this point and we will find that exactly at that point. The situation will be search that, you know it is continually from this end to this end and for values r less than 1 by cos hyperbolic m l, you will find that this is going beyond the physical boundary of the fin. So here, in this case, we will have the fin extremum located at this point. So, is it making any kind of changes in the fin equations? Or in what does if physically mean? You will find that it is meaning that how is the fluid configuration here in this case? The fluid is at a temperature T 0, this wall is at a temperature T A.

Now, the heat is getting transfer from wall to this fluid and also T B is higher than the fluid temperature. So, the heat is getting transferred from this wall to the fluid. So, both the wall are giving fluid to this temperature in this configuration, now if we look into that q A and q B expression and if we look into it is configuration, I mean it is expression and there if we look into situation, where r is less than cos hyperbolic m l under this condition, you will find that you know in this configuration, what you will find is that in this situation we will find that, it is q A, it is positive or I mean heat is coming from this wall to the fluid, but this will have a change in the values. So, here instead of you know heat going from the wall to the fin, you will find that, the fin is you know giving heat, the fluid is giving heat to the wall. So, this is in contrast to the earlier I mean situation and it will lead to a kind of direct heat transfer from this wall to this wall and by passing the fluid stream and we often call it as the transfers heat conduction or we have to in I mean this has to be either taken care of separately or we have to define a fin efficiency to take care of this by pass heat transfer. This is often also called by pass heat transfer as because; the heat is as if by passing from one plate to the other and by passing the fluid. So, as if directly this heat is q A is going from the fluid to the other separating plate.

So, now if we look in to this situation, there may be another situation, where this fluid stream is such that you know this is like this, where this is T A and this is at T B and the fluid temperature, I mean sorry the intermediary fluid, which is the fluid flowing the convective fluid flowing over the fin, you will have a temperature T 0, which is intermediate at to this T A and T B. So, in this configuration also we will find that the fin temperature is at the I mean there is no extremum located at this point.

So, at some point or I mean in between say, we have said that e to the power 2 m x is equals to that 1 is equals to e to the power m l minus r by r minus e to the power minus m l, that is what we have found, what that you know fin equation where we got an extremum, but for this situation where I mean at some point, you know this fin temperature profile will have same temperature as that of T 0. So here, in this case that will correspond to a configuration where we will find that this situation is similar to that e to the power 2 m x.

So, at some point theta I mean at some point for this situation, where we have talking about see here, this is T A and this is say T B and this is where we have T0. So, at some point x, we will find that x, this will correspond to a situation, where the theta x itself will manages I mean, because it will be similar to the T 0 and now in this case, this will be e to the power 2 m x and this will lead to the situation e to the power m l minus r divided by e to the power minus m l minus r.

So, this is different from the previous situation and this will also corresponding to a different situation, which giving thus different kind of heat transfer and particularly that will lead to I mean all the earlier cases also, where we have talked about that when it the physical, it lies the maxima or the minima lies beyond the physical boundary of the condition or when you know the situation is such that the fluid temperature is in between T A and T B that kind of situation.

How do it taken into account in real situation, we which we do not know, I mean when there multiple fluid streams, I mean in different layers and there may be different situations analyzing and during that time the analysis of the fin analysis should be such that, there will be automatically taken into account of those situations. So, all though we have estimated q A and q B, now we will look for whether it is sufficient to take account of the heat transfer from this wall to the fluid or from this side to this is adequate.

Or we have to look for something else, which is the taking account of the actual sorry the transfers heat conduction of the bypass heat transfer and then we will be able to we should not be worried about whether it is already taken care or not in the analysis. So, that we will look in the next class and.

Thank you for your attention