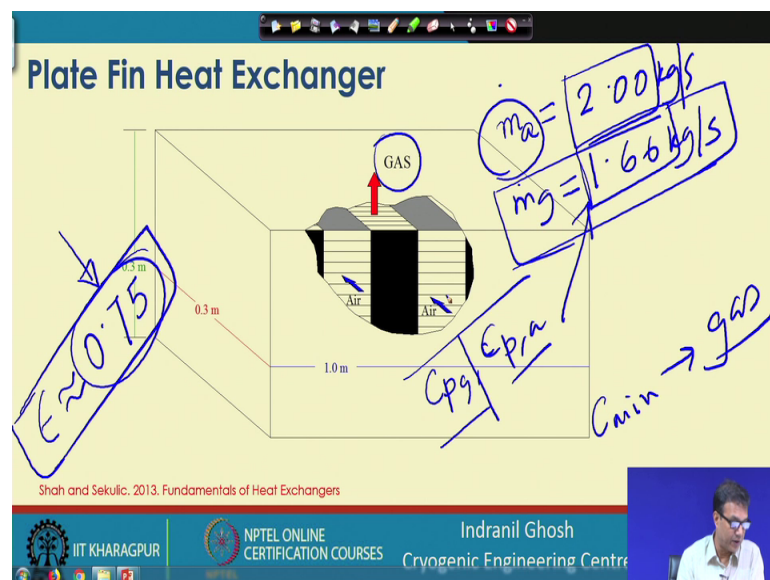


Heat Exchangers: Fundamentals and Design Analysis
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Lecture – 31
Plate fin heat exchanger : Numerical (Contd.)

You are welcome to this lecture. We are trying to solve the numerical problem related to Plate fin type heat exchanger; this rating problem or the performance evaluation of the heat exchanger. So, until the last class we have tried to estimate the heat transfer surface area, and also we have evaluated flow rate mass flow rate corresponding to the airside and the gas side.

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So now if we look at we will find that we have estimated the mass flow rate of air, and that is equals to 2.0 kg per second. And then we had the corresponding value for the gas side was 1.66 kg per second. So, this mass flow rate of air and the mass flow rate of the gas when we multiply it with the corresponding the C_p . C_p values we will be able to find out the C_{min} or C_{max} or determine the C_{min} or C_{max} . Here at this moment we will, we do not know exactly what is the exit temperature of the air or what is the exit temperature of this gas.

So, that is what we are supposed to find out, and as we have seen in the earlier cases also, we have to make an rough estimation of this gas exit or the air exit so that we can

take the fluid properties, or evaluate the fluid properties at the mean temperature. So, should we go for any wild guess or should we go for some kind of I mean reasonable estimate of the mass flow rate of the gas; that will determine I mean the how close we are to the actual result.

So, later on we may have to you know, look for an iterative solution if we are making very wrong guess about the exit temperatures. So now, we can understand that this mass flow rate of air is slightly higher, and then the mass flow rate of the gas. So, if we assume that they are not having much dependence of these C_p of the gas or the air or the C_p of gas or C_p of air their not very different or there not really I mean dependent on the temperature; in that case, we can say that the minimum capacity fluid or C_{min} will correspond to the gas.

And accordingly we have to we can try to find out the fluid properties based on than assumption that, let us assume that the heat exchanger effectiveness is 0.75. So, for if this cross flow type heat exchanger, this is the assumption we have we are starting with. So, this assumption will lead to the estimate of the exit temperature on the gas side as well as for the air side. And once we know the exit temperature of the gas and the air, we would be able to find out the mean temperature at which we have to evaluate the fluid properties. And then once we know the fluid properties, we will try to calculate the Reynolds number and the other fluid I mean other properties.

So now, if we assume epsilon to be 0.75, we would be able to correlate them with the exit temperature.

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$$\epsilon = 0.75 = \frac{(T_{g,i} - T_{g,o})}{(T_{g,i} - T_{a,i})}$$

$$\Rightarrow T_{g,o} = 375^\circ\text{C}$$

$$\epsilon = 0.75 = \frac{(T_{a,o} - T_{a,i})}{(T_{g,i} - T_{a,i})}$$

$$\Rightarrow T_{a,o} = 635.8^\circ\text{C}$$

So, epsilon is equals to 0.75, and since we have assumed similar specific heat for the air and the gas we know that the gas is the minimum capacity fluid. So, we can say that $T_{g,o}$ of the T sorry T gas of the inlet minus $T_{g,o}$ at the exit. This is the minimum capacity fluid, and this is $T_{g,i}$ of entry minus $T_{a,i}$ at the entry this should be the epsilon or the heat exchanger effectiveness. So, from here what we know is the entry temperature of the gas, this gas entry and this air entry temperature is known.

So, from this relation we can calculate the gas exit temperature, and this will come out to be 375 degree centigrade. Now similarly if we look into the temperature corresponding to the air exit, so, you will find that epsilon that is equals to 0.75 and that is equals to the $T_{a,o}$ minus $T_{a,i}$ into mass flow rate of the air divided by the $T_{g,i}$ minus $T_{a,i}$ that is the maximum difference in temperature multiplied by the minimum capacity fluid.

And we have assume the C_p of the air and C_p of m_g are the same. And so, this is here in this relation we will find that we know the air entry $T_{a,i}$ gas entry $T_{g,i}$ and m_a by m_g ratio is already known to us. So, we would be able to find out $T_{a,o}$ air exit temperature and this will come out to be 635.8 degree centigrade. So, we have now an idea about the air and gas exit temperatures so, we can now calculate the mean temperatures.

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The image shows handwritten calculations on a whiteboard. On the left, a blue scribble contains the labels $T_{g,m}$ and $T_{a,m}$. To the right, the following calculations are written:

$$T_{g,m} = \frac{900 + 375}{2} = 637.5^\circ\text{C}$$
$$T_{a,m} = \frac{200 + 635.8}{2} = 417.9^\circ\text{C}$$

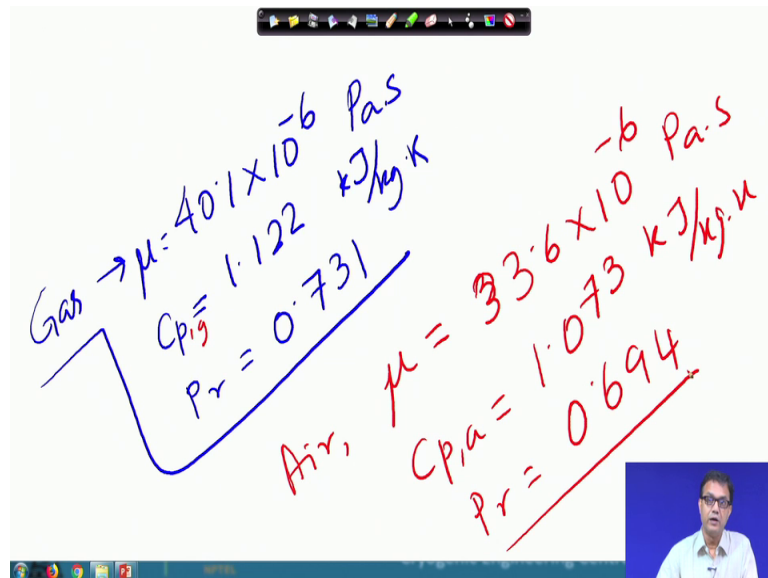
To the right of these calculations, the following fluid properties are listed with arrows pointing to the right:

$$\mu \rightarrow$$
$$C_p \rightarrow$$
$$Pr \rightarrow$$

And the gas side mean temperature will now come out to be $T_{g,m}$; that is equals to 900 plus 375 divided by 2, this is arithmetic mean 0.5 degree centigrade.

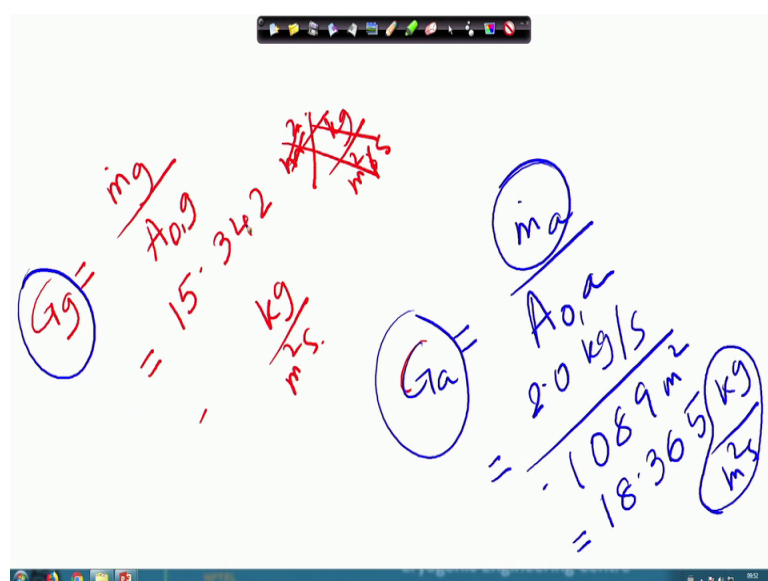
And for the air side we have the mean temperature as 200, it was it is entry temperature and 635.8 is the estimated exit temperature based on the epsilon equals to 0.75. So, that gives you 417.9 degree centigrade as the average temperature. So now, based on this temperature mean temperature, we would be able to calculate the fluid properties of the I mean, different fluid properties like what are the fluid properties we need, then μ we need, the C_p we need, then we need, the Prandtl number and Prandtl number; obviously. These are the parameters what we will now require for.

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And for gas we have the mu equals to 40.1 into 10 to the power of minus 6 Pascal second is the unit. And then we have this is the mu. Then we have C p that is equals to 1.122 kilo joule per kg Kelvin. Then we have P r is equals to 0.731, this is for the gas. And for the similar parameters for the air, we have mu is equals to 30, I am sorry 33.6 into 10 to the power minus 6 Pascal second . Then we have C p of air this is C p of gas so, C p of air is equals to 1.073 kilo joule per kg Kelvin. And then we have P r for this one is 0.694. So, these are the values known for the gas and the air. So, based on this value, we would be able to now calculate the gas velocity.

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What is gas velocity for the G_g or mass velocity for the gas side? So, this is equals to $m \dot{g}$ by air free flow, area for the gas sides. So, both the parameters are known so, we can now able to find it to be this is 0.3 meters kg, sorry, this is kg per meter square per second. I am sorry, this is unit is kg per meter square second. And similarly we can also calculate the mass velocity for the air.

And the mass velocity for air will come out as $m \dot{a}$ divided by A_0 . So, both the parameters are known already we have able to calculate, the mass flow rate that is equals to 2 kg per second. And this is just nothing but 0.10, 0.1089 meter square.

So, that gives you 18.365 kg per meter square second. So, these are the mass velocities of air and the gas side already known to us. So, while we know about this gas side mass velocity and the air side gas velocity, we would be able to calculate the hydraulic diameter.

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$$Re,g = \frac{G_g D_h}{\mu_g}$$

$$= \frac{15.342 \times 0.00154}{40.1 \times 10^{-6}}$$

$$= 589$$

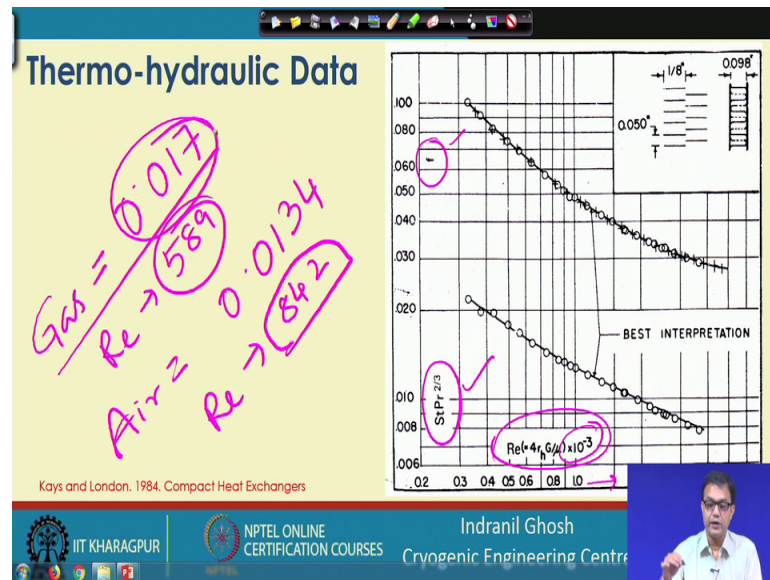
$$Re,a = \frac{G_a D_h}{\mu_a}$$

$$= 842$$

Corresponding to the gas side Re_g , this will come out to be $G_g D_h$ by μ_g , and this is the μ of the gas. So, this will come out to be if we put all this parameters, this is equals to 15.342 multiplied by 0.00154 divided by μ of the gas has been taken as 40.1 into 10 to the power minus 6. So, this will come out to be 589. Similarly, if we try to estimate the Re for the a side, we will find that this is $G_a \mu$ by, I am sorry, D_h by μ of the air side. And if we put all the values it will come out to be 842.

So now we have the knowledge about the Reynolds number air side and the gas side. So, corresponding to this values, we will you know try to find out, will go to the diagram where we will be able to locate the; this is nothing but the Stanton number or multiplied by Prandtl number to the power 2/3.

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Basically this is the (Refer Time: 13:34) and this is the friction factor; at this moment we will not look for the friction factor we will look into the j factor to calculate the heat transfer and this is the Reynolds number. So, we have the Reynolds number here, and please note that this is multiplied by 10 to the power minus 3. So, corresponding to that 589 and 842, we have to find out the j factors; so, the gas we will have a j factor given as corresponding to 0.0170 corresponding to R e value of 589.

This for this R e value, if you look at we will have this value. And for the gas and the air side, we have 0.0134 corresponding to an R e value of 842. So now, we have the j value for the gas and the air.

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The whiteboard contains the following handwritten notes and equations:

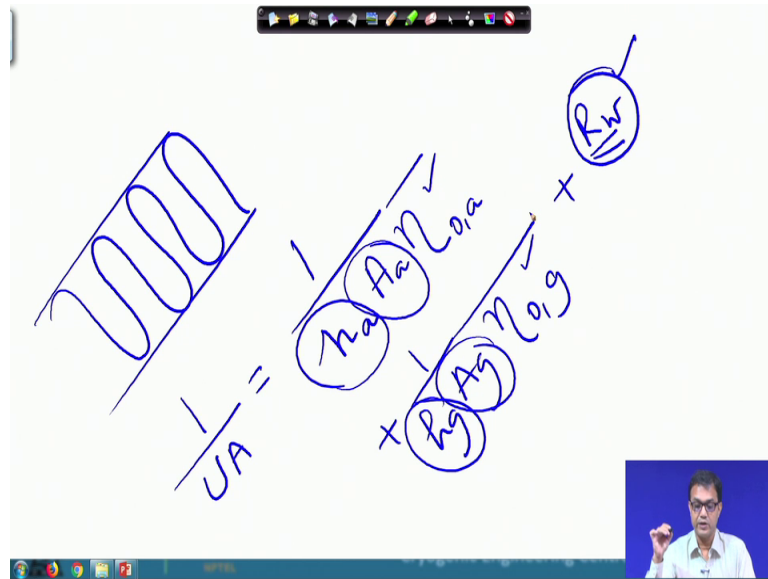
- Stanton number definition: $j = \frac{h}{G C_p} Pr^{2/3}$
- Given Stanton number: $j = 0.017$
- Given Prandtl number: $Pr = 0.7$
- Calculated gas side heat transfer coefficient: $h_g = 360.83 \text{ W/m}^2\text{K}$
- Calculated air side heat transfer coefficient: $h_a = 336.8 \text{ W/m}^2\text{K}$
- Another Stanton number value: $j = 0.0134$

Now if we have the j value j we have seen as Stanton number into Pr to the power 2/3. This Stanton number contains h by $G C_p$ and Pr to the power 2/3. So, since we know this j we have already estimated $G C_p$ all these values are known. So, for j corresponding to 0.017, and we can try to find out the heat transfer coefficient of this is for the gas side. And you will find that h_g will come out to be 360.860 watt per meter square Kelvin.

Similarly, we need to put the value of the heat transfer coefficient corresponding to air side; where we have the j value given as 0.0134. And if we put all those values, it will come out to be 336.81 watt per meter square Kelvin. So now, we have the heat transfer coefficient for the air side and the gas side, and though we are using the similar kind of heat exchanger surfaces, we have different heat transfer coefficient as the mass flow rates or the volumetric flow rates were different.

And that corresponds to different Re value, and that corresponds to different heat transfer coefficient for the heat I mean gas side or the air side. So now, we have the knowledge about the heat transfer coefficient. Next is the we need to find out what is the overall fin efficiency.

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Since the fins are connected between the plates, we need to also know what is the fin efficiency so that we can put that 1 by $U A$. Basically if you remember that in the last few I mean in the previous classes we have told that this is the heat transfer coefficient multiplied by the heat transfer area multiplied by the overall heat transfer coefficient. Plus this will be 1 by h_g and then A_g and $\eta_{o,g}$ for the g and also we have the wall heat transfer coefficient.

I mean, the resistance estimated for the wall. So, for this one we will try go later. And already we have evaluated h_a we have already evaluated A_g and h this all this parameters are known. And we now need to find out this overall heat transfer coefficient for the gas and air side. So, for that what we need to do is that, we need to calculate the m_g .

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The diagram shows a fin of length l and thickness δ . The heat transfer coefficient is h and the thermal conductivity is k . The fin efficiency is given by $\eta = \frac{\tanh(ml)}{ml}$. The parameter m is defined as $m = \sqrt{\frac{hP}{kA}}$ and is further simplified to $m = \sqrt{\frac{2h}{k\delta}} \sqrt{1 + \frac{\delta}{2l}}$.

That is, if we remember that this is not the flow rate, this is related to the fin where that m you know the fin efficiency will be termed as \tanh hyperbolic m into l divided by $m l$. And here this m we now try to calculate, because we know that m is equals to root over hP by kA . h is the heat transfer coefficient, P is the perimeter, k is the thermal conductivity of the fin material, and A is the cross sectional heat transfer cross sectional area. So, if we put these values for this particular situation, we will find that this is going to be $2 h$ by k of the fin material, and then we have the δ that is the thickness of the fin. And then we have $1 + \frac{\delta}{2l}$ that is the lance length of the fin. And then we have whole root over.

So, this is what is the heat I mean this m parameter for this η or the fin efficiency. And we have both for the gas side and the air side, m we can calculate from this correlation.

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The image shows handwritten calculations and a diagram on a whiteboard. The calculations are as follows:

$$m_g = 634.94 \text{ m}^{-1}$$
$$m_a = 615.37 \text{ m}^{-1}$$
$$l_a = l_s = \frac{b/2 - \delta}{2} = \frac{249}{2} - 0.102$$
$$= 1.143 \text{ mm}$$

The diagram shows a cross-section of a fin with a central vertical line. The width of the fin is labeled as $b/2 - \delta$. The word "half" is written below the diagram, indicating that the calculations are for half of the fin.

So, that will give this m_g to be 634, 634.94 and m_a will come out to be 615.37 meter inverse. So, here this both l_a and l_s that has been taken as b by 2 minus δ ; that is equals to 249 by 2 minus 0.102, and that comes out to be 1.143 millimeter. Now the question is why did we take this b by 2 minus δ ? If you remember that in some of the previous classes we have talked about the half fin idealization. They are that half of the fin is as if attached to this plate and half of the fin is as if attached to this the upper plate.

And thereby you know this half factor is coming as if this adiabatic plane is there at the middle, and that gives half of the fin. And in fact, for his symmetric configuration a like where it 2 stream exchanger is concerned, we will find that the half of the fin is really attached to this upper fin and the half of the fin is related to this lower other fin other plate side; so, will get into this in details when we talk about the multi stream plate fin heat exchanger. At this moment, we will we know assume that this half fin idealization works good for I mean the 2 stream heat exchanger cases. And we have this l_a equals to, I am sorry; l_g is equals to b by 2 minus δ .

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Handwritten equations on a whiteboard:

$$\eta_{fg} = 0.8581$$

$$\eta_{fa} = 0.8657$$

$$\eta_{og} = 1 - (1 - \eta_{g0}) \left(\frac{A_f}{A} \right)$$

$$\eta_{oa} = 0.8886$$

$$\eta_o = 0.8946$$

So, accordingly we have the ma value, and we can now calculate the eta f or the g. And this will come out to be 0.8581 and eta a for the air side will be 0.8657. So, from there, we can now calculate eta overall for the gas side, it will come out to be 1 minus 1 minus eta a, and then sorry this is eta g 0 and there comes that A by f minus by A.

So, this is nothing but this parameter is also given in our calculation, I mean fin specification. So, this comes out to be 0.88, 8886 and for the eta 0 of the a that comes out to be 0.8946. So, you can evaluate it using this correlation. For this it will be basically 1 minus, 1 minus eta g, sorry, eta a overall multiplied by A f by A and that will come out as eta 0 for the air side. So now, we have the idea about these overall heat transfer efficiency.

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The image shows a whiteboard with handwritten calculations in red ink. The first equation is $R_w = \frac{\delta_w}{K_w A_w}$, where δ_w and A_w are circled. This is followed by the substitution $= \frac{0.5 \times 10^{-3} \text{ m}}{18 \text{ W/m}\cdot\text{K} \cdot 30.24 \text{ m}^2}$, which results in $= 9.186 \times 10^{-7} \frac{\text{K}}{\text{W}}$. The second equation is $A_w = L_1 L_2 \times (2N_p + 2)$, which is then calculated as $= 30.24 \text{ m}^2$.

Now we can go to calculate the R_w so, this R_w is basically the δ_w divided by K of the wall divided by A of the wall. And this δ_w is the thickness of the fin. So, that is equals to 0.5 into 10 to the power minus 3 meter. And K_w is already given, 18 watt per meter Kelvin. And this A_w is the nothing but A_w is equals to how many number of layers were there? L_1 into L_2 multiplied by $2N_p$ plus 2 number of layers we had.

So, this comes out to be 30.24 meter square. So, this is by substantial and this is 30.24 and this comes out to be this many meter square. So, R_w will come out as 9.186 into 10 to the power minus 7 Kelvin per watt. So, this is 9.186 into 10 to the power 7 watt per Kelvin per watt.

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$$\frac{1}{UA} = \frac{1}{(\eta_0 hA)_h} + R_w + \frac{1}{(\eta_0 hA)_c}$$
$$\Rightarrow UA = 12985 \text{ W/m}^2\text{K}$$

So, we have now the overall estimation of this $1/UA$ is equals to $1/(\eta_0 hA)$ for the hot side plus R_w plus $1/(\eta_0 hA)$ for the cold side. And if we put all the values, we will find that from here UA is coming to be 12985 watt per meter square, sorry, watt per Kelvin that meter is already here.

So, 12985 if it is this many UA it is there. So now, we have to calculate the ratio between the C_g is equals to $m \cdot C_p$ of the gas.

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$$C_g = (m c_p)_g \rightarrow 1863 \text{ W/K}$$
$$C_a = (m c_p)_a \rightarrow 2146 \text{ W/K}$$
$$NTU = 6.97$$
$$CR = 0.868$$

And c_a is equal to $m \cdot c_p$ of the air. And we have this to be 1863 watt per Kelvin. This comes out to be 2146 watt per Kelvin. And then the NTU comes to be $U A$ by C_{\min} . So, this is just nothing but 6.97. And the ratio between this $2 C_R$ by C_{\min} by C_{\max} , that is equal to 0.868. So, we have now these values for the NTU and C_R , and then based on this relation we would be able to calculate what is called the epsilon.

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Thermo-hydraulic Data

$$\epsilon = 1 - e^{-\frac{1}{C_R} (Ntu)^{0.22} \{e^{-C_R(Ntu)^{0.78}} - 1\}}$$

$\epsilon = 0.75$

$\epsilon = 0.8328$

$q = \epsilon (T_{g,i} - T_{a,i}) C_{\min}$

$\approx 0.8328 (900 - 200) \times 1863 \text{ w/K}$

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And we know this relation for the cross flow heat exchanger. And from this correlation if we estimate this epsilon will come out to be 8328.

And since we know this epsilon value, we can now calculate the exit temperatures and also we can calculate the overall heat transfers. So, q would be epsilon times the $T_{g,i}$ minus $T_{a,i}$ into C_{\min} this is the, this is from the basic definition of the heat exchanger effectiveness. And if we put all these values so, this is supposed to come nearly to 0.8328 multiplied by this is the epsilon value T , this is equal to 900 minus 200 and C_{\min} was whatever value for the C_{\min} we had, 1863 watt per Kelvin.

So, if we now evaluate all these things, this will give you some value for the q . And here you will find based on that q value, you would be able to find out the T gas side exit temperature and the air side temperature. And you will find that they are not matching with the assumptions that we have made. As you can understand that we have estimated an or assumed an effectiveness of 0.75 so, already we have deviated from there.

So now, based on this new value of the heat exchanger effectiveness we will have 2 sets of the exit temperature, and based on that value we can now again make an another estimation of the heat exchanger effectiveness. And then we can make an estimate of the total heat transfer, and then we have to find out that when we reasonably close value or predicted, then we can stop our retardation. So, that is how we have to go ahead with the design of the simulation of the heat exchanger problem.

Thank you for your attention.