Heat Exchengers: Fundamentals and Design Analysis
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Lecture – 27
Plate fin heat exchanger: Analysis

Welcome to this lecture, in this lecture on Plate fin heat exchanger we are going to talk about the analysis of plate fin type heat exchanger.

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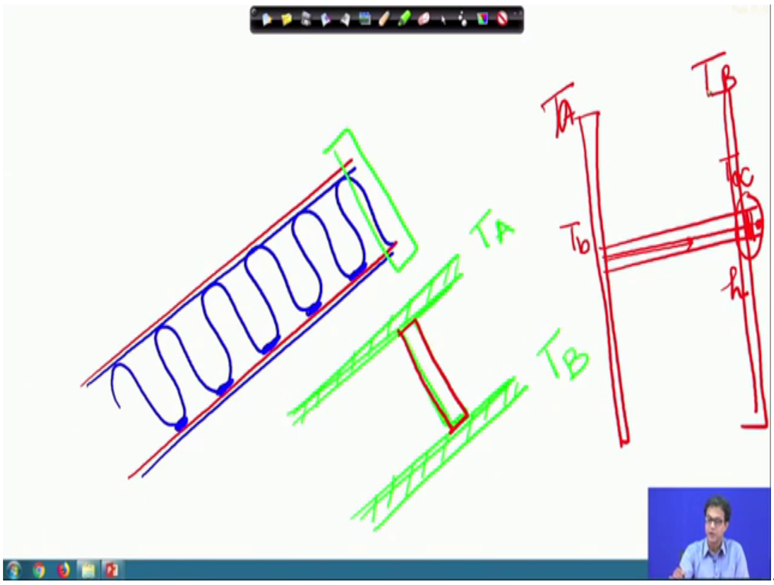

And in this one we will first start with the fin equation where you have might have noticed that we have so far in case of plate fin type exchanger, we have talked about the separating plates and in between the plate what we have is basically some fin connected between this one, between the 2 separating layers, we have fins joint and we have talked about the brazing between this plate and separating I mean separating plate and the fin and we have ensured that there is a good thermal contact between the plate and the fin.

So, now if we look into with I mean carefully what we have is basically separating wall and each individual this layer we can think it as a fin connected between the 2 separating plate. So, what we need to analyze finally, or what it eventually comes as this is some plate at T temperature T A and some plate at temperature B depending on the type of fluid and we have like 2 plates connected by a separating I mean this a fin which is connected between the 2 plates. So, we need to analyze this one, already if you have analyzed this kind of fin equation.

When most of the time we see that the you know we have a plate and we have a fin and the one end of the fin is at a particular either it is at particular temperature or we often considered that this fin is having an adiabatic tip that is very common and sometimes we often you know assume that the fin at this end it is communicating with some fluid and it is a thermally communicating with some fluid and there is some heat transfer h between this fin tip and the surrounding. We also often use that this fin is also at a particular temperature some temperature T infinity or some other temperature.

So, this is typically T base and the fin is connected to T base and this is how it is looking like. So, here in contrast to this one we have another fixed plate and we have you know designated as T A and the T B to different temperatures.

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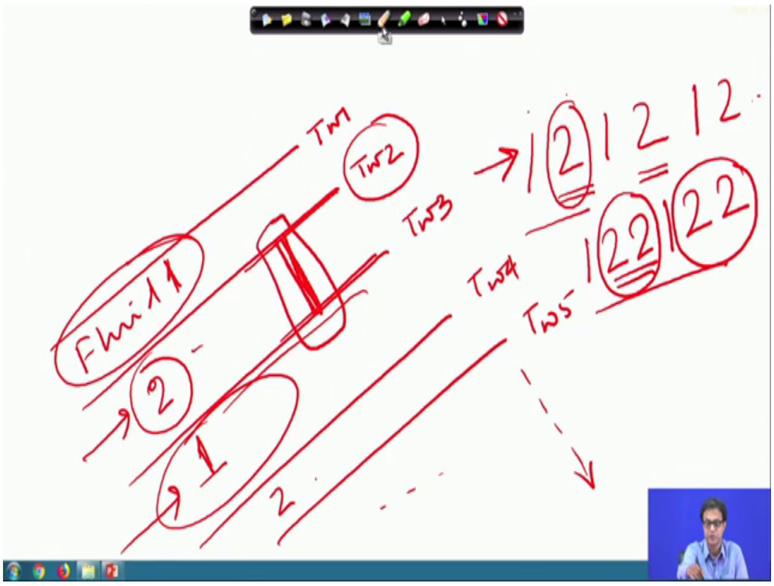

So, similarly if we look at the complete assembly, we will find that there are different type of layers of fins and I mean this is like this say this is designated for fluid 1 and this is fluid 2, this is again fluid 1 and fluid 2 and so on, but eventually if you look at you will finally be solving at anytime and I mean say there will be say T wall 1, T wall 2, T wall 3 and so on T wall 4 and T wall 5 like that it will continue at the actual exchanger.

But for a particular layer I mean for this particular layer on one side we have fluid 1 and this other side we have I mean I mean between fluid 2 I mean if you consider this fluid 2 on one side it is having fluid 1, on the other side it is having fluid 1 and both the sides it is having fluid 1. Similarly if we consider this fluid 1 on either side of it is having the fluid 2, this is particularly the case in a symmetric configuration of 2 stream heat exchanger where most of the time you will find that we have I mean combination like 1 2 1 2 1 2 and so on, but often it may so happen that depending on the amount of fluid we may not be able to the equally distribute it among the which I mean 2 layers.

So, that will consider later on where sometime we need to put it like 1 2 2 1 2 2 is where you know if you have more amount of fluid you know, so we know put it in a double bank we call it. So, that configuration we are not considering at this moment. So, basically we are looking into this configuration where alternatively fluid 1 fluid 2 fluid 1 fluid 2 like that we have I mean they are arranged. So, basically it balls down to a situation where we have 2 separating plate and fin connecting between the 2 plates.

So, if we have to analyze it, we have to analyze these basic phenomena that a fin is connected between the 2 separating walls at different temperature. Now if we have to do that one.

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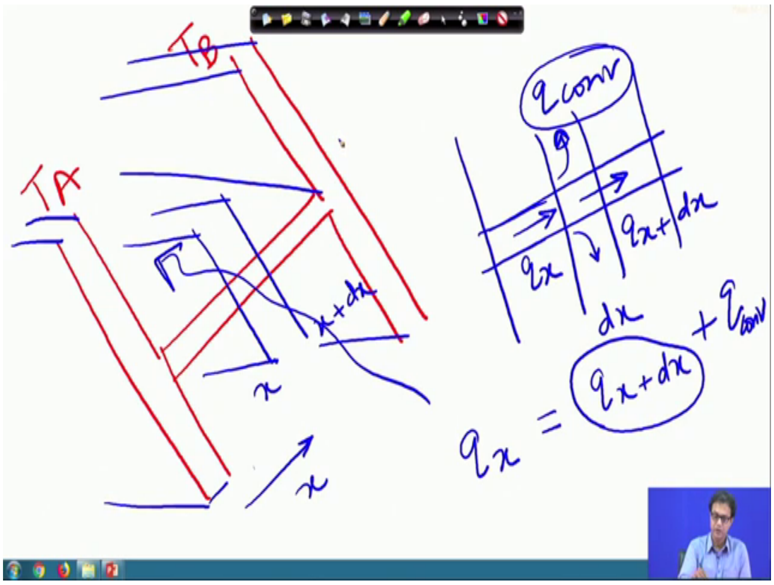

Let us look into that in details, then we have a configuration like this, we have separating walls and this is say this side is this is the fin and this fin is connected to a temperature T A and this side is connected to temperature T B.

Now, if we try to solve that fin equation we what we do it is we take a small section of this one and we its actually 3 dimension this dimension we are not drawing, this is how it is looks like this is the separating plate and this how it look like. We have taken the small section on the of this final in this is a x direction and this is between x and x plus d x. So, between the small element of this fin we have considered a small element d x between x and x plus d x.

So, we had q x amount of it conduction heat getting transferred and this is going through this one this element q x plus x d x and in between you know we have some kind of fluid which is taking this heat. So, we have quick on break tip. So, whatever heat that is coming through the fin the conflictive fluid flowing on top of it on top of this fin is taking that amount of it.

So, if we now have to make an energy balance what we find is that q x amount of heat is getting distributed to q x plus d x and we have also q convective term. Now if we look into this q x plus d x you will find that q x plus d x can be written as, q x plus d x we can write it as q x plus d q x d x into delta x.

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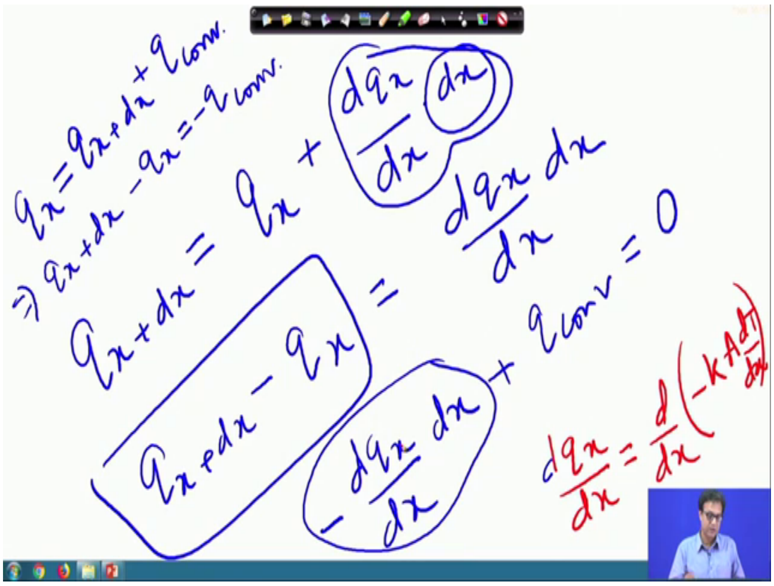

So, we have considered small element delta x. So, we have this is how it is getting distributed.

So, we have already talked about q x is equals to q x plus d x plus q convective. Now from here what we getting is q x plus d x minus q x q x plus d x minus q x is equals to d q x d x into d x, this what we are getting. Now this q x plus d x minus q x is nothing, but d q x d x with the negative term. So, this will become d q x d x multiplied by d x with the negative term and that will become plus I mean q x plus d x minus this will become q convective is equals to 0. This is q x plus d x minus q x and q x plus d x from here if we look at q x plus d x minus q x is equals to minus q convective.

So, that is what we have written here q x minus d x is equals to minus convective and then we have the d q d x part. So, this d q d x with the negative this part can also be written as, this can be written as, this can be written as d q d x we can write it d I am sorry this d q q x d x is equals to we can write it as d d x of minus k into A into d T d x. So, this is what is the convective heat transfer and it will put it in this equation what we will find is that q convective on is, that q convective plus d 2 T d x square with k and A.

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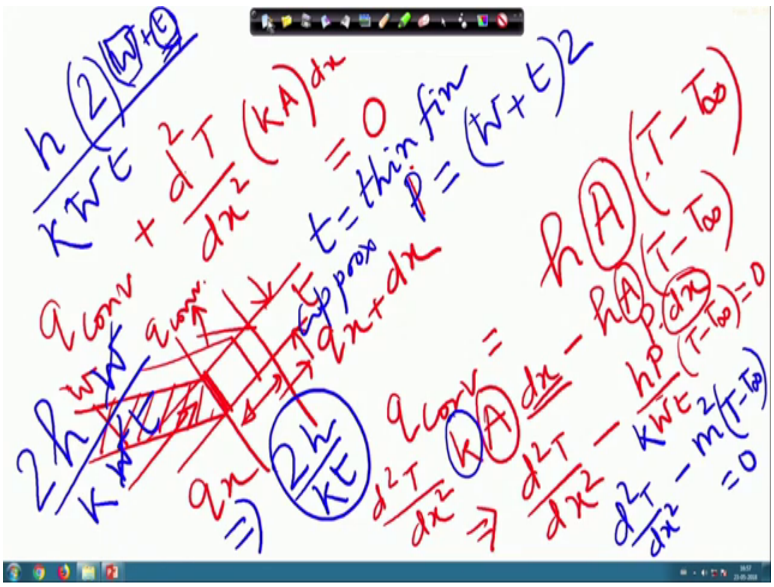

These are the 2 terms we will have for this is equals to 0. So, if you have this terms what is that fin equation we are looking at this is for this cross section, we have where the heat is going q x and this is q x plus d x and this is where we have q convective.

So, how much is the q convective heat transfer, q convective is basically h into the area, what is that area and h A T minus T infinity T infinity is the fluid temperature and this area is basically nothing, but the perry meter multiplied by this d x. So, here also we have that d x part. Now if we put into this equation we will find that this is giving you d 2 T d x square into k A and then you have that delta x term and then we have then we have that h into A into T minus T x infinity and that A we can write it in terms of the sperry meter multiplied by the d x.

So, this will make the total equation d 2 T d sorry this is d x square is equals to this is h upon p by this k into A, this A is nothing, but that cross sectional areas through which the conduction heat was taking place. So, that is about if you talks about this direction as W then we have W plus this is the thickness of the fin T. So, we have W into T as the one and we have then T minus T infinity and both sides this d x and this d x are coming out. So, this is equals 0.

So, that is about the sign we have that this is equals to if we put the sperry meter p if we put this sperry meter p sperry meter p if we look into this p is nothing, but W plus thickness multiplied by 2 and if we arrange it here the you will find that h into p is nothing, but 2 and then we have W plus t divided by W into t and of course, we had that k part here, so there has to be a k here and we should have k here.

Now in often we make an assumption that this thickness of the fin is very I mean small as compare to the this W. So, we call it thin fin approximation, thin fin approximation we call it approximation. So, in that case we neglect t as compared to W. So, in that case we will have twice h w by k W and t. So, this W and W will cancel. So, we will have twice h by k and t. So, this will call it as d 2 T d x square minus m square into T minus T infinity that is equals to 0.

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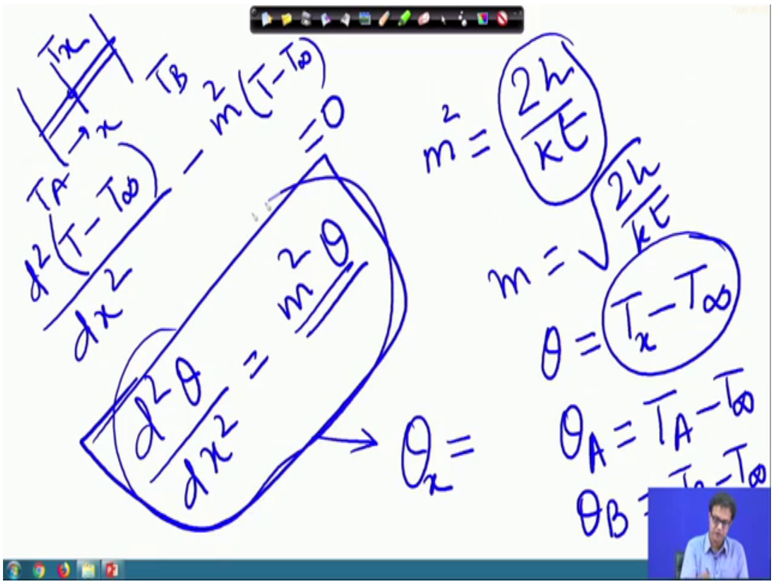

So, now if we look into this equation, we have d 2 T and we put deliberately this is the constant the fluid temperature flowing over the fin we assume at to be constant. So, we have this is minus m square and T minus T infinity is equals to 0 where this m square is nothing, but 2 h by k t are in general it is h p by k a or for thin fin thin, fin assumption this is rooted by I mean m becomes rooted by 2 h by k t.

So, this equation we can shortly write it to be d 2 theta d x square is equals to m square theta and this is the fin equation where this fin is subjected to accommodative fluid temperature I mean dissipating heat through the convective heat transfer. Now if we have to solve this equation we know the solution of this equation in general equation theta x what is theta, theta has been considered to be T minus T infinity.

So, T is basically the temperature at any location in that fin this is at T A and this is at T B anywhere between this one at location x we considered this temperature to be T x. So, this is the temperature T x minus T infinity. So, that this theta A we call it as T a minus t infinity and theta B it would be T b minus T infinity.

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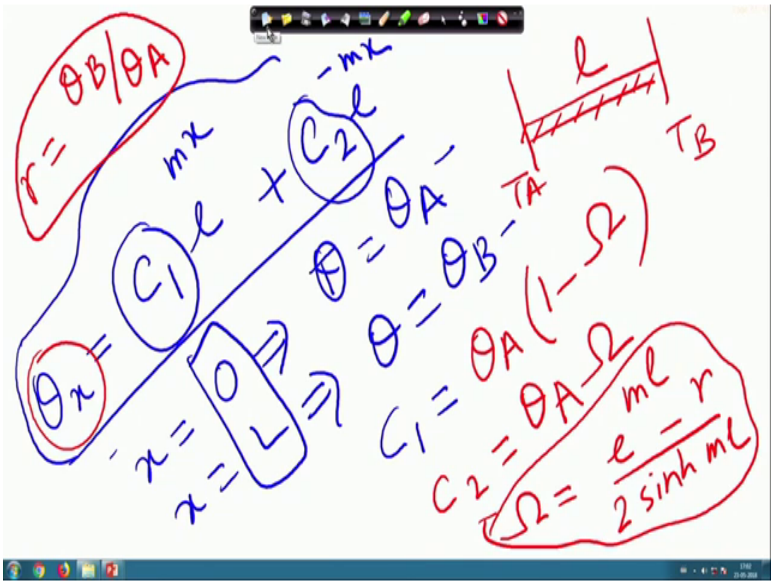

So, now we are trying to look for solution of this equation and we know what is the solution, the theta x or I mean theta basically is equals to some constant into e to the power m x plus some constant of e to the power minus m x. Now we have to boundary conditions appropriate for this equation and from here we have to get this constant C 1 and C 2 and already we have to learned how to do that in your earlier classes probably where we have assumed that one of the fin is that constant temperature T A the other end gives at adiabatic condition.

So, in this case where we have at x equals to 0 we considered it to be T theta equals to as we have said theta A and at x is equals to L. We have theta is equals to theta B this is what we have assumed that between the 2 I mean end the temperature is T A and T B. So, if we solve this equation we will find I mean if we apply this boundary conditions we will find that C 1 comes to be C 1 comes to be c one comes to be theta A multiplied by 1 minus omega and C 2 becomes theta A multiplied by omega where this omega is nothing, but is e to the power m l minus r by 2 sin hyperbolic m l where l is length between the 2 I mean the fin length. So, this is the fin between T A and T B and we have this omega is equals e to the power m l by minus r by 2 sign hyperbolic m l and C 1 C 2 are like this these are the constants.

So, we have evaluated for this equation theta x. So, the overall equation and what is r, r is nothing, but a ratio between theta B by theta A. So, there are other forms of expression this a fin equation. This is this particular expression of fin I mean solution of fin equation, solution we have taken from B S P fresher's paper.

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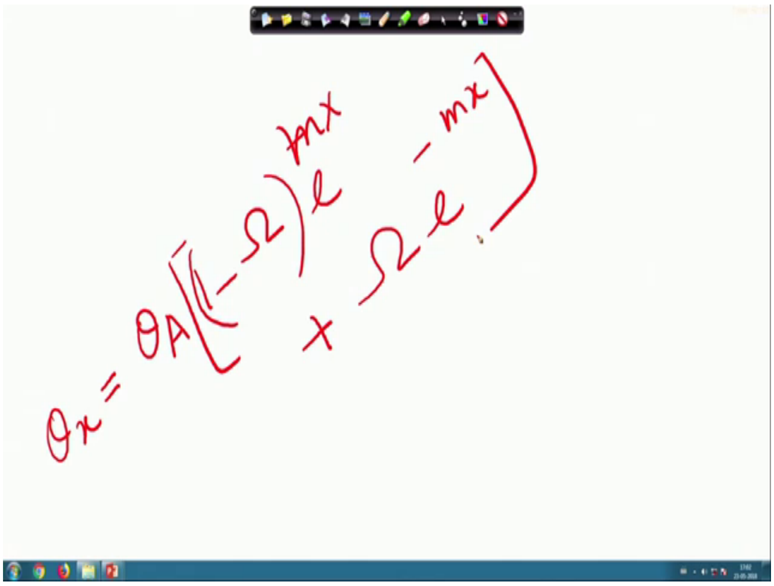

So, now if we look at we have the complete solution. Theta x is equals to theta A multiplied by 1 minus omega and that multiplied by e to the power m x, this is e to the power m x plus omega into e to the power minus m x, this what is the complete solution. Now if we have the knowledge about the fin temperature profile over the fins.

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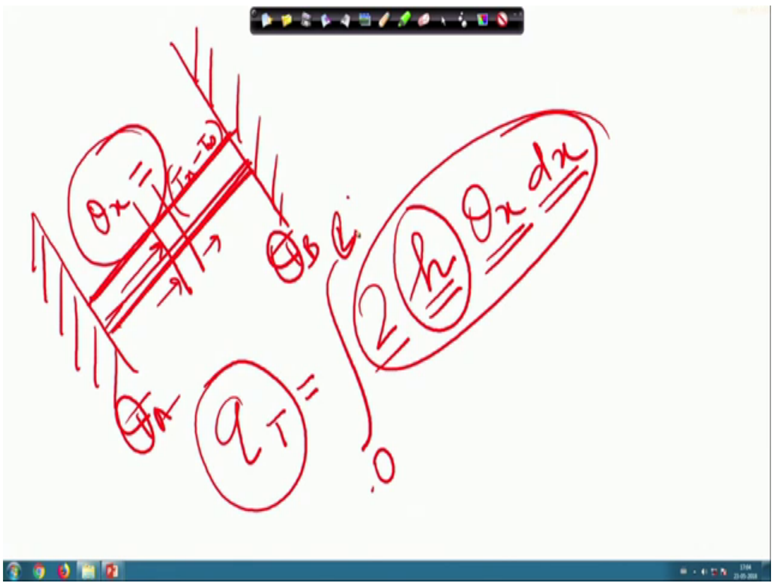

So, this is what we have the T A and T B and we have the knowledge about the fin temperature profile theta x known and this off course not T A we call it theta A and theta B.

So, this at T A and this is at T B and we know now the temperature profile over this fin. So, once we have the temperature profile known to this fin, we can try to estimate what is the total amount heat being dissipated through this fin. So, if we try to do that one we find that total amount of heat getting dissipated over this fin when these are 2 into h into theta x into d x and this is intrigued over the length 0 to l by this 2 factor is coming.

Because we have this surface this surface and over this surface we have the heat transfer coefficient h and what is theta x, theta x is basically is the temperature T x minus T infinity so, that T x minus T infinity heat transfer coefficient and the T x. So, that is how you know that if we consider an element d x over this length and that length has to be you know instigated over the length 0 to l. So, that how you will get total amount of heat getting transfer through this fin over this length l of this is basically a small l.

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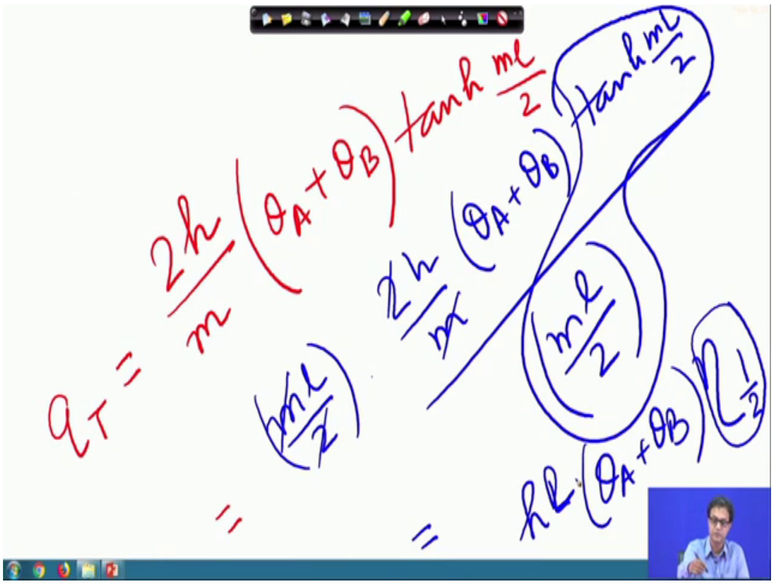

So, this is if we trying to evaluate this one, we will have an expression of q t is equals to 2 of h by m into theta A plus theta B if you put that value of theta x then you interrogate you will have an expression of ten hyperbolic m l by 2. So, now, what we do is that we divide and multiplied both sides by m l by 2. So, that we what we will get is m l by 2 on this side and then we have 2 h by m then you have theta A plus theta B this is the term there and ten hyperbolic m l by 2 and here we have multiplied it and divided it, but this ten hyperbolic m l by m l this is nothing, but the fin effect fin efficiency.

So, we can write it as this 2 and this m they are going out. So, you have h into L then you have theta A plus theta B and then we write it as eta. This eta as if is for l by 2. So, we call it fin efficiency of the half of the fin.

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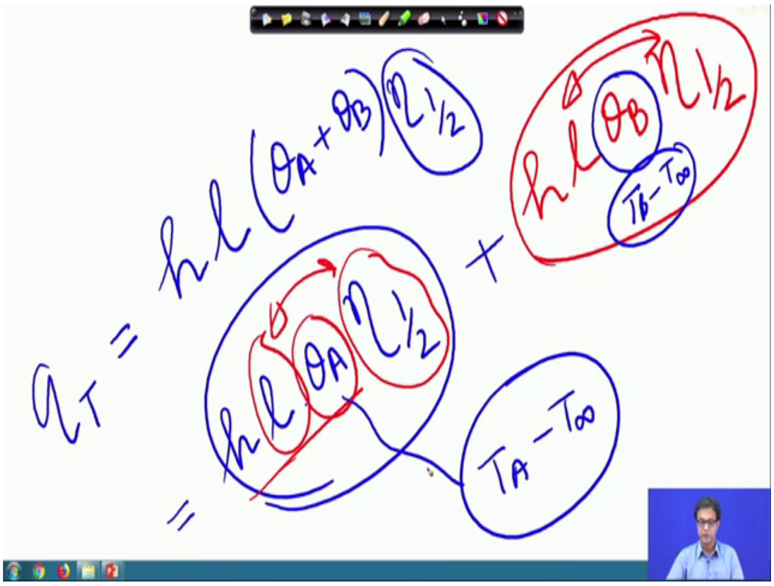

So, now, if you look at we have the heat transfer the total amount of heat transfer that q T the total amount of heat getting dissipated through the fin is h into l and then you have theta A plus theta B and multiplied by eta half half fin efficiency.

Now, if you carefully look into it we can write as if this is h into l and then theta A and then eta half and so, this is one part and the other part if you look at it is just nothing, but h into l and then theta B and then eta half. So, as if we have this contribution of half of the fin this l and then half of it is contributing with theta A and this is where we have the theta B. So, this is just nothing, but T B minus T infinity this theta B and theta A is nothing, but T A minus T infinity. So, as if we have half of the fin contributing to the fluid a and the other half is connected to the other fin. So, this is basically nothing, but the half fin idealization of the plate fin heat exchanger.

So, this again this is I mean good assumption particularly for a 2 stream heat exchanger, but when we have multiple streams are we have a different asymmetric fin we find that there is a I mean good deal of halation of this one and we will then you know we need to look into different analysis during that time. So, the total heat transfer is a getting I mean half of that fin connected to surface a T A and the other one is connected to T B.

Thank you.