

**Heat Exchangers: Fundamentals and Design Analysis**  
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**Lecture - 23**  
**Finned tube heat exchange (Contd.)**

Hello everyone, we were discussing Finned Tube Heat Exchanger. In last few lectures we have seen; how to calculate the fin efficiency for finned tube heat exchanger. How to calculate different geometrical parameters particularly area of finned tube heat exchangers and ultimately we have calculated heat transfer coefficient for finned tube heat exchanger. Once again I like to remind you for finned tube heat exchanger, where fluid passes through tube side and the gas passes over the fins surface of the tubes; the challenge lies in the designing of fin side that is mean; the gas side of the heat exchangers.

Both the heat transfer coefficient and the pressure drop, that will depend on the geometry of the finned surface, tube arrangement, tube layout, number of row, number of columns and; obviously, the physical dimensions. And most the cases these are correlations obtained from experiment. So, really we should not bother nah too get them by heart, only thing is that a familiarity is needed. So, that is why we will take up some problem or example, which a typical example of finned tube heat exchanger.

Nothing has to be get by heart only the method of solution or rather the logic we will follow to solve them that has to be understood so, that when another problem comes we will be able to solve it. So, this is what I like to convey, and certain generality is there. So, those generalities I am pointing out as we proceed with this course. So, analysis of finned tube heat exchanger continued; we will continue with this thing and let us go to the next slide.

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**Pressure drop in finned tube arrays :**

$$\sigma = \frac{p_1 - D_r - \left\{ \frac{2LW}{(w+S)} \right\}}{p_1}$$

- ❖ The value of  $K_f$  depends not only on the geometry of the tube array.
- ❖  $K_f$  is a function of Re based on maximum fluid velocity and tube diameter and is recommended for Re in the range  $10^3$  to  $10^5$

$$K_f = 4.71 Re^{-0.286} \left( \frac{L}{S} \right)^{0.51} \left( \frac{p_1 - D_r}{p_2 - D_r} \right)^{0.536} \left( \frac{D_r}{p_1 - D_r} \right)^{0.36}$$

- ❖ High fin tubes in staggered arrays with Re in the range  $5 \times 10^2$  to  $5 \times 10^4$

$$K_f = 4.567 Re^{-0.242} \left( \frac{A}{A_T} \right)^{0.504} \left( \frac{p_1}{D_r} \right)^{0.376} \left( \frac{p_2}{D_r} \right)^{0.546}$$

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So, next slide. So, next slide gives pressure drop in finned tube arrays, but before doing that let me do one thing I like to give certain overview so, that you will be able to understand what we are going to do.

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Diagram showing a tube array with flow direction indicated by arrows. The equations shown are:

$$\Delta p = (k_a + k_f) \frac{1}{2} \rho V_{\max}^2$$

$$k_a = 1 + \sigma^2$$

$$\sigma = \frac{\text{Min}^m \text{ flow area}}{\text{Total frontal area}}$$

A small video inset shows Prof. P. K. Das speaking.

So, let us say this is the tube array, as I have told number of times that; flow of fluid is through the tube and flow of gas or air, which is the gas in many cases is in this direction. So, length of the tube is normal to the diagram I have shown. So, I have discussed the heat transfer aspect of heat, let me discuss the pressure drop aspect of heat.

So, pressure drop aspect you see the when the air passes through this; it will have different cross section of area.

So, when the air is passing through this portion. So, it is passing through some sort of restricted phase, and when it is passing through this then it has got more cross sectional area. So, air will go through experiencing or air will experience some sort of acceleration and deceleration. So, this will give some amount of pressure drop; plus of course, it is passing passed solid bodies which are plugged bodies and these tubes are having fins etcetera.

So, there will be some sort of pressure loss due to that, which is called friction loss, but it could be both your friction loss and your loss due to form drag, loss due to the presence of a black body. So, all this thing generally it is done in some sort of a correlation form. And one can write  $\Delta p$  the pressure drop that is equal to due to 2 component,  $K_a$  acceleration plus  $K_f$  friction.

So, these 2 are coefficient and contribution from acceleration and friction as I have told. And then half into  $\rho V_{max}^2$  so this simple formula is used; so, all the complexity of the pressure drop that has been taken care by this simple formula. Then how it is it has been taken care of, all the complicities have been clubbed into 2 coefficients. One is  $K_a$  which is due to acceleration and another is  $K_f$  which is quote unquote due to friction.

So, as I have told that friction, and form drag, and all these losses due to viscous nature of the fluid that will be plugged into  $K_f$ , and then  $K_a$  is equal to  $1 + \sigma^2$ , again I will get another relationship for  $K_a$  and then sigma, what is the sigma? Sigma is some sort of a geometrical parameter. So, this is the minimum flow area. Some of these things have been used earlier and divided by the total frontal area.

So, from there I will get sigma. So, sigma is geometrical parameter and then I will get  $K_a$  and I will get also  $K_f$ ,  $K_f$  is given by some sort of a correlation. with these let us go back to our slide now we will be able to understand what is there. So sigma, how I am calculating this sigma? So, I have given some sort a of a tube array p th etcetera have been described.

So,  $\sigma$  comes from the geometrical feature. So,  $\sigma$  will be given by as you can see the  $p$  the  $1/p$  th will come,  $D_r$  root diameter of the fin will come. And then this is the height of the fin, this is the width of the fin, this is the width of the fin and the space between 2 fins. So, all these things will come here; the value of  $K_f$  depends not only on the geometry of the tube array, but it also depends on other factors. So,  $K_f$  is a function of Reynolds number, based on maximum fluid velocity and tube diameter. And is recommended for  $Re$  in the range  $10$  to  $10^3$  to  $10^5$ .

So,  $10^3$  to  $10^5$  the  $Re$ , and  $Re$  is defined or rather  $Re$  is specified based on the bare tube diameter and maximum velocity. It covers the range in which these finned tube heat exchangers are generally used. So, the correlations that we will be showing today, though I have told that they need not be memorized; these correlations will hold good for most of the finned tube heat exchangers used in practice. So,  $K_f$  now we can see  $K_f$  is  $Re$  to the power some factor. Then  $L$  by  $S$  then the pitch that will come into picture and then the root diameter etcetera will come into picture.

So, basically it is dependent on the Reynolds number diameter of the tube, diameter of the  $p$ , fins etcetera. Then high fin tubes in staggered array with  $Re$  in the range of  $5$  into  $10^2$  to  $10^5$  we have got some other formula. A and AT I have defined in our earlier class. So, you see you have not to bother regarding the form formula because these formula are available. Unfortunately, if somebody uses some sort of a tube layout or some sort of a fin geometry for which formula is not there, then of course it is a matter to be over it. And in that case one has to again conduct experiment to get this kind of correlations.

Otherwise if we pick our selection from the available a geometry which are used in the industry, then this kind of formula are available and one can just plug in. Only one has to know: what is the logical procedure of designing, what is the logical procedure of analysis and that is what I am going to describe here. So, as I have told  $\Delta p$  depends on  $K_a$  and  $K_f$ .  $K_a$  depends on  $\sigma$  so we have shown in this particular slide how to calculate  $\sigma$ . And we have also given some probable correlations, some existing correlations for  $K_f$ 's; now let us move to the next slide.

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Where,  $\frac{A}{A_T}$  is the ratio of the total surface area of the finned tube to the surface area of the equivalent bare tube.

$$\frac{A}{A_T} = \frac{\frac{1}{2}(D_i^2 - D_o^2) + D_o w + D_o s}{D_o(w+s)}$$

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So, next slide here I have shown  $A$  by  $A_T$  is the ratio of the total surface area of the finned tube to the surface area of the equivalent bare tube. So, that is what and with this we can calculate the required quantities.

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A heat exchanger consists of four rows of eight steel tubes [thermal conductivity 15 W/(m.K)] in equilateral triangle array (TLA=30°) fitted with corbels, or dummy half tubes, to reduce bypass flow. The tubes have roll-formed rectangular cross section fins and the following dimensions:

$$L = 0.5 \text{ m}, D_i = 1.64 \times 10^{-2} \text{ m}, D_o = 2.46 \times 10^{-2} \text{ m}, L = 4.1 \times 10^{-3} \text{ m}, w = 1.0 \times 10^{-3} \text{ m}, s = 2.0 \times 10^{-2} \text{ m}, p_1 = 3.13 \times 10^{-2} \text{ m}, p_2 = 2.71 \times 10^{-2} \text{ m}$$

The tubes are heated on the inside by a condensing vapor that maintains a uniform tube wall surface temperature of 343 K and cooled on the outside by cross-flow air initially at 288K. Flowing at a rate of 0.914 kg/s. What is the total rate of heat transfer and pressure drop across heat exchanger?

Fluid Properties:  $\rho = 1.217, \eta = 1.8 \times 10^{-5} \text{ (N.s)/m}^2, c_p = 1007 \text{ J/(Kg.K)}, k = 2.53 \times 10^{-2} \text{ W/(m.K)}, Pr = 0.71$

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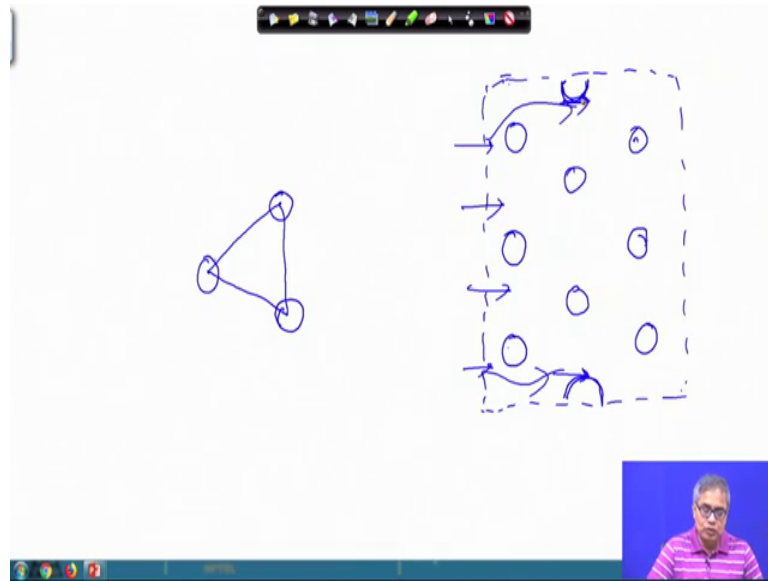
Now let us take up a problem and if the problem we will see how we can solve this, and how we can use the things which we have learnt. A heat exchanger consists of 4 rows of 8 steel tubes thermal conductivity of the steel tube that has been given and in equilateral triangular array fitted with corbels or dummy half tubes to reduce the bypass flow. The

tubes have roll form rectangular cross section fin, rectangular cross section fin and the following dimensions. length has been given, root diameter has been given, and then the then the tip diameter has been given, then actually this first length one should take as the length of tube and second L this is typographical mistake over here.

So, this is the length of the fin or height of the fin. The  $W$  is the width of the fin, and  $S$  is the gap between 2 fins, then  $p_1$  is the pitch 1 pitch in the direction of flow, and  $p_2$  sorry,  $p_1$  is pitch normal to the flow direction, and  $p_2$  is the pitch in the direction of flow. Let me explain  $L$  which is 0.5 meter that is the length of the tube,  $D_r$  is the root diameter of the base fin or the diameter of the tube bare tube,  $D_t$  is the outer diameter of the fin. So, from here you can calculate using these 2 you can calculate the fin height. So, this is giving the fin height, and this is giving the fin width  $w$  small  $w$  is the fin width,  $s$  is the gap between 2 fins and  $p_1$  is the pitch you please consult our earlier diagram there you will get  $p_1$ .

So,  $p_1$  actually is a direction normal to the direction of flow and  $p_2$  is in the direction of flow. The tubes are heated on the inside, by condensing vapor that maintains a uniform tube wall surface temperature of 343 k. And cooled on the outside by cross flow of air initially 288 k. Flowing at the rate of 0.914 kg per second, what is the total rate of heat transfer and pressure drop across the heat exchanger. So, there are a few things which I like to mention. So, let us say this is this these tubes are the tubes are arranged in equilateral triangular array so the tubes are arranged like this.

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So, this is basically if we join the centre of the tubes then they are in equilateral triangular array. And then you can know that this angle this angle included angle will be 60-degree half of that angle will be 30 degree. So, this is how tubes are arranged so this is kind of a staggered arrangement. So, let us think of a staggered arrangement so this is the staggered arrangement. next row of column of tubes comes like this.

Now, you see what happens that; what happens that here we are having more number of tubes, in the next column we are having less number of tubes. Let us say this is my control volume or this is the envelop of the heat exchanger. So, what one can see if the air passes through this then air will get larger vacant space. So, if air passes through this, let us say air is passing through this and here air will get larger vacant space. So, most of the air that will try to bypass through this portions; so, this problem will be there, towards the end this problem will be there when there is staggered arrangement of tubes.

So, what one can do here one can put half tubes which are dummy tubes. So, this what one can put and then the airflow distribution will not be left sided, and we have selected the staggered arrangement of tube, and we will get a good distribution of air through the staggered arrangement of the tube. So, this is one thing which one has to appreciate. So, you see here it is told that fitted with corbels or dummy half tubes. So, what is dummy half tubes that I have explained. Then the tube is heated from inside with the help of a condensing steam.

So, if it is heated from the inside by a condensing steam, generally in condensation heat transfer coefficient is very high. So, what has been done in this problem that; we have neglected the resistance inside the tube due to condensation because that resistance is very small compared to the other resistance. So, this is also one thing you have to remember. And in many cases one can also neglect the resistance of the metallic tube because metal has got high heat transfer coefficient conductivity high conductivity.

And a low thermal resistance compared to the resistance offered by the gas which is outside the tube. So, basically then the all the resistances are in series, but the resistance on the air side that becomes the deciding factor ok. And fluid properties have been give given, so the density have been given, here viscosity have been given and CP has been given, k thermal conductivity of air that has been given, and then Prandtl number that have also been given.

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- Surface area of fins,  $A_f = \frac{N L_t \pi}{(s+w)} \left\{ \frac{1}{2} (D_f^2 - D_r^2) + D_f w \right\} = 2.542 \text{ m}^2$
- Surface area of the tubes between fins,  $A_w = \frac{N L_t \pi}{(s+w)} \{ D_r s \} = 0.550 \text{ m}^2$
- Total surface area of finned tube,  $A = A_f + A_w = 3.092 \text{ m}^2$
- The total tube surface area without fins removed) is of course  $A_T = N L_t \pi D_r = 0.824 \text{ m}^2$
- Minimum cross section flow area,  $S_{min} = n_t L_t \left\{ p_1 - D_r - \frac{2wL}{w+s} \right\} = 5.172 \times 10^{-2} \text{ m}^2$
- For Reynolds Number,  $V_{max} = \frac{\dot{M}}{S_{min} \rho} = \frac{0.914}{1.217 \times 5.172 \times 10^{-2}} = 14.5 \text{ m/s}$

$$Re = \frac{V_{max} D_r \rho}{\eta} = 1.61 \times 10^4$$

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So, with these let us go to the next line the surface are of fin, that is A f we can calculate the surface area of the fin. Then surface area of tubes between the fins that we can calculate by Aw, this have been explained earlier. And I again like you to derive this relationship of your own so that you can understand this, how they have how these formula have been a write at. And what you can do, you can also solve this problem of your own though it has been given so taking this problem statement you can solve it.



So, then the total surface area this is the outside surface area which is responsible for heat transfer, but again here I like to mention that; this surface area is not equally effective in heat transfer. The bare tube surface outer surface that is more efficient or more effective, where as the fin surface there are there will be a variation of temperature. So, fin efficiency also you have to take into consideration.

The total tube surface area without the fins or fins removed is we have got, without the fins, that is we will get 8 point sorry 0.824-meter square. Then minimum cross section area of flow from where we will get V max, V max is needed in many calculations so we will get like this. Please recheck this numbers you can calculate and check with our number. For Reynolds number we will get this is our Reynolds number, getting the V max we will get Reynolds number. This is 1.61 and into 10 to the power 4, and you can note oh all these correlations I have mentioned earlier so it falls within the range of those correlations. So, let us go to the next slide.

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- The average heat transfer coefficient can now be calculated using the appropriate correlation for  $\overline{Nu}$  versus Re.
- Average heat transfer coefficient for low fin tubes :
 
$$\overline{Nu} = 0.183Re^{0.7} \left(\frac{S}{L}\right)^{0.36} \left(\frac{P_1}{D_t}\right)^{0.06} \left(\frac{L}{D_t}\right)^{0.11} Pr^{0.36} \hat{f}_1 \hat{f}_2 \hat{f}_3$$

$$= 91.6 \hat{F}_1 \hat{F}_2 \hat{F}_3$$
- Correlation for high fin tubes:  $\overline{Nu} = 0.242Re^{0.658} \left(\frac{S}{L}\right)^{0.292} \left(\frac{P_1}{P_2}\right)^{0.09} (Pr)^{(1/3)} \hat{F}_1 \hat{F}_2$ 

$$= 100.9 \hat{F}_1 \hat{F}_2$$

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Next slide the average heat transfer coefficient now can be calculated. So, Nusselt number we can calculate the average Nusselt number these kind of formula I have given earlier. So this is your Nusselt number, and then there are 2 factors ok. So, these 2 factors how do we calculate these 2 factors correlation for high fin tubes we have used, correlation for high fin tubes low fin tubes is this formula and high fin tube is this

formula. So, we have to decide which one we can use and already we have given this formula earlier. So, you are familiar what are the terminologies are used in this formula.

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•  $\overline{Nu} = 91.6 \times 0.9 = 82.5$   
 •  $h = \frac{\overline{Nu}_k}{D_r} = 127.3 \text{ W/(m}^2 \cdot \text{K)}$   
 • Fin efficiency,  $\eta_{f,R} = \frac{\tanh m\psi}{m\psi}$   

$$\psi = \frac{D_r}{2} \left( \frac{D_t}{D_r} - 1 \right) \left( 1 + 0.35 \ln \frac{D_t}{D_r} \right) = 4.68 \times 10^{-3} \text{ m}$$
  

$$\eta'_f = 0.89$$

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So, then we have selected the actual relationship and from there we have got the average Nusselt number, from there we have got the heat transfer coefficient which is 127.3 watt per meter square Kelvin. Now, this value I mean one can, one does not have to get it by heart, but 100, 120, 100 up to 150 watt per meter square Kelvin, that is the heat transfer coefficient for air in force convection. At least this much one can remember and this kind of through gut feeling of a number is very important, when we are designing certain heat transfer equipment. Then fin efficiency has to be calculated last day also I have shown how to calculate fin efficiency using the same formula we are getting the fin efficiency 0.89.

This is also one thing that fin efficiency should be on the higher side. If the fin efficiency is not on the higher side then we have not selected a fin properly, we have not selected a fin justifiably. So, here we are getting almost 90 percent fin efficiency which is acceptable. Sometimes even go for lower fin efficiency, and in case of some high end heat exchanger high effectiveness heat exchanger we are having higher fin efficiency.




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• Hence the effective average heat transfer coefficient is,


$$\bar{h} = \left( \frac{\eta_f A_f + A_w}{A} \right) h = 115.8 \text{ W/(m}^2 \cdot \text{K)}$$

Log mean Temperature difference,  $\Delta T = (T_c - T_h) \left\{ \frac{1 - \exp\left\{ \frac{-A\bar{h}}{M c_p} \right\}}{\left\{ \frac{-A\bar{h}}{M c_p} \right\}} \right\}$ ;  $\frac{A\bar{h}}{M c_p} = 0.389$

$$\Delta T = 45.6 \text{ K}$$

$$\dot{Q} = \bar{h} A \Delta T = 1.63 \times 10^4 \text{ W}$$




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So, with this we will move on. So, as I have told once we have got the fin efficiency what we can do there are 2 ways, either we can get some sort of reduced area by the use of fin efficiency or we can get some sort of reduced heat transfer coefficient. So, here effective average heat transfer coefficient we have calculated and that is 115.8 watt per meter square Kelvin. Then log mean temperature difference we have to calculate the log mean temperature difference. Here there is one catch please try to understand that; the catch is this is a phase change heat transfer device where condensation is taking place and it has been cooled outside with air.

So, air will air will change its temperature or rather there will be a change in temperature of air, but from the problem statement; the condensing fluid so condensing fluid side there not be any change in temperature. Very high heat transfer coefficient on the condensing side. So, the tube wall will remain at a constant temperature, this is very important to understand. So, this is very important to understand that a tube wall will remain at a constant temperature, and using that constant temperature which has been given in the problem we can calculate the log mean temperature of the heat exchanger. Basically then this is a heat exchanger where one side temperature remains constant other side temperature changes.

And then we will discuss about this kind of heat exchanger in details, may be from next lecture onwards. And then there is only one resistance because other thermal resistances are negligible compared to the air side resistance. The formula here which I have given  $\Delta T$  is equal to this complex or not so complex expression. this has come through a

number of steps. I would request all of you already you are familiar with the LMTD, already you are familiar with the how to do the temperature variation estimation.

So, please try to get this expression. So, once you get this expression then it is very easy delta T is 45.6 k, what is this delta T? This is the delta T experienced by air which is passing outside the condenser coil. So now your Q dot is equal to h A delta T, that is this is the this amount of heat transfer. So, total amount of heat transfer we are getting from this formula all right. So, basically what we have calculated; we have calculated a few areas from there, we have also calculated the Reynolds number, and the heat transfer coefficient and using those values what we have done; we have calculated the total rate of heat transfer.

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• The ratio of minimum free flow area to total frontal area is,

$$\sigma = \frac{p_1 - D_r - \left\{ \frac{2Lw}{(w+S)} \right\}}{p_1} = 0.39$$

• One velocity head based on maximum velocity is,  $\frac{1}{2} \rho V_{\max}^2 = \frac{1}{2} \times 1.217 \times (14.52)^2 = 128.3 \text{ N/m}^2$

$$K_a = 1 + (\sigma)^2 = 1.15$$

$$K_f = 4.71 Re^{-0.286} \left( \frac{L}{S} \right)^{0.51} \left( \frac{p_1 - D_r}{p_2 - D_r} \right)^{0.536} \left( \frac{D_r}{p_1 - D_r} \right)^{0.36} = 0.52$$

• Total pressure drop is therefore

$$\Delta P = (1.15 + 4 \times 0.52) 128.3 = 414 \text{ N/m}^2$$

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Well now we have to calculate the pressure drop. The ratio of minimum free flow area to total frontal area, that is sigma at the beginning of the lecture I have described so point that is 0.3 one 39. The half rho V max square that is needed for pressure calculation V max we have calculated. So, this is what we can get, K a acceleration coefficient I have defined as a plus sigma square. And then I can get it as 1 plus sorry 1.15 so; obviously, you can understand as it is 1 plus something, which is a positive quantity. It will be more than one and it depends on sigma. Then K f K f calculation is not straightforward, it depends on what kind of correlation we pick up.

So, we picked up a correlation suitable for this; based on Reynolds number and the geometrical parameter so we have calculated  $K_f$ . Then  $\Delta p$  is equal to  $K_a$  this  $K_a$  plus  $K_f$  this is your  $K_f$ , 4 into  $K_f$  we have to see how rows are there. So, that is why this 4 or number of rows we have to give. So, that is what we have given, and then  $\frac{1}{2} \rho V_{\max}$  sorry  $\frac{1}{2} \rho V_{\max}^2$ . So, that is  $\frac{1}{2} \rho V_{\max}^2 \times 128.3$ . So, that has also come and giving everything it is 414 Newton meter square Newton per meter square. So, this is what is the pressure drop across the finned tube heat exchanger.

So, in a nut shell what we have done, we have first calculate first describe; we have first describe the method of pressure drop calculation in a finned tube row and column, where there are number of finned tubes. And then we have taken up a problem which is fin tube condenser, outside air is flowing for that; we have calculated the overall heat transfer. Total heat transfer rather and then we have calculated the pressure drop. So, this should give you some idea that how to do calculation for a finned tube heat exchanger. So, with this we will we will try to make some sort of a conclusion that that we have started with, with augmentation of heat transfer.

And then we defined also what are compact heat exchangers, I have told that fins are very useful method for augmenting heat transfer and they are very extensively used in heat exchangers. So, that is why we have spent some time on finned tube heat exchanger different methods of specifications calculation etcetera I have described. And finally, we have taken up a problem where we could calculate the total amount of heat transfer and the pressure drop.

So, there are many such problems, and the books which I have described I mean; which we given referred as reference. So, those books also you can see. And if any queries are there you are welcome to put up those queries, but it is important that whatever I have given the derivation those derivation you check those are very simple geometrical derivation. And sometimes it is derivation from LMTD, please do this derivations of your own.

Thank you.