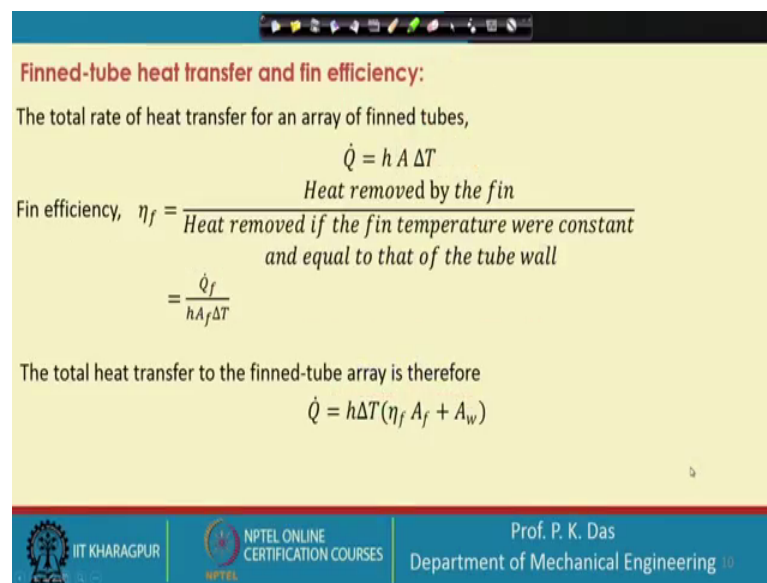


Heat Exchangers: Fundamentals and Design Analysis
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Lecture – 22
Finned tube heat exchanger (Contd.)

If we recall, we were discussing Finned Tube Heat Exchanger. So, we were actually preparing ourself with the analysis which will help us to design if finned tube heat exchanger if there is a need. Now for that what we want to do or what we need to do is, do certain estimation, estimation of area etcetera; particularly on the fin side. And that is what has been done in our previous lecture. We will continue with the results of the previous lecture.

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Finned-tube heat transfer and fin efficiency:

The total rate of heat transfer for an array of finned tubes,

$$\dot{Q} = h A \Delta T$$

Heat removed by the fin

Fin efficiency, $\eta_f = \frac{\text{Heat removed by the fin}}{\text{Heat removed if the fin temperature were constant and equal to that of the tube wall}}$

$$= \frac{\dot{Q}_f}{h A_f \Delta T}$$

The total heat transfer to the finned-tube array is therefore

$$\dot{Q} = h \Delta T (\eta_f A_f + A_w)$$

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So, you see the finned tube heat transfer and fin efficiency we have to now consider. As I have told that heat transfer that will be given by $h A \Delta T$ that is actually in the fin side. And h is the heat transfer coefficient. And this is one of the aim to determine h from the analysis which is now we are continuing to do. A is the area; we have seen some calculation of area in our previous lecture. So, h we have not done yet, A we have seen some calculation in our previous lecture.

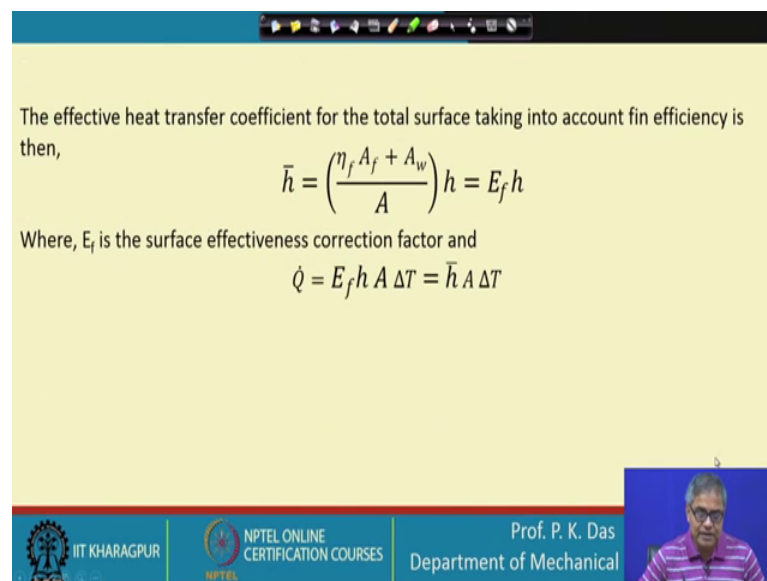
η_f that is your fin efficiency, as fins are there we have to have fin efficiency. So, this is heat removal by the fin, and heat removal if the fin temperature were constant; that

means, at the root temperature and equal to that of the tube wall. So, this is how we can get our η_f . So, this also you have to calculate if we have to calculate the heat transfer. Because, the area what we will get that is not equally participating or it is not participating in heat transfer with equal effectiveness. So, we have to have the idea of fin efficiency.

Then the total heat transfer to the fin tube array is therefore, \dot{Q} is equal to h into ΔT . Fin area has to be multiplied by your fin efficiency. And then this is the bare area this is having the temperature of the bare tube. So, we will take it as it is. So, this is how in the next cell will be our formula probably you know it, but as this is important. So, we have again repeated it.

So, let us go to the next slide.

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The effective heat transfer coefficient for the total surface taking into account fin efficiency is then,

$$\bar{h} = \left(\frac{\eta_f A_f + A_w}{A} \right) h = E_f h$$

Where, E_f is the surface effectiveness correction factor and

$$\dot{Q} = E_f h A \Delta T = \bar{h} A \Delta T$$

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The effective heat transfer coefficient for the total surface taking into account of the fin efficiency is then \bar{h} , that is equal to $\eta_f A_f + A_w$ divided by A into h , or some factor E_f into h . See it is like this whenever fin is there; we are getting more surface area for heat transfer. So, more surface area we are getting for heat transfer, but unfortunately the temperature difference for heat transfer that varies along the length of the fin along the surface.

So, if it is so, either we can take care of it, but it is always easy to deal with a constant delta T. We will not consider a variable delta T along the surface of the fin. We will consider only a constant delta T along the surface of the fin. So, if we have to do this then what we have to do? The area we have to multiplied by some factor. Or heat transfer coefficient we have to multiply by some factor. In both the cases, these factors are less than 1. So, this is one we have already defined the fin efficiency.

We have defined the overall surface efficiency. So, basically we use the maximum temperature difference, but the maximum temperature difference is not available to the entire surface. So, the surface area is reduced by multiplying it with a factor which is less than 1. So, this is what you are familiar with. Alternatively, what you can do that, we can think of that heat transfer coefficient for the entire surface is not equal, and that heat transfer coefficient we are multiplying it with a factor called E f effective heat transfer coefficient.

So, then E f is the surface effectiveness correction factor, and Q dot is equal to E f h A delta T or h bar a delta T. So, basically we can we could have used our concept of overall surface efficiency. But this is another concept followed by some of the references some of the books. So, that is why I wanted to make you familiar with this particular concept. With these let us go to the next slide.

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Low-Fin tube heat transfer

The average convective heat transfer coefficient may be calculated from a correlation of Average Nusselt Number for the whole tube array, \overline{Nu} , as a function of Reynolds Number (Re), geometry and fluid properties.

$$V_{max} = \frac{\dot{M}}{S_{min} \rho}$$

$$Re = \frac{V_{max} D_r \rho}{\eta}$$

$$\overline{Nu} = \frac{\bar{h} D_r}{k}$$

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Low fin tube heat transfer. Actually I have told that there are 2 types of fins on the outer surface of the tube, low fin tube that is the fin height is small. And in this case what we do? The average convective heat transfer coefficient may be calculated from a correlation. So, in all the cases we will use some correlation ok. Fin tube heat exchangers all the cases whether it is low tube low fin tube or high fin tube, we will use correlation and there are number of correlations.

I am not going to burden you with correlations; there are handbooks available from where you can pick up the correlations. Only for the sake of completeness, one or 2 2 correlations very important correlations we will discuss during the course. May be sometimes we will give you a list of correlation, but that is not for memorizing; that is, as some sort of a reference information for you people. Because heat exchanger design cannot be completed unless I mean we it cannot be done unless we use these correlations. Many cases we have to use correlations and large numbers of correlations are available. Correlations are available for heat transfer; coefficient correlations are available for the estimation of pressure drop. So, here let us see how this correlation has been developed.

So, average convective heat transfer coefficient may be calculated from a correlation of average Nusselt number of the whole tube array. So, the average Nusselt number is denoted by \overline{Nu} as a function of Reynolds number and geometry of the tube. Reynolds number how are we going to calculate? One thing you have to understand that the fin tube they create passage for air flow which is not constant throughout. Throughout the fin tube the area of air flow path cross sectional area of air flow path is not fixed. So, what we do? We generally use 2 area.

One is frontal area of the heat exchanger sometimes it is called face area. And we also use the minimum area available for the air to flow. The minimum area available for the air or gas to flow that is gives us the maximum velocity. As we have considering steady state operation of the heat exchangers. So, minimum area will correspond to maximum velocity. So, here also we will do the same thing.

Let us say we have got mass flow rate of gas is \dot{M} , V_{max} maximum velocity we can calculate divide it dividing \dot{M} by S_{min} into ρ , ρ is the density of the fluid, gases fluid. S_{min} is the minimum cross sectional area which we have already calculated in our

previous lecture we have shown how to calculate S_{min} . I will again ask you to refer it back, and calculate S_{min} or get the expression of S_{min} by your own by your own calculation by your own derivation so, please derive it.

Then Reynolds number according to the equation one can get Reynolds number we have got V_{max} . We will take the outside area sorry, outside diameter of the tube as the characteristic length ρ and then μ with viscosity. So, this is how we will calculate the Reynolds number. Average Nusselt number, average Nusselt number will come from average heat transfer coefficient; the characteristic length that is the outer diameter of the tube, outer diameter of the bare tube which we have also used for the calculation of Reynolds number.

And then we will use the conductivity of the flowing fluid, that is air or any other gas. So, up to these things are simple, only thing is that we have to be careful what characteristic length we are taking, what characteristic velocity we are taking.

So now, our problem boils down to calculation of \bar{h} or rather, calculation of \bar{Nu} which is a function of Reynolds number and which will be also a function of your tube arrangement.

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The recommended correlation (Engineering Sciences Data Unit, 1984) is

$$\bar{Nu} = 0.183 Re^{0.7} \left(\frac{S}{L}\right)^{0.36} \left(\frac{P_1}{D_t}\right)^{0.06} \left(\frac{L}{D_t}\right)^{0.11} Pr^{0.36} \hat{f}_1 \hat{f}_2 \hat{f}_3$$

Where fluid properties are based on bulk mean fluid temperature and

- \hat{f}_1 = factor for fluid property variation
- \hat{f}_2 = factor for number of tube rows
- \hat{f}_3 = factor for tube arrangement

This correlation is applicable to liquids and gases with Reynolds numbers between 103 and 8×10^5

And is based on experiments covering the ranges $0.19 < \frac{S}{L} < 0.66$, $1.11 < \frac{P_1}{D_t} < 4.92$ and $0.058 < \frac{L}{D_t} < 0.201$

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So, the recommended correlation is something like this, Nusselt number is equal to Re to the power 0.7 S by L to the power 0.36.

P_r by D_t to the power 0.06 L by D_t to the power 0.11. P_r to the power 0.36 and then there are 3 factors correction factors. F_1 , F_2 , F_3 . So, what are their Nusselt number will be a function of Reynolds number and Prandtl number. So, these 2 non dimensional numbers are there. They this is not kind of a simple geometric. So many geometrical parameters are there. So, this geometrical parameters are there in the correlation.

What do you geometrical parameters would be important? The length of the fin or height of the fin that will be important, then the spacing between the fin that will be important so, that has been given. Pitch of the fin normal to the direction of flow that will be important and the outer diameter of the fin that is also or tip diameter of the fin that will be also important. So, all these things are there. And then we know Prandtl number take cares of the properties so, Prandtl number will come here.

Then of course, there are 3 factor F_1 , F_2 , F_3 ; where fluid properties are based on the bulk mean fluid temperature. This correlation Prandtl number we have calculated or even in Reynolds number there will be ρ and μ . So, these have been calculated based on bulk mean fluid temperature. F_1 is the factor for fluid property variation. So, this is one thing we routinely use for heat exchanger. Actually in heat exchanger there will be heat transfer between the solid surface and the liquid or the gases fluid; that means, between the solid surface and the fluid. So, bulk of the fluid and the solid surface will have different temperature.

This difference in temperature may give raise to property variation, and that property variation has to be taken care of. So, F_1 this is a factor it takes care of fluid property variation, because the bulk fluid temperature and the solid wall temperature tube wall temperature they are different. F_2 factor for number of tube rows how many tube rows are there. Generally, more the tube rows F_2 will come closer to 1, and F_3 factor for tube arrangement. Tube can be arranged in many different ways.

So, I mean depending on tube arrangement we can have some factors. So, generally if that to F_1 that is factor for fluid property variation, it is calculated as a ratio of Prandtl number calculated at the wall temperature and calculated at the bulk temperature. F_2 and F_3 generally the values are close to unity. This correlation is applicable for liquids and gases with Reynolds number between 10^2 to 10^8 into 10 to the power 5. So, you see it covers a very large range covering. So, called laminar and the turbulent range. And is

based on experiments covering the range see the range has been given for what range, this has been developed this correlation has been developed.

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Factor \hat{F}_1 is given by,
$$\hat{F}_1 = \left(\frac{Pr_b}{Pr_w} \right)^{0.26}$$

Pr_w is based on the mean surface temperature \bar{T}_w

$$\bar{T}_w = T_b - \frac{\dot{Q}}{\bar{h}A}$$

Where, \dot{Q} is the total rate heat transfer to the tube array,
 \bar{h} is the effective mean heat transfer coefficient taking into account fin efficiency,
 A is the total surface area

Factor \hat{F}_2 takes into account the number of tube rows, approaches unity for large number of rows

Factor \hat{F}_3 takes into account the geometrical arrangement of low fin tubes, unity for common tube arrangements

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F_1 as I have told it takes care of the property variation due to temperature variation. So, Pr_b that means, Prandtl number calculated at bulk temperature. Pr_w that is Prandtl number calculated at wall temperature to the power 0.26 Pr_w is based on the mean surface temperature T_w . T_w is equal to T_b minus this one. So, bulk temperature also one can get, where Q dot is the total rate of heat transfer to the tube array.

So many time what happens depending on how the problem is posed; so, you will find that the entire design calculation cannot be done explicitly cannot be done in one go without taking the help of any iteration. Particularly, when property variations are involve, because for calculating the heat transfer coefficient we need the property values, and then only we can calculate the wall temperature. But for knowing the property values wall temperature should be known and bulk mean temperature should be known.

So, sometimes you will find that a few steps of iterative calculation is required. Then factor F_2 takes the account of the number of tube rows approaches unity for large number of rows, and factor F_3 takes into account the geometrical arrangement of low fin tubes unity for common tube arrangements. Common kind of tube arrangement one can take it as unity.

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High-Fin tube heat transfer

$$\overline{Nu} = 0.242 Re^{0.658} \left(\frac{S}{L}\right)^{0.292} \left(\frac{P_1}{P_2}\right)^{0.09} (Pr)^{1/3} F_1 F_2$$

This correlation is applicable to liquids and gases with Reynolds numbers between 2×10^3 to 4×10^4 and $0.13 < s/L < 0.57$ and $0.15 < \frac{P_1}{P_2} < 1.72$

High-Fin tube heat transfer, In Line arrays:

$$\overline{Nu} = 0.3 Re^{0.625} \left(\frac{A}{A_T}\right)^{-0.375} Pr^{0.333}$$

This correlation is applicable for Reynolds numbers between 5×10^3 and 10^5 , and for all values of A/A_T between 5 and 12.

$$\overline{Nu} = \frac{h D_o}{k}$$

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So, with these let us go to the high fin tube heat transfer. High fin tube heat transfer is again giving some sort of a correlation. So, this correlation again in terms of Reynolds number and Prandtl number, and the geometrical parameter like the inter fin distance and pitch, transverse pitch and longitudinal pitch. P_1 is called transverse pitch, which is normal to the direction of flow. And P_2 is in the direction of flow, it is called the longitudinal pitch. So, these 2 pitch have been important, and then there are 2 factors F_1 and F_2 . This correlation is applicable to liquid and gases with Reynolds numbers range has been given, and then some other geometrical parameters have also been given.

So, this is one kind of correlation we get. And then we can have different correlations also. There are many many correlations available. So, another correlation has been shown below; where the correlation is applicable for Reynolds number between 5 into 10 to the power 3 to 10 to the power 5 and all values of A/A_T between 5 and 12. Let me explain these correlations little bit. So, second correlation also you can see that it is based on Reynolds number and Prandtl number. So, you see whenever flow is external flow and whenever flow is high Reynolds number flow. So, then we get this kind of relationship where Nusselt number is some sort of a coefficient constant.

Ah So, it is like this let me most of you know it, but in then let me write it. Nu is equal to $C Re^{C_1} Pr^{C_2}$. And then probably it will be multiplied by some sort of a function of geometry, some sort of correction factor. You see already

we are familiar with heat transfer coefficient or Nusselt number, but here with whatever we have done from there we can see that Nusselt number is in general given by this kind of a formula. Some constant R_e to the power C_1 and P_r to the power C_2 , C_1 and C_2 are again constants.

Then if it is a complex geometry then some geometrical parameter will come; that means there are more links which are important, I mean one can have though one select one characteristic length, but other lengths are also important; so then one have to have some geometrical parameters. And then correction factor like temperature correction. Like correction for some other geometry. Sometimes even we give correction factor if some sort of natural convection is also associated with the flow con force convection sometimes if the flow is not fully developed etcetera. So, do we give some sort of correction factors? So, these correction factors are there.

So, 2 correlations I have shown with high fin tube for high fin tube heat transfer; obviously, the correlation at the top is more complex the correlation at the bottom is not that complex. But at the bottom there is one term which needs explanation A by A_T . A is the total area A is the total area of the finned and unfinned portion of the tube; that means, for the fin tube certain fin area is there, and certain unfinned tube area is there.

So, A is calculated taking all these area. And A_T is the bare area of the fin, sorry bare area of the tube. Suppose the same length of tube is there, but it does not have any fin, then whatever area we will get that is the bare area, and that is given by A_T and here we have taken this. So, you see if you remember in our earlier lecture, we have calculated the area of the tube bare tube; so that is important. In certain cases, that will be needed for calculating the heat transfer coefficient. So, this is one example we have got. So, basically then what we have discussed?

We have discussed how the Nusselt number is to be calculated for low fin tube heat transfer surface and high fin tube heat transfer surface. So, in all the cases the definition of Nusselt number is the same; that is, Nusselt number let us call it. So, this definition we have sorry, this definition we have used so, this definition we have used a \bar{Nu} \bar{Nu} , let us see so, this is k only. So, $\bar{Nu} \bar{Nu}$ is equal to $\bar{h} D_r$ by k \bar{h} is the average heat transfer coefficient. And D_r is the root diameter or the outside diameter of the tube and k is the, k is the conductivity of the gas which is flowing. So, with these one

can calculate the h bar. So, both in case of your low fin tube and high fin tube we can calculate the h bar.

So, this gives us, suppose from the external surface of a fin tube we like to calculate what is the heat transfer coefficient, sorry, what is the rate of heat transfer we can do this analysis. We can also do this analysis or we can also use this analysis, if the fin tube heat exchanger is to be designed and we have to calculate the overall heat transfer coefficient u . Because in that case the gas side h bar u need to know, and gas side h bar u are getting from this formula. So, you have to get the gas side h bar from this analysis. Now gas side h bar is dependent on gas side area or finned area.

That is why the previous lecture we have spent, because we have shown you how to calculate the fin side area; so, that is important. So, with the previous lecture and this lecture with this 2 lecture we have some idea that how to calculate the fin side area and how to calculate the fin side heat transfer coefficient. Obviously, heat transfer coefficient will come from some correlation, but that correlation will require certain inputs, will require certain parameter which probably we have to estimate from the geometry, and that is how that is what I have taught you.

So, next what we will take up? As I have told that due to the complex geometry of the fin we had to have these complications and calculation requirement of calculation etcetera. Similarly, for pressure drop calculation also this fin side is bit unique compared to tube side. So, next we will see how the pressure drop calculation for the fin side can be done. Now let me tell you one word of caution. I have taken up some example, but there could be other examples and there could be other correlation.

Particularly, there are many many correlations, and geometric and vary to a very great extent and we can have let us say we can have serpentine kind of tube we can have, different kind of tube layout etcetera. So, one has to be one has to take some sort of a judgment. One has to take some sort of a judicial action while selecting the correlations and while calculating the geometrical parameters ok. So, what I have told given it is just some sort of a guideline, but it is not suitable for all the cases. This is what you have to keep it in mind.

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The efficiency of the radial fin can be determined by modifying the efficiency of the longitudinal fin.

$$\eta_{f,L} = \frac{q_f}{q_{max}} = \frac{\tanh(ml)}{ml}$$
$$\eta_{f,R} = \frac{\tanh m\psi}{m\psi}$$

Where,

$$\psi = \frac{D_r}{2} \left(\frac{D_t}{D_r} - 1 \right) \left(1 + 0.35 \ln \frac{D_t}{D_r} \right)$$

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With these we go to the fin. The efficiency of the radial fin can be determined by modifying the efficiency of the longitudinal fin. This is again I am giving some sort of a new input. Earlier I have told that the radial fins are there, calculation of the efficiency of radial fin involved Bessel function. So, what you do? We do not calculate the Bessel function, there are charts or graphs from the graphs we can determine the radial fin efficiency.

But even looking in to graph that is sometimes little bit time consuming and cumbersome. So, what we are telling? That longitudinal fin efficiency which comes as a function of tan hyperbolic, so that can be used with some sort of modification. So, you see I have given $\eta_{f,L}$ longitudinal fin efficiency that is given by $\tanh ml$ by ml . All the symbols have their usual meaning; l is the length of the fin. Probably for circular fin I have used a capital L , but l is the length of the fin.

Now for radial fin what we will do? $\tanh M \sigma$ we will use $M \psi$ we will use. So, this symbol we will use, and where this is given by the geometry of the fin ok. And in this if you provide d_t / D_r or d_t by D_r close to 1, you will find that it is becoming a longitudinal fin. So, basically then we have not 2 we have got 2 ways now. Either we can use the chart for the efficiency of radial fin. Or we can use this kind of a formula and go ahead. So, the formula this is quiet handy to use.

With these I come to an end. So, we have learned quite a few things that we have learnt how to take how to do the calculation for the fin side. Both the geometric calculation and how to estimate the heat transfer coefficient. And then again I have told you a an easier way to calculate the fin efficiency, calculate the efficiency of circular fins.

Thank you. And then, we will proceed with our discussion where we will do the pressure drop calculation for the fin side.