Mechanism and Robot Kinematics

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Lecture - 09

Geometry and Representation of Vectors

In the course of our discussions on kinematic analysis, we will draw upon concepts from Vector Algebra. So in this lecture, I am going to discuss some of these concepts that you will use in the forthcoming lectures. Most of the material that I will cover today is known to you, but to have uniformity in our understanding I would like to go through or revise these concepts.

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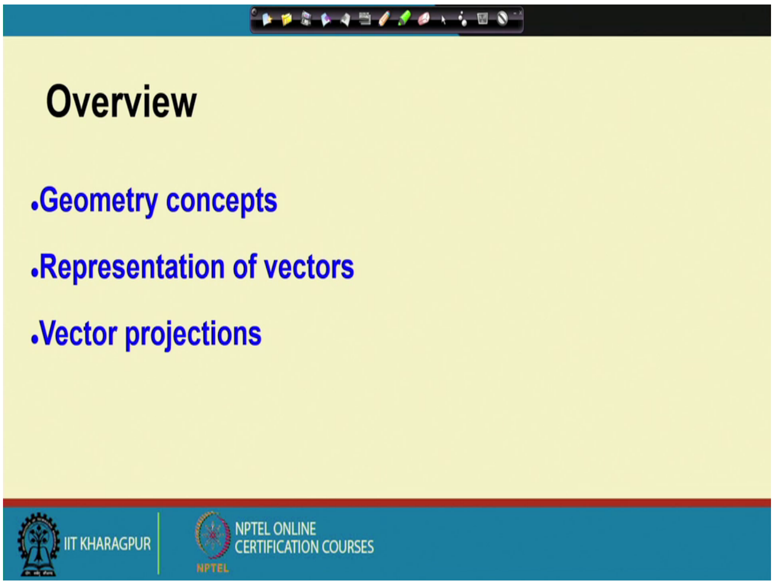

So, here is the overview of what we are going to discuss today: we are going to look at some geometric concepts, representation of vectors, vector projections etcetera.

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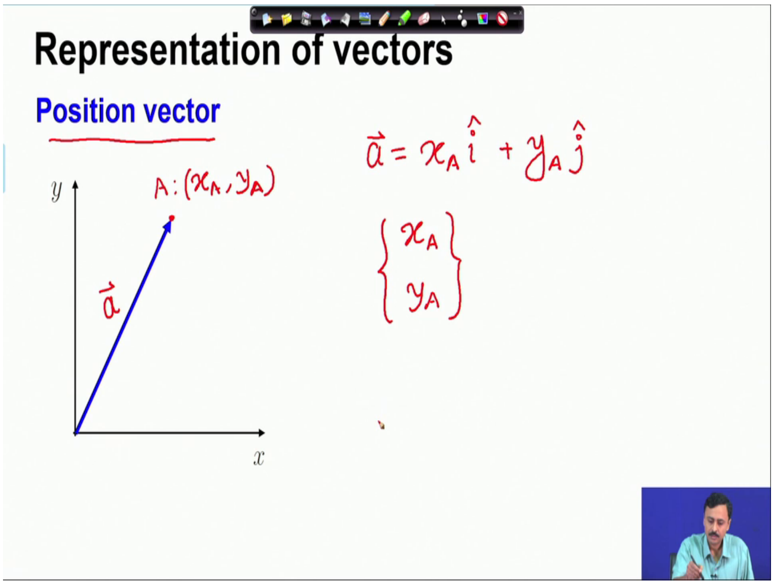

So, let us begin with the position vector as you know the position vector gives us the position of a certain point let us say this point a whose coordinates I can write in this form x A comma y A an ordered pair.

So, the position vector is a vector that represent the position of this point a in a certain coordinate system here I have chosen the planar Cartesian coordinate system. So, let me represent this position vector by this symbol lowercase a with a vector. So, as you know that this vector can be represented as x A i cap plus y A j cap. So, this is the position vector this is the position vector of point a.

Now, we sometimes represent vector also by a column vector. So, that also represents a vector which contains the coordinates of point a consider 2 points a.

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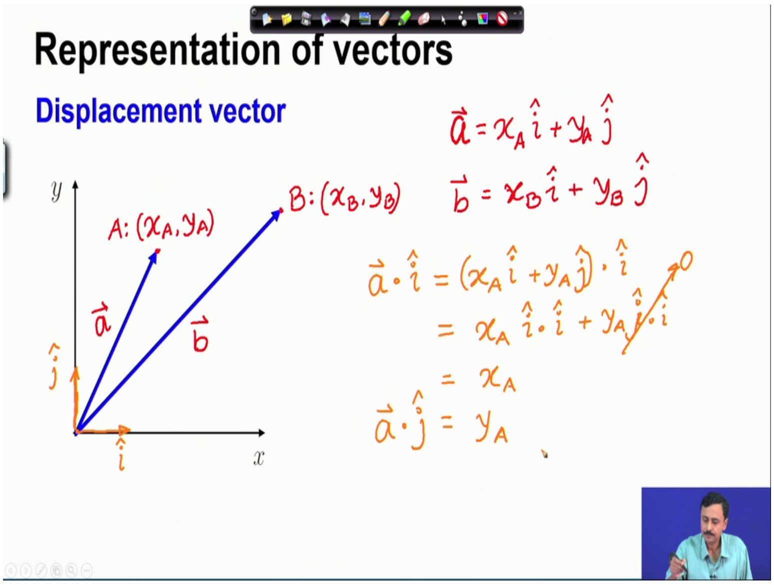

With position vector lowercase a coordinates x A y A and another position vector corresponding to point b with coordinates x B y B and represented by this lowercase vector b.

So, therefore, I have these representations for vector a and this is for vector b. So, here of course, you know that I cap and j cap are unit vectors which are along the directions. So, I cap is along x and j cap is along y.

So, you can find out the coordinates of the point by taking projections for example, if you want to find out the coordinates of point a then you need the position vector and if you dot it with I that will give us this. So, using distributive property I will get x A i cap dot i cap plus y A j cap dot i cap i cap dot i cap is one because these are unit vectors. So, this becomes x A and since i cap and j cap are orthogonal, this is 0 so I have x A. Similarly if i dot it with j cap you can work out that you get the coordinate y A. So, this way you given the position vector you can find out the coordinates of the point.

Now, let us look at the displacement vector.

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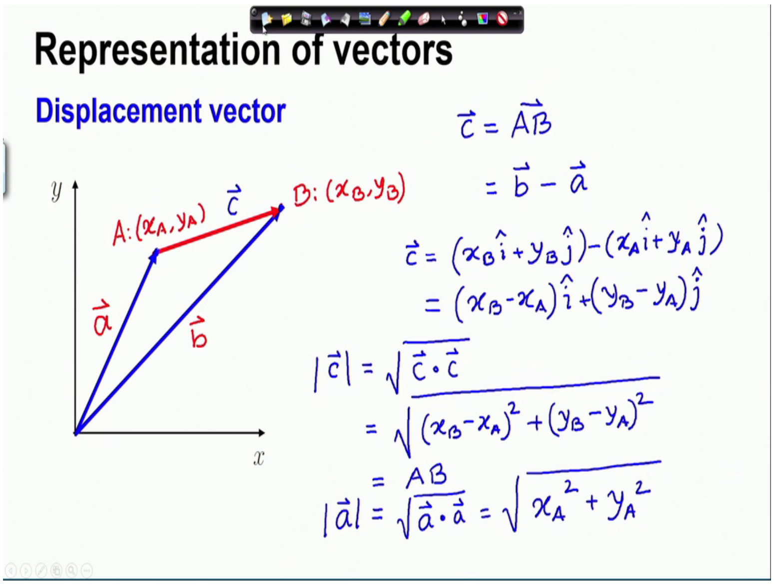

So, once again I have this point a and point b. So, these are the vectors vector a and b consider the vector joining points a and b let us call this the vector c. So, c goes from a to b. So, I will represent c as a vector from a to b. So, I can write c as a b vector.

So, this can be written in terms of this lowercase a and b vectors in this manner. So, b minus a since capital B is the final position and capital A is can be considered to be the initial position. So, c goes from initial position to final position. So, therefore, is the final position vector minus the initial position vector. So, it is b minus a. So, therefore, vector c can be written as x B i cap plus y B j cap minus x A i cap plus ya j cap. So, this is the expression of vector c. So, this is the displacement vector

Now, if you want to find out the magnitude of vector c you want to find out the magnitude of vector c. So, that you know is square root of c dot with itself. So, this turns out to be and under root of x B minus x A whole square plus y B minus y A whole square. So, that is the magnitude of vector c which is the length. So, this is equal to the length a b.

So, you can also find out the magnitudes of vectors a and b. So, for example, magnitude of vector a is nothing but x A square plus y A square. So, that is the magnitude.

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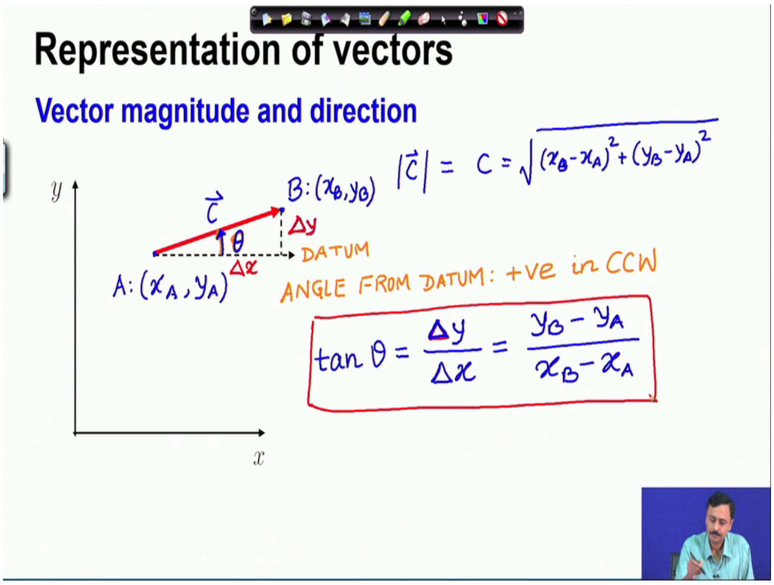

So, we have discussed vector magnitude. So, consider that this is vector c again. So, the magnitude of vector c. Let us now indicate that by lowercase c. So, we represent the magnitude of this vector c by lowercase c.

Next comes the direction. So, let me mark out the positions once again. So, these 2 points a and b and c is the vector from a to b. So, we have talked about the magnitude. So, that is the magnitude of c vector c which I represent as lowercase c.

Next the direction: so direction is represented by the angle that this vector makes with one of the axis. So, here I am considering the x axis as my datum and I am measuring all angles from this datum one thing you need to remember is you have to measure angles only in one direction. So, that the sign of the angle is taken care of. So, what I mean is I will measure any angle a vector makes with the datum as positive in the counterclockwise sense. So, angle measured from the datum is positive in the counter clockwise direction.

So, here I have shown an arrow. So, from the datum I am measuring theta is positive in the counterclockwise direction. So, how do we now represent or find out this angle n. So, this angle is determined from the expression of tangent theta or tan theta. So, tan theta is nothing but this delta y by delta x. So, tan theta is delta y by delta x where I have shown delta y and delta x in the figure.

So, therefore, this is nothing but y b minus y A by x B minus x A. So, remember when the vector goes from a to b and I am measuring this angle theta, in this manner in the positive in the counterclockwise sense. So, tangent theta is delta y divided by delta x and delta y is the y of the final position point minus y of the initial point that is delta y divided by x coordinate of the final point minus x coordinate of the initial point. So, this is important.

Now, from here I can calculate theta now this definition will remain unchanged whatever happens to this vector. So, this is how we are going to calculate the angle and remember this angle is measured from the datum in the counterclockwise sense. So, what are the implications of this?

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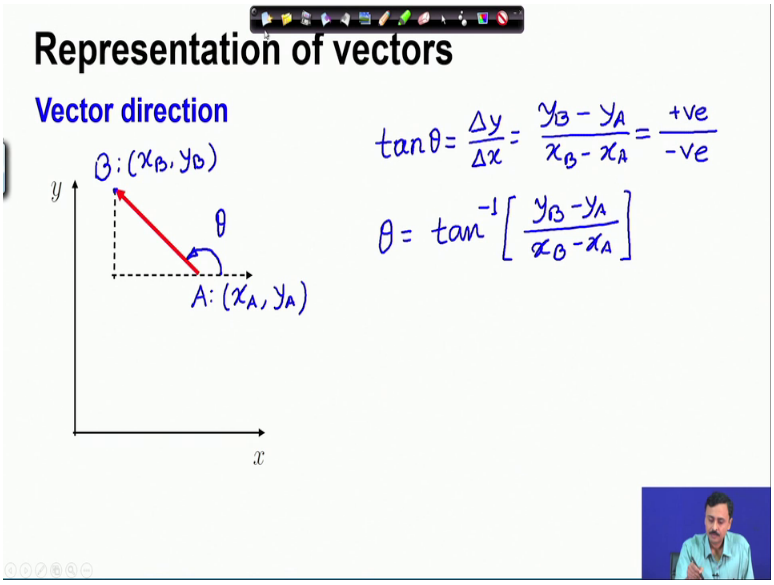

Let us look at this vector. So, this is point a and here is point b, I have defined tangent theta as delta y by delta x is equal to y coordinate of the final point minus y coordinate of the initial point divided by x coordinate of the final point minus x coordinate of the initial point.

Now, you will find that this ratio has numerator as positive and the denominator as negative numerator is positive and the denominator is negative. So, when you do tangent inverse to find out theta when you do this tangent inverse to find out theta maybe on a calculator and if you are not careful, it is going to give you a negative angle, but as we have noted this angle is definitely not negative and what is our angle this is our angle this is measured from this datum which is the x axis the horizontal axis is our datum.

So, therefore, you need to be careful and take into account whether the numerator is positive or negative or the denominator is positive or negative when you get this ratio as positive or negative specifically when it is negative; if the ratio is negative you have to be careful about which is negative numerator or denominator there can be another case where the ratio is positive yet then both the numerator and the denominator are negative the ratio can still be positive is positive.

So, once we have this we need to take care of the sign of the numerator and denominator and then decide upon the angle. So, as you know tangent theta to tan theta is sin theta by cosine theta. So, sin theta is positive in the second quadrant while cos theta is negative in the second quadrant that is why with this vector that I have shown the numerator is positive while the denominator is negative.

So, you will have to look into the sign of numerator and denominator in order in order to decide whether this angle this vector direction or the angle of the vector is in the second quadrant or the fourth quadrant both will give you a negative ratio.

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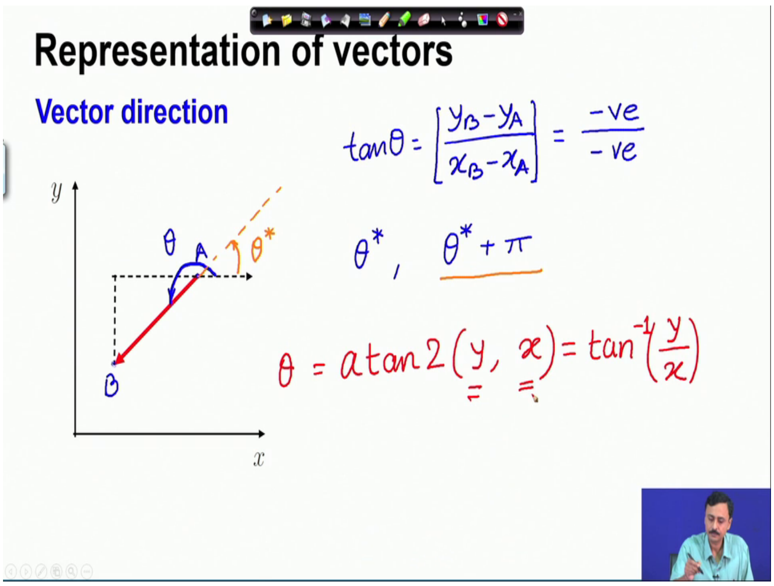

So, you have to look at the sign there can be further cases like this. So, this is a, this is b. So, in this case tangent theta is again y B minus y A divided by x B minus xA, in this case, you will note that both the numerator is negative and the denominator is negative. So, the ratio is positive. So, if you are not careful it will give you a wrong theta when you do tangent inverse just with the ratio.

So, you will have to look into the sign of the numerator and denominator. So, both are negative sin is negative and cosine is negative because tangent theta is ratio sin theta and cos theta. So, both are negative in the third quadrant. So, in this case your angle should be this. So, when you determine these angles by taking tangent inverse you should also note the sign of the numerator and denominator which will give you the proper quadrant. So, if suppose theta star is a solution of tan tangent theta equal to this ratio, then you know that theta star the spy is also a solution.

So, if you get a positive ratio like this here the ratio is positive if you calculate the value of this ratio it is positive if you do tangent inverse of that positive quantity then what you are going to get is this angle, if you do the tangent inverse of the positive quantity you are going to get this angle theta star, but because numerator and denominator both are negative this is the solution. So, this blue angle theta is the solution.

In order to take care of such things there is a function called a tan 2 which is nothing but tangent inverse of y by x in most programming languages and in computational softwares, you have this function a tan 2 and it takes the numerator and denominator separately for obvious reasons, it will look at the sign while giving you the angle and when you do use this a tan 2 function it does tangent inverse and gives you the angle in the proper quadrant. So, the angle that you will get will be in the proper quadrant. So, it will look at the sin of the numerator and denominator and give you the angle in the proper quadrant. So, you should be aware of this function a tan 2.

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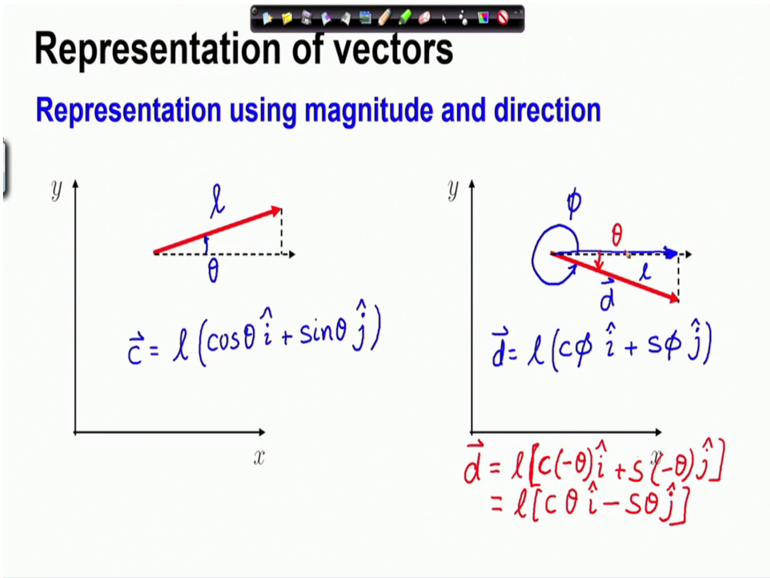

So, when you have vectors of a specific magnitude and maybe a direction. So, you have been specified a vector of a certain magnitude and certain direction, then the representation of this vector, let me call it c is magnitude times the unit vector in this direction. So, that this is the representation of this vector.

Now, when it comes to a vector as I have shown you on the right this should actually be the angle. So, from the datum measured positive in the counterclockwise sense therefore, suppose this vector is d, then d has a representation of let us say the magnitude is l, then l cosine phi i cap the sin phi j cap.

Now, this defining angle like this will automatically take care of the sign as you know cosine is positive in the fourth quadrant. So, this will turn out to be positive as expected because it has got a positive projection along the x direction and it has got negative projection along the y direction sin phi is negative in the fourth quadrant, but you can also sometimes represent this sometimes it is quite tempting to use this angle let me call this angle theta, but now you see from the datum I have to measure in the clockwise sense. So, therefore, you can also represent d as l cosine of minus theta i cap plus sin of.

So, I have represented cosine theta as c cosine of minus theta is c of minus theta I cap plus s of minus theta j cap. So, sin of minus theta j cap which turns out to be l cosine theta I cap minus sin theta j cap. So, that is also a representation of d, but now using an angle measured in the clockwise sense.

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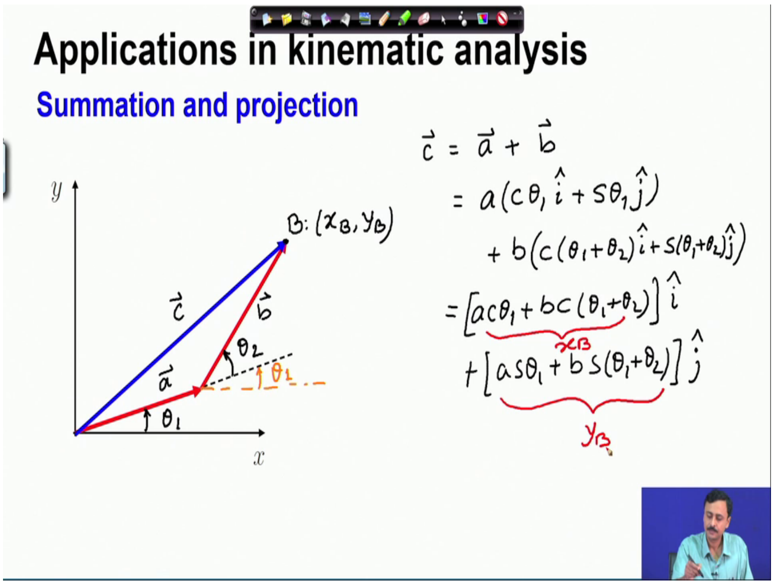

Now, let us look at summation of vectors sometimes in applications you will have situations like this where this is one vector let us call it a and here is another vector let us call it b and the angles are specified in this form. So, theta 1 is the angle that a makes with the x axis theta 2 is the angle that b makes with the a vector. So, such situations arise let us call this point b whose coordinates are x B, y B, I would like to find this vector c which is the position vector of point b.

In terms of the magnitudes and directions of vectors a and b. So, vector c is nothing but vector a plus vector b vector a is magnitude of a times cosine theta 1 i cap, I am writing c theta 1 for cosine theta 1 plus sin theta 1 j cap. So, again s theta 1 for sin theta 1 plus b times now cosine the angle this vector b makes with the x axis I have to find that because this is our datum now here this is theta 1. So, therefore, this becomes cosine of theta 1 plus theta 2 i cap the sin of theta 1 plus theta 2 j cap . So, therefore, this becomes a cos theta 1 plus b cos theta 1 plus theta 2 i cap plus a sin theta 1 plus b sin of theta 1 plus theta 2 j cap.

So, now you can read out the coordinates x B and y B of point b. So, this is x B and this is y B just by taking projections of c with I cap and j cap. So, you will get the coordinates of point b; now let us look at another situation where it is of this form.

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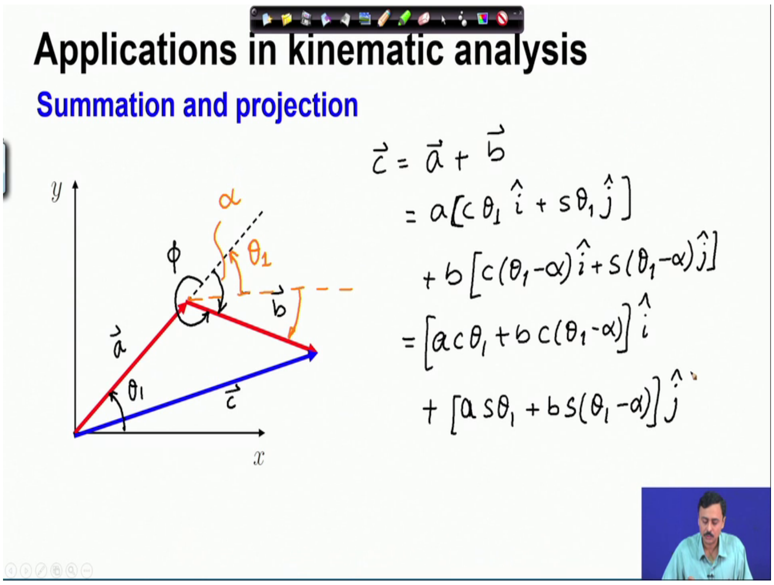

So, once again I have these angles let us say theta 1 now this angle should be measured from the from this line, let us say phi or you can also alternatively use theta 2 in the clockwise sense.

In that case, I still have the same vector equation. So, a cos theta 1 i cap plus sin theta 1 j cap plus b, remember I have to measure this angle of vector b from the datum. So, I have to measure this angle. So, if I measure this angle and instead of calling this here you can see that; so this angle is theta 1 and suppose I call this angle as some alpha then this expression becomes cosine of theta 1 minus alpha I cap. So, theta 1 goes positive minus alpha I cap plus sin of theta 1 minus alpha j cap. Therefore, this becomes. So, b in theta 1 minus alpha j cap. So, that will be the representation of c.

So, what I want to underline is you should always measure angles in the counterclockwise sense as positive if it goes in the clockwise sense, then it should be taken as negative.

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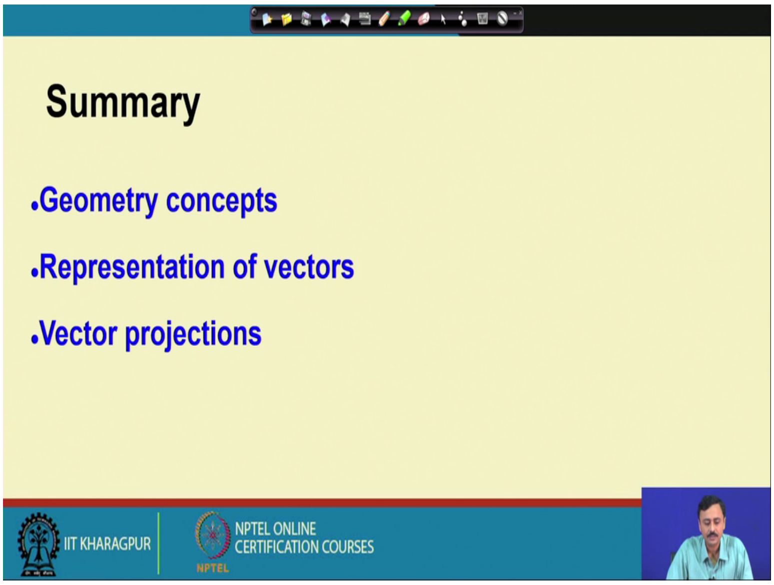

So, with that let me summarize; what we have discussed. Today, we have looked at some geometric concepts we have looked at representation of vectors and we have also looked at vector projections.

With that I close this lecture.