Mechanism and Robot Kinematics
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Lecture - 08
Grashof Criteria - II

So, we have been discussing about Grashof Criterion and the presence of a completely rotatable link in a kinematic chain, we have looked at the 4R chain. So, today we are going to go further in this discussion.

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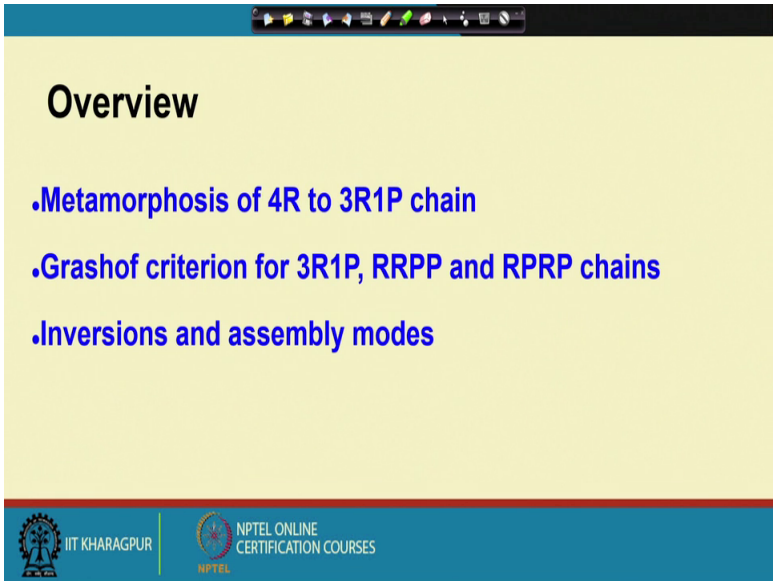

So, here is the overview of today's lecture, first I will show you how 4R chain can be converted to a 3R1P chain, this is called metamorphosis; a gradual chain; a change of a 4R chain to a 3R1P chain and then I can extend the Grashof criterion for the 4R chain to various other chains, as I will show you in this lecture then we will talk of inversions and assembly modes.

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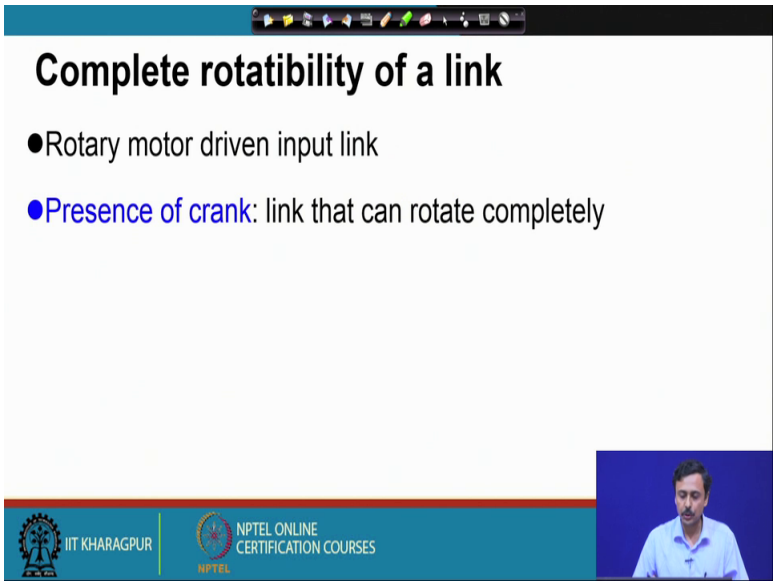

So, just to review what we have already discussed we were looking for a completely rotatable link because in many many applications you have a rotary motor driving the input link. So, we look for presence of crank; crank, this is a link that can rotate completely and hinged with the ground and the motor is fixed to the ground.

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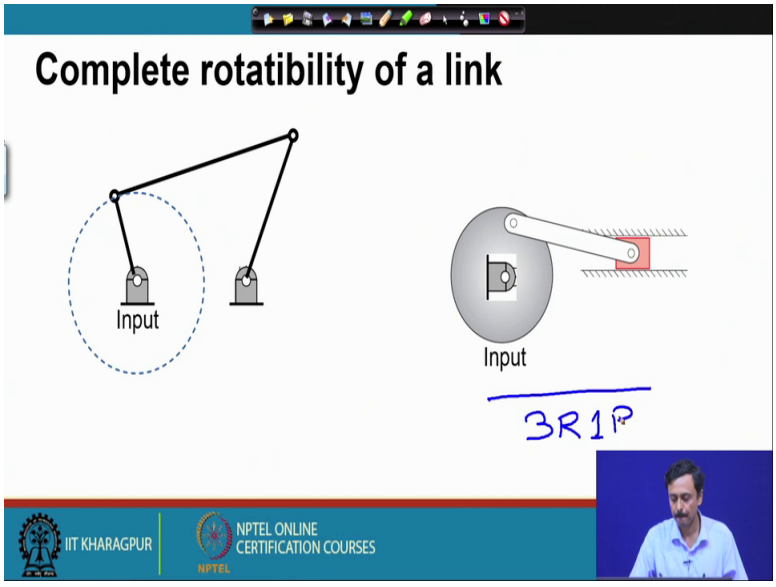

So, we are looking at the presence of crank in a kinematic chain we have looked at these examples the 4R chain. And today, we are going to look at this 3R1P chain which I have shown here.

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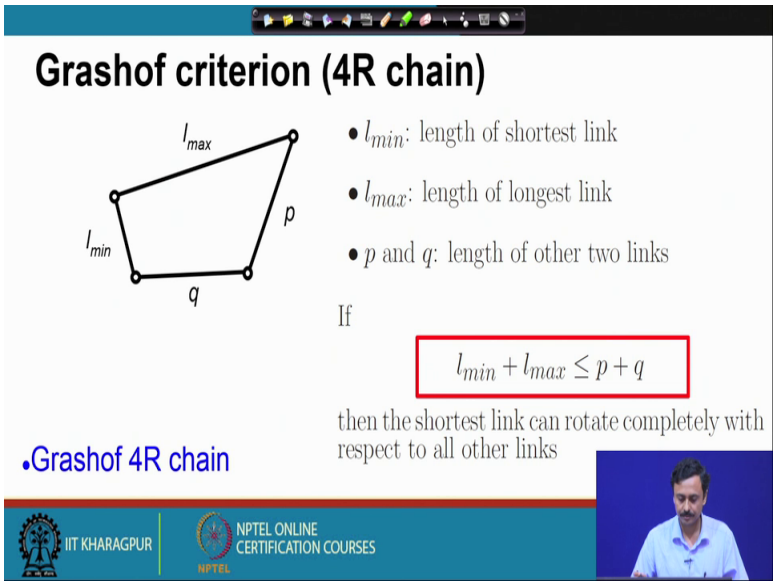

Just to review of review the discussions we had on Grashof criterion if l min is the length of the shortest link l max is the length of the longest link and p and q are the lengths of the intermediate links then Grashof criterion says that if l min plus l max is less than equal to p plus q, then the shortest link can rotate completely with respect to all other links. So, this was essentially the Grashof criterion for 4R chain and we say 4R chain to be a Grashof 4R chain if it satisfies this Grashof criterion.

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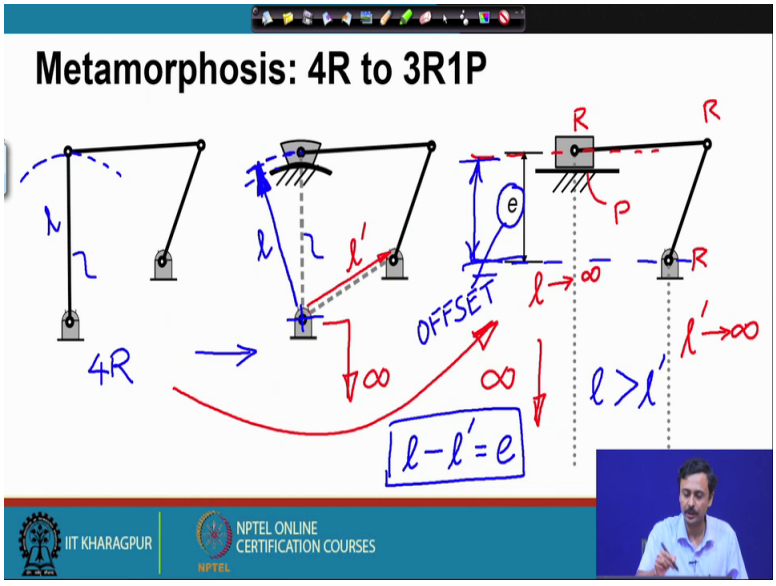

Now, let us come to this what I call metamorphosis from a 4R chain to a 3R1P chain. So, I have shown all the stages in one go. So, here we start with a 4R chain which I can think of as. So, so this link I can think of as a scurve slider rotating on a sliding surface whose radius of curvature is same as the length of this link. So, the length of this link is equal to the radius of curvature of this path on which the slider is moving. So, essentially it is the radius of curvature of this pin. So, it is the radius of curvature of this pin path of this pin because this pin is going to have the same radius of curvature l. So, I can put it on a slider such that the pin describes a circular trajectory circular path with this point as its center.

Now, I start taking this further and further till it goes to infinity. So, I start taking this ground hinge down further and further till this goes to infinity. So, which means the radius of curvature is going to infinity. So, curvature becomes zero. So, this becomes a straight path the path of this pin now becomes straight. So, this I will call metamorphosis of a 4R chain to a 3R1P chain. So, this is R, there is 1 R here and P here.

Now, here I have a definition you see that there are these link lengths. So, one is this length l and let me call this as l prime. So, l is this link length and l prime. So, this is l I have mark it and this is l prime; now as I take this hinge to infinity this l goes to infinity and what is l prime? L prime connects this hinge to this hinge. So, that also goes to infinity, but then if I look at their difference imagine that infinity is actually a very large number if I look at their difference the difference is exactly this which I called e; the difference in these 2 infinities is e; e is known as the offset the offset of a 3R1P chain, e is known as the offset. So, offset is the difference between the 2 infinities 2 link lengths that are now infinity and you can see that one is definitely larger than the other and this difference is the offset.

So, we have l larger than l prime. Now this is an important observation, l must be larger than l prime that is why l prime l minus l prime l minus l prime is e to offset now this allows us to extend the Grashof criterion for 4R through 2 3R1P this is what we are going to see.

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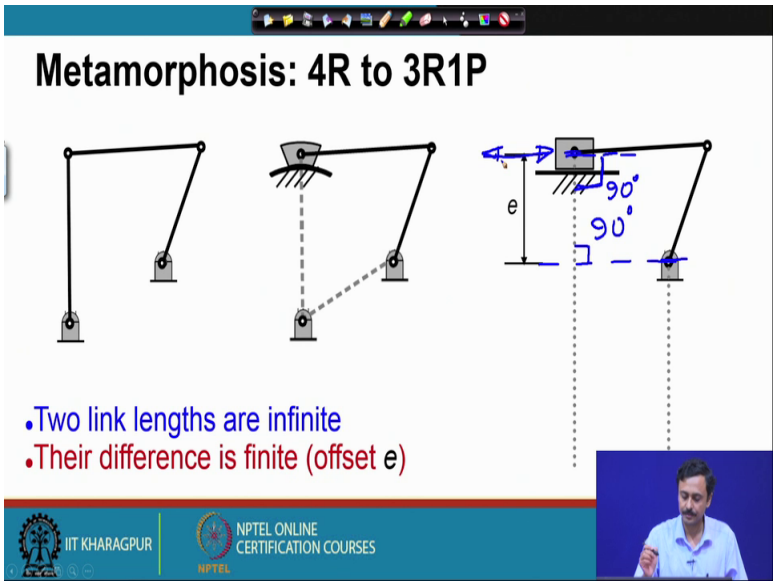

So, as I mentioned 2 link lengths are infinite, but the difference is finite which is known as the offset. So, remember that the offset is being measured perpendicular the offset is being measured perpendicular to the direction of travel. So, this is the direction of travel of the slider and offset is being measured perpendicular to the direction of travel.

So, these are offset is the distance perpendicular distance between the ground hinge and in this particular case the ground hinge here and the slider hinge measured perpendicular to the direction of travel, I can define distances between the ground hinge here and the slider hinge in various ways, I can join them and measure a distance, but this distance please note is measured perpendicular to the direction of travel the distance between the ground hinge here and the sliding hinge on the slider. So, measure perpendicular to the direction of travel.

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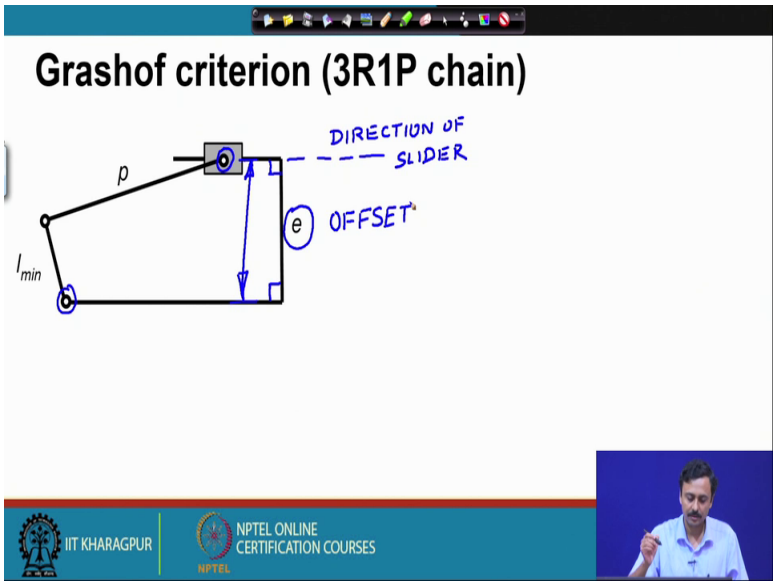

So, we come to the Grashof criterion for 3R1P chain here I have redrawn the chain in a slightly different manner now you can see this once again the definition of e here I have purposefully drawn them as ninety degree, but they can be connected in arbitrary manner. So, the direction of travel and the slider is this. Therefore, this is the sliding hinge on the hinge on the slider and this is the other link other hinge. So, the perpendicular distance between them the distance between them perpendicular to the direction of sliding is the offset.

So, with this explanation let us now move forward what was Grashof criterion for 4R changes.

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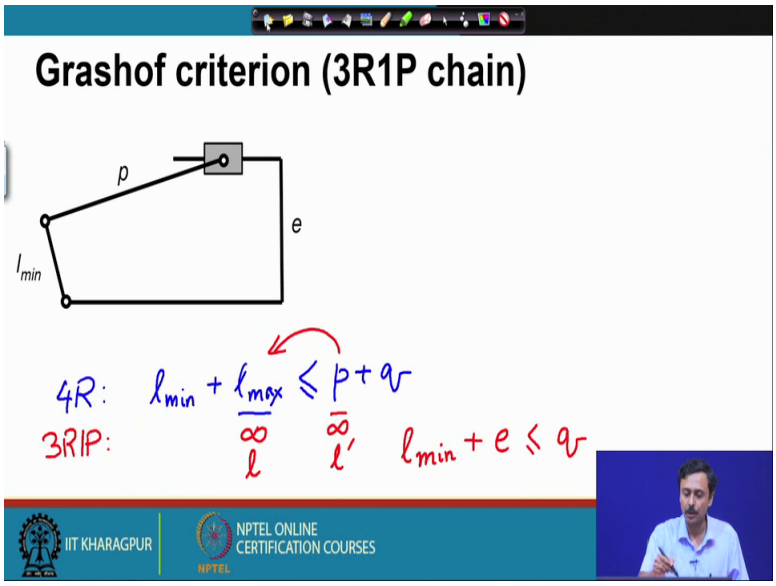

Just let me reconnect. So, l min plus l max less than equal to some of the other 2 links this was Grashof criterion for a 4R chain. Now I have we have seen that l max is infinity in a 3 or one p chain the longest link is infinity that we had written as l there is another infinity. So, let us choose; let us say this is also infinity. So, this was l prime if you remember from the previous slide. So, l max was infinity which we indicated by l and there is another infinity let us say p that is indicated by l prime. Now l minus l prime is e that is how offset is defined.

So, therefore, if I bring this p on to this side what I get for a 3R1P chain is l min plus l max minus p is e and that must be less than equal to the other link. So, this is a; this is what it is this is what crush off criterion for a 3R1P chain is let us look at it formally.

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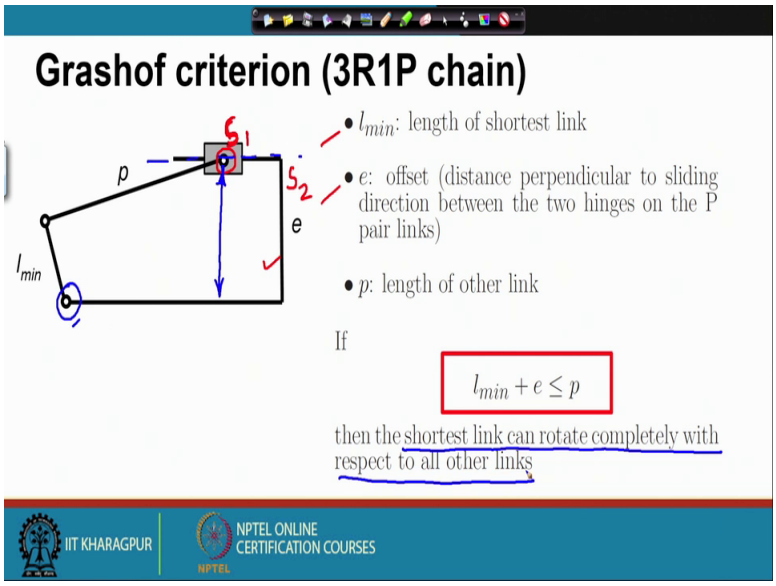

So, if l min is the length of the shortest link e is the offset as I mentioned I have defined this distance perpendicular to sliding direction between the 2 hinges on the p pair links. So, if this is one of the sliders this is another slider. So, slider one and slider to link one these 2 links the slider is a link and this is a link I call them S 1 and s two. So, the hinge on S 1 is this while the hinge on S 2 is this.

So, in another way I can define offset as the distance perpendicular to the sliding direction this is a sliding direction distance perpendicular to the sliding direction between these 2 hinges one hinge belongs to slider one the other hinge belongs to slider 2 both are sliders with respect to one another if something slides with respect to another the second one slides with respect to the first. So, both are sliders and these are the hinges on the 2 sliders.

So, that is offset again- and let us say p is the length of the other link then Grashof criterion says for a 3R1P chain states that if l min plus e is less than equal to the other link which is p here then the shortest link can rotate completely with respect to all other links. So, that is Grashof criterion for 3R1P chain.

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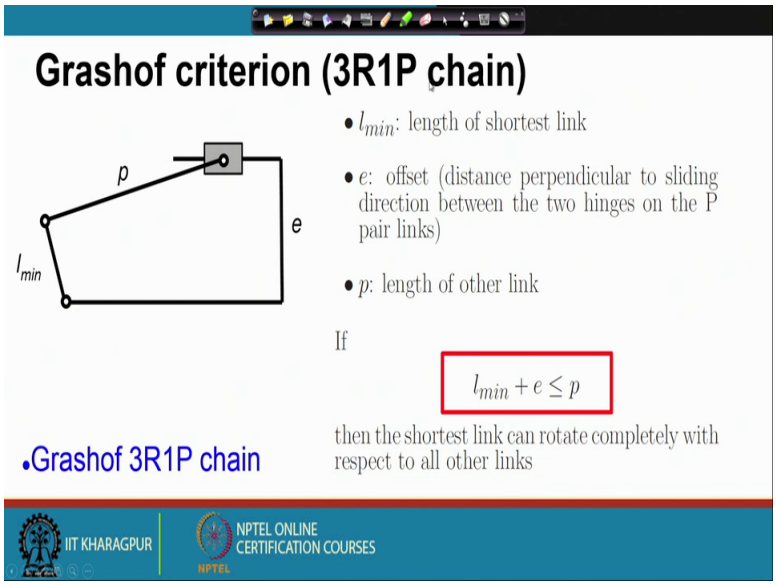

And we call this a Grashof 3R1P chain if it satisfies this Grashof criterion.

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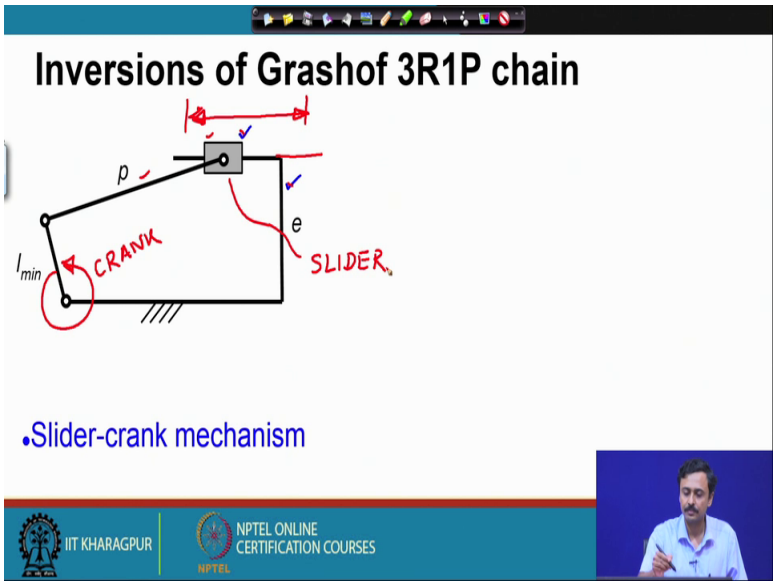

Now, let us look at inversions of the Grashof 3R1P chain, if I fix this slider as I have been mentioning that there are one is this slider block the other is this the slider block and this link is also in sliding with respect to that slider block. So, there are both sliders. So, if I fix this link as ground and if it satisfies the Grashof criterion then l min can rotate completely and relative motion between any other pair of links may be this slider and this slider or this slider and this link they are all in oscillatory motion. Therefore, this

slider block can only slide oscillate the sliding motion will be oscillating between certain limits of course, this needs to be extended.

So, when l min rotates completely all other links are in relative oscillation. So, this slider block will slide this is called therefore, a slider crank mechanism. So, this is the crank and this is the slider; so slider-crank mechanism.

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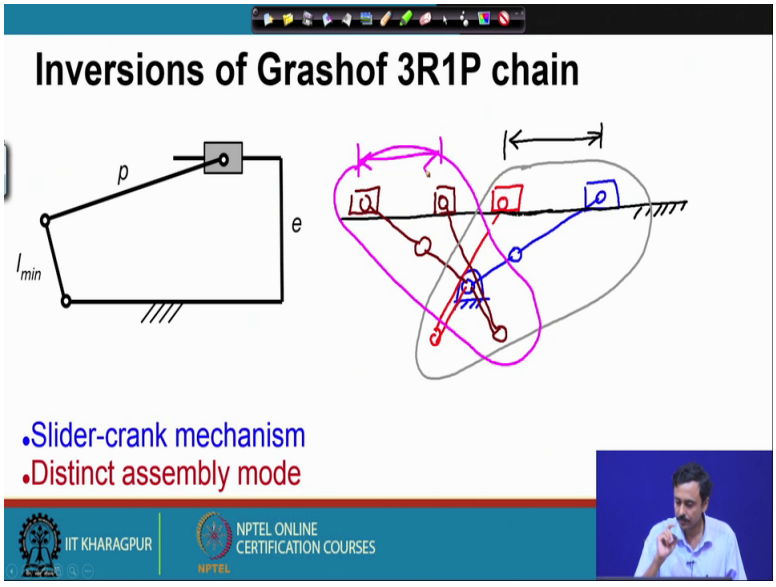

Now, this has distinct assembly modes now how do we understand the assembly modes. So, let me draw. So, this is a slider sliding on the ground. So, this is the ground. So, this is one extreme configuration of the slider this is another extreme configuration of the slider. So, the slider oscillates between these 2 configurations.

Now, I can also assemble the mechanism like this I can put the slider on this side and assemble the mechanism in that case it might oscillate between these 2 limits. Now, it cannot go from this assembly configurations to these assembly configurations this cannot go as you can see the slider can only oscillate between these 2 here, it can oscillate between these 2. So, there are 2 distinct assembly modes of a Grashof chain as we have seen before also for a 4R chain the slider cannot cross over from one to the other.

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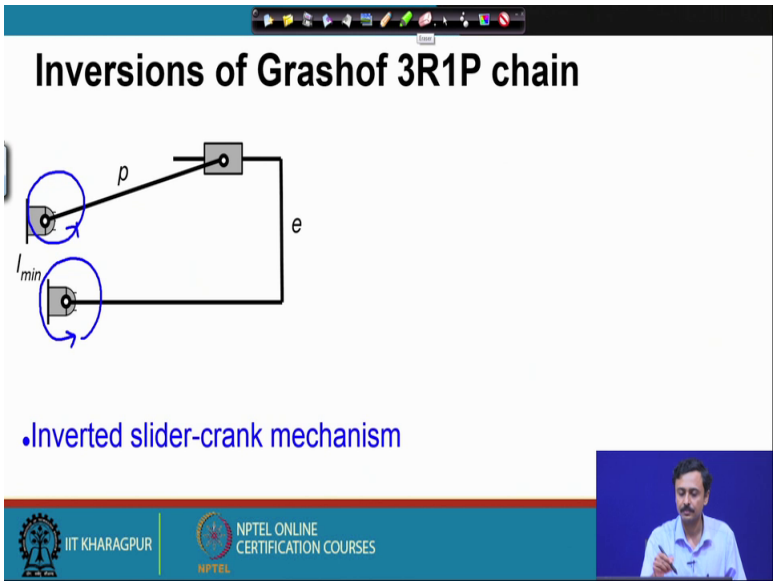

Now, if I fix the shortest link which we have seen the shortest link and rotate completely with respect to other links; so if I fix the shortest link all links should be able to rotate completely with respect to the shortest link. So, this can rotate completely, this can also rotate completely, but the motion between these 2 sliders. So, it has got sliding motion that will remain sliding. So, this is called an inverted slider crank mechanism inverted slider crank mechanism if I fix this link p.

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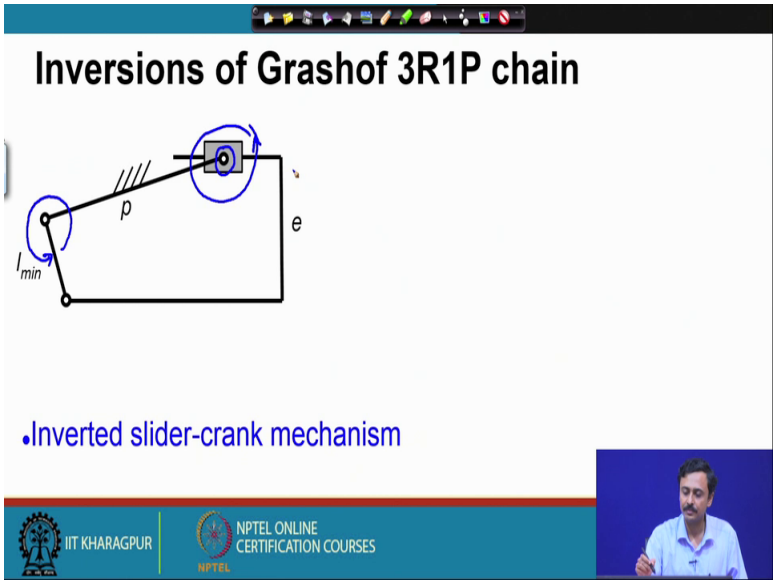

So, this is the situation that this can rotate completely with respect to. So, p is the ground. So, l min can rotate completely with respect to the ground.

Now, this is also an inverted slider crank mechanism the thing is you should note that here this sliding direction can rotate because of this hinge on the ground the slider can rotate the slider can rotate with respect to the ground it will it will of course, oscillate, but the direction of sliding is changing if you see here; here also the direction of sliding is changing because this link is rotating completely. So, the direction of sliding can change continuously. So, this here the slider can rotate completely here the sliding direction remains fixed because this link is fixed. So, sliding direction remains fixed here the sliding direction can rotate completely here the sliding direction will oscillate, but the sliding direction will change it can change.

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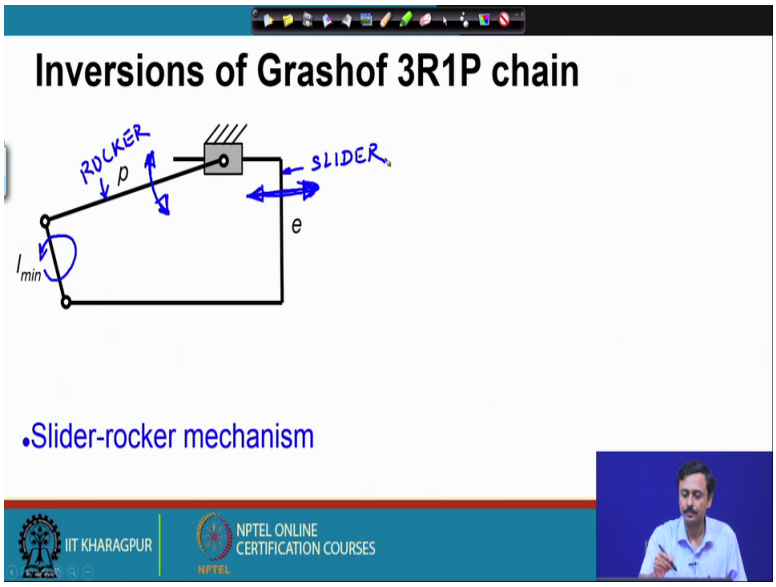

Now, if I fix the slider itself the slider block again the sliding direction becomes fixed now this becomes a slider rocker mechanism it is the l min which can rotate completely you see the l main can rotate completely, but others only oscillate. So, this is translational motion oscillate the translational oscillatory motion this is rotational oscillatory motion this is a slider rocker. So, this is the rocker and this is the slider.

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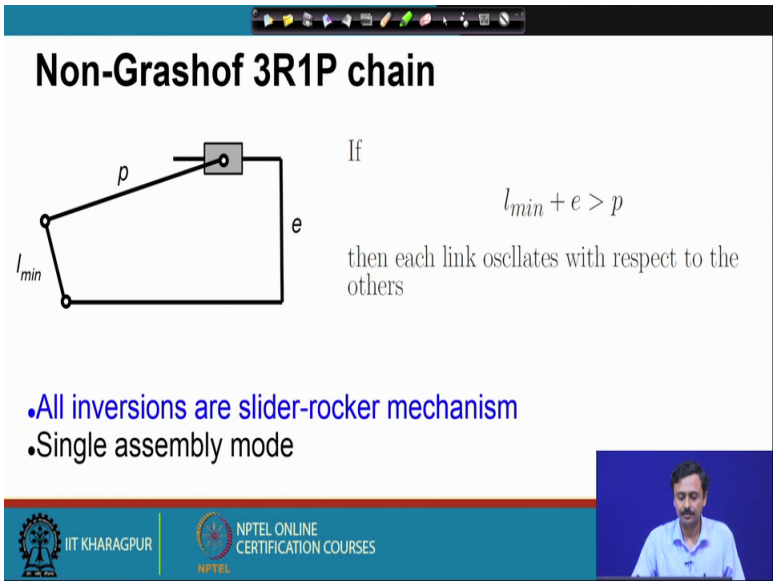

So, let us look at non Grashof 3R1P chain. So, if l min plus e is greater than p then each link oscillates with respect to the others. So, Grashof criterion not satisfied means this must be the condition for a non Grashof 3R1P chain all inversions of a slider rocker of this 3R1P non Grashof 3R1P chain are slider rocker mechanisms all inversions of the non Grashof 3R1P chain are slider rocker mechanisms there is a single assembly mode as we have seen before.

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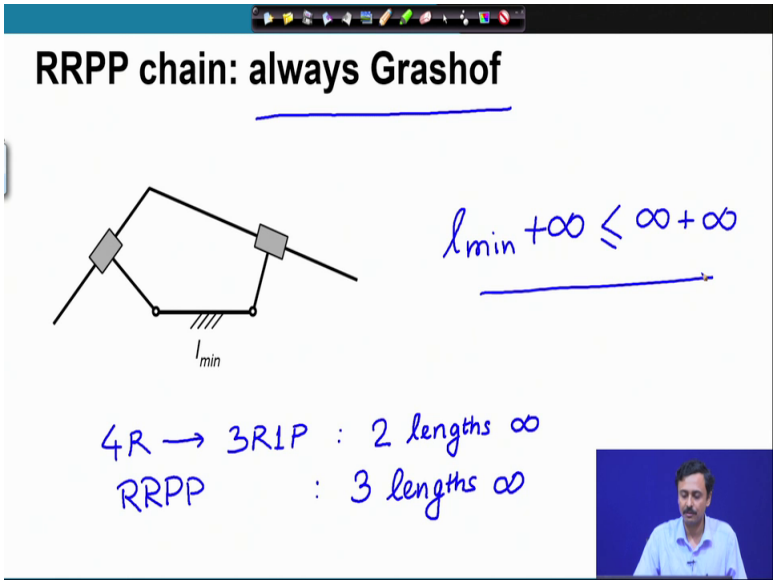

Now, let us look at the RRPP chain. Now when we go from 4R to 3R1P. So, 4R to 3 are one p, but metamorphoses that we have been discussed there were 2 link lengths which were in fine infinity 2 link lengths were infinity; now when we introduced another slider like we have done here its RRPP. So, we are having 3 link lengths as infinit,y there are 3 link lengths that are infinite in an RRPP chain. So, what do you expect l min plus infinity now this has to be less than or equal to the other 2 infinities some of other 2 infinities.

So, therefore, an RRPP chain is always Grashof can because it satisfies this Grashof criterion it satisfies the Grashof criterion always. So, this is always a Grashof chain.

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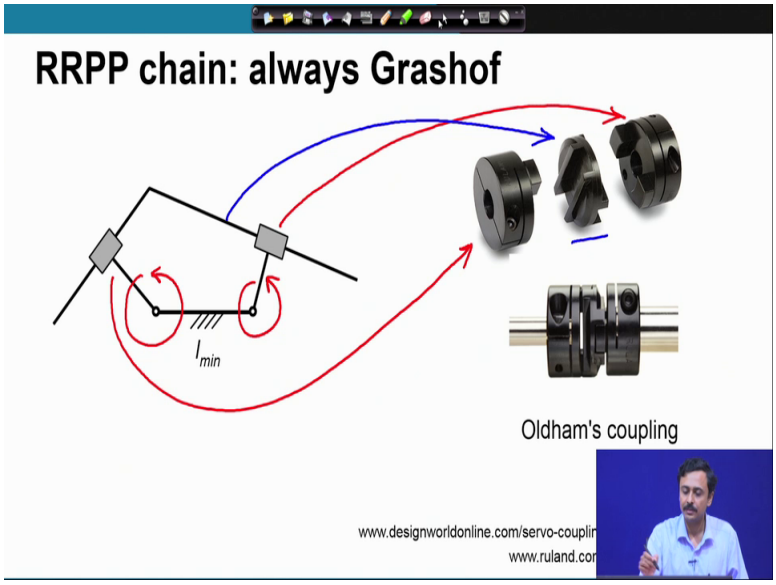

Let us look at an application we have seen this application before the Oldham's coupling let me quickly tell you this is one shaft this is the other shaft both can rotate completely. So, this is here this is connected to the other shaft and the coupler is this middle piece. So, this can rotate completely both shafts can rotate completely had it been longer how obviously it would be useless.

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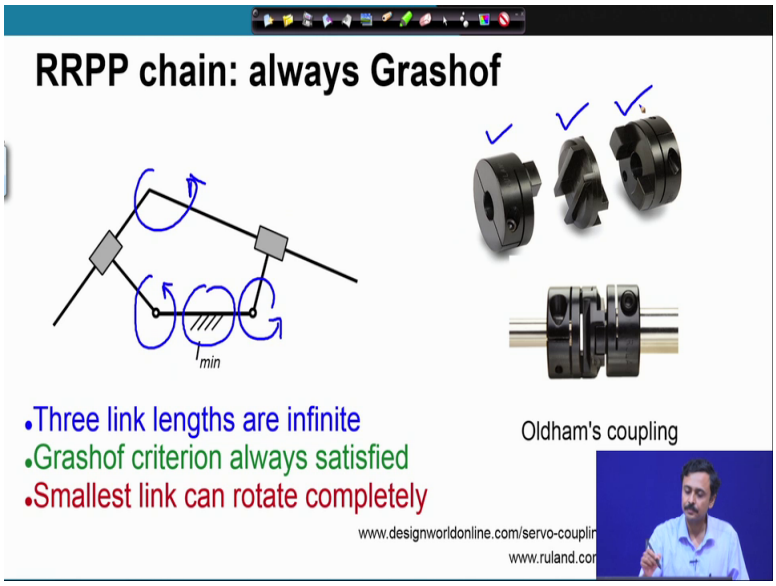

So, 3 link lengths are infinite Grashof criterion is always satisfied and the smallest link can rotate completely now if I fix the smallest thing as I have done here; I have fixed the smallest link therefore, all links should be able to rotate completely with respect to the ground link. So, all of them are can rotate completely that is why you can connect 2 shafts all of them can rotate completely.

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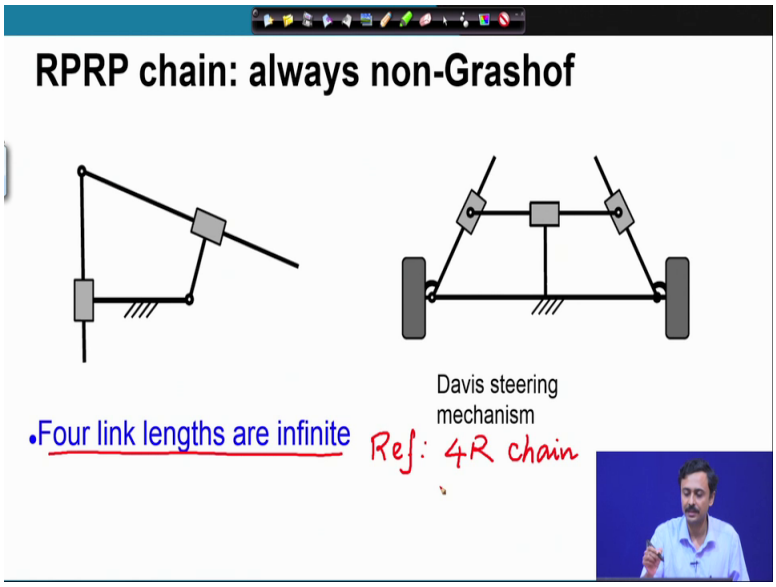

Now, RPRP chain is always them non Grashof now you can guess you have four link length as the infinity here it is in such a way the p is have been replaced in such a way

that four link lengths go to infinity. So, this is never satisfies the Grashof criterion and one of the applications we have seen before the dav is steering wheel mechanism.

So, four link lengths are infinite whenever we say four link lengths are infinite always this is in reference to a 4R chain; always remember this is in reference to a 4R chain. So, with reference to a 4R chain; we say that these are all four link lengths are infinite Grashof criterion is not satisfied all links have relative oscillatory motion.

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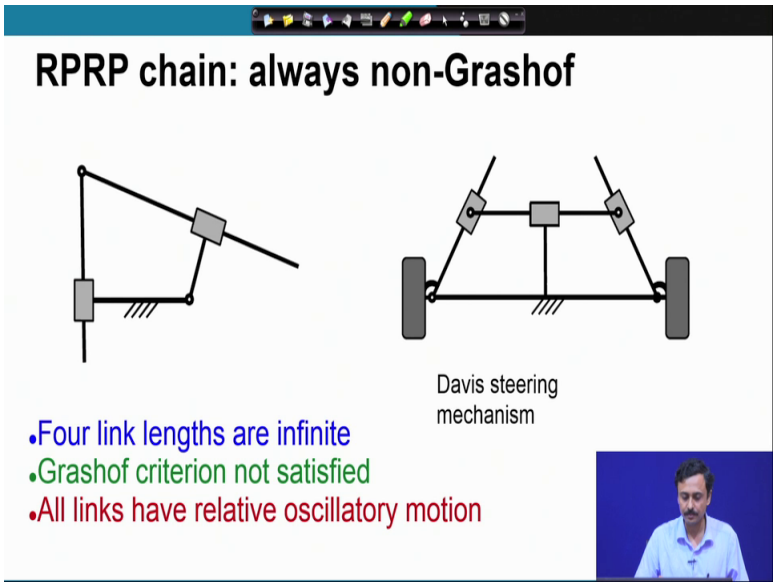

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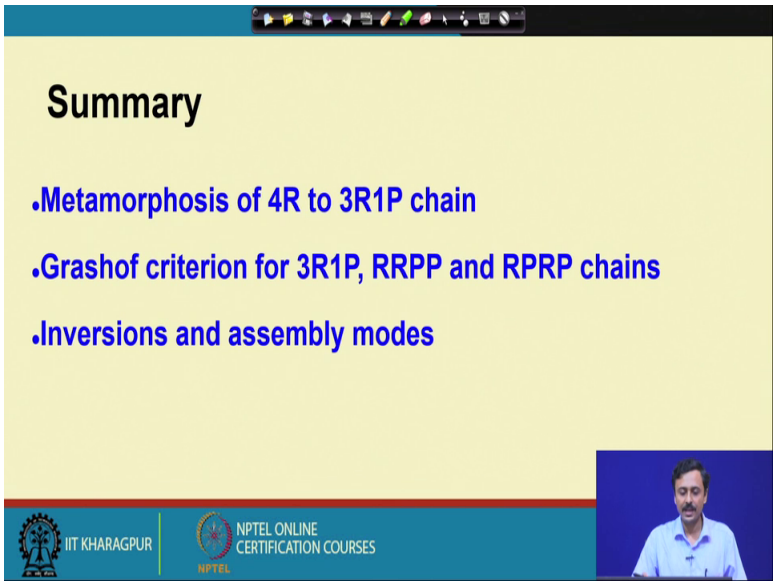

So, let me summarize we have we began with the metamorphosis of a 4R to a 3R1P chain then we extended the Grashof criterion for the 4R chain to the other chains. And we have looked at the inversions and assembly modes of these various chains. With that I will close the lecture now.