Mechanism and Robot Kinematics
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Lecture – 07
Grashof Criteria – I

In a number of applications of constrained mechanisms, we need one of the links; in particular the link that is hinged with the ground, we need a link which can rotate completely for various applications and we are going to look at some of these examples.

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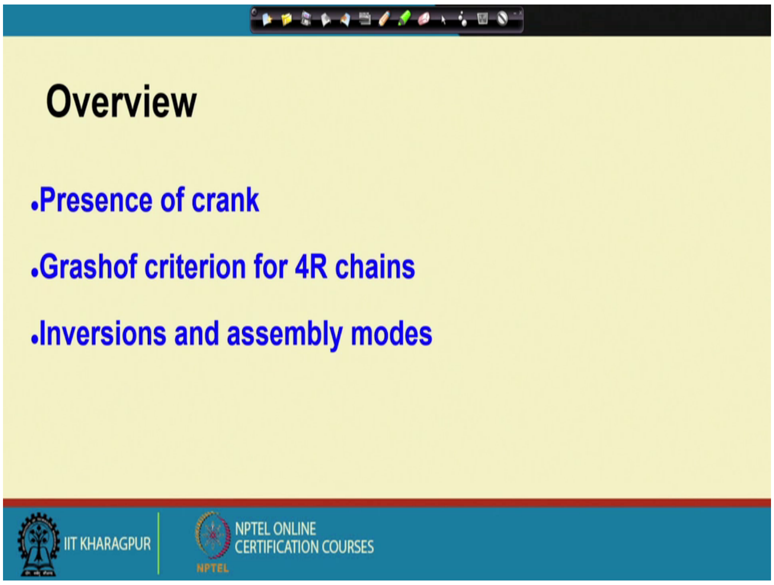

So in today’s lecture, we are going to look at the presence of crank. So, crank is a link which is hinged with the ground and which can rotate completely. So, for this we have what is known as the Grashof criterion; we are going to look at Grashof criterion for 4R chains today and then we will go to inversions.

So, I will tell you what inversions are and assembly modes of mechanisms.

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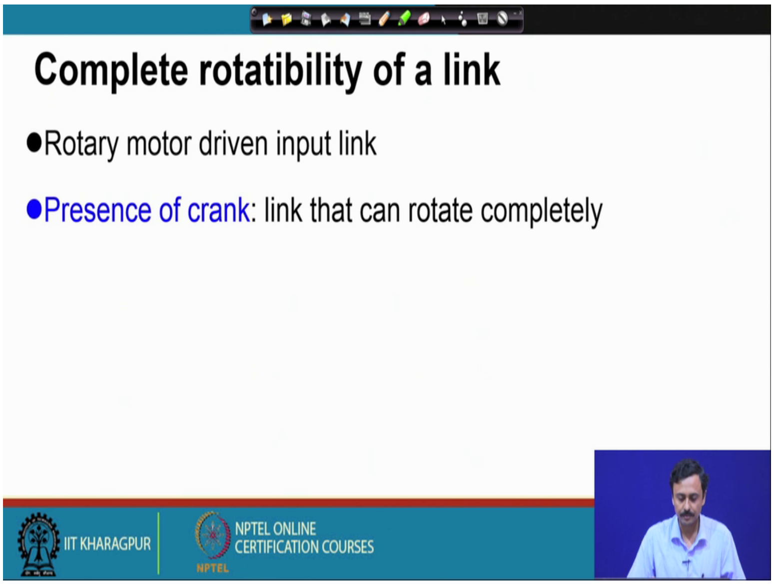

So, to begin with as I mentioned in certain applications we need a link to rotate completely with respect to the ground. Why? Because that link might be connected to a rotary motor and that is driven continuously. So, we are looking for the presence of crank in a mechanism; so a link that can rotate completely with respect to the other links.

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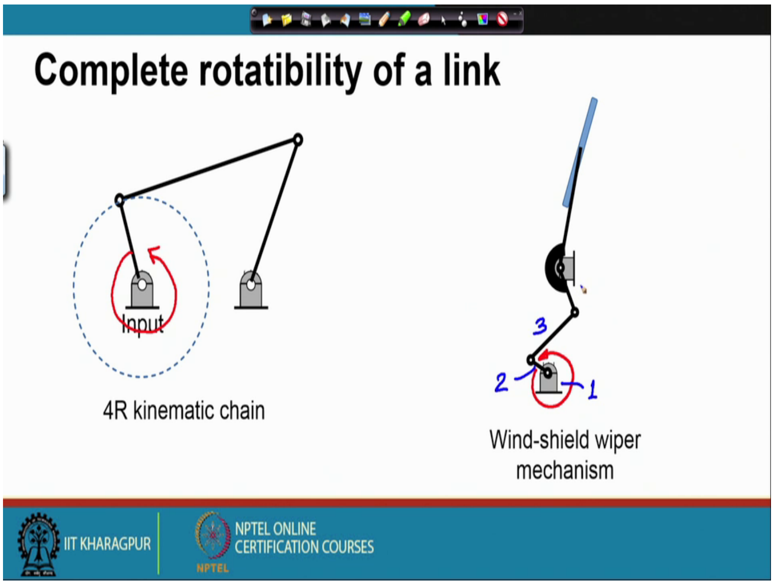

So, here I have an example; so on the left, I have shown a 4R kinematic chain in which the input can rotate completely or should be able to rotate completely. On the right I have an example for that, where it is used; I have shown this example before.

So, this is a wind shield wiper mechanism here there is a motor which rotates, this small link. So, let me number the links; so this is ground is 1, this small link is 2, this coupling link is 3.

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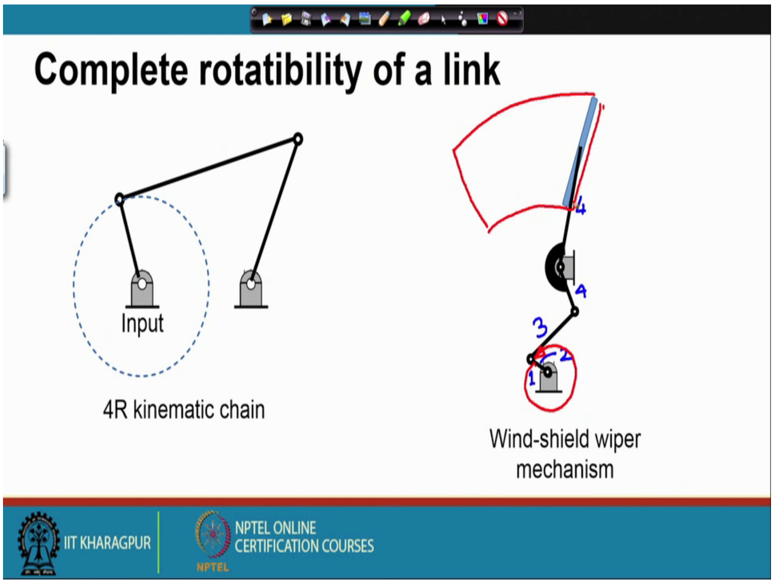

So, this ground is 1, this link is 2, the coupling link is 3 and this link is 4; which is connected with the wiper, so this is the 4. So, this link 2 is connected to a motor and it drives it continuously so that, this wiper can wipe to given area. So, the wiper oscillates while, link 2 rotates continuously.

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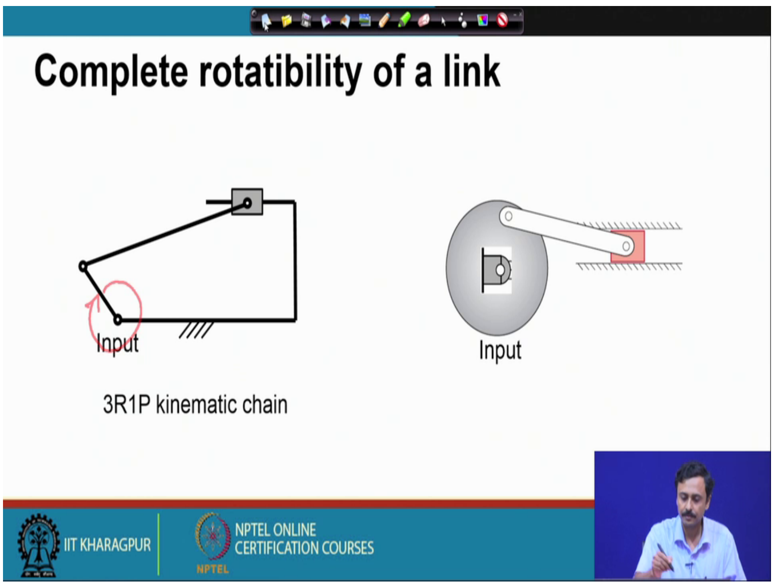

Here is this 3R1P chain which has applications in IC engines in punches. So, I have shown one on the right; so, I might need complete rotatability of the input which might be connected to a motor and correspondingly this will oscillate.

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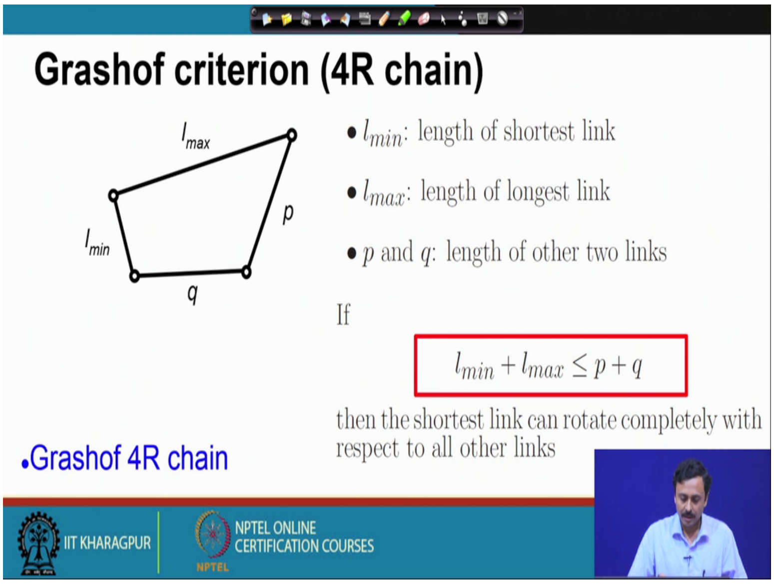

Now, let us come to what is known as a Grashof criterion; now this is possibly the most elegant result in all of kinematics. So, for a 4R chain let us start with this 4R chain which is quite common. Here I have shown a 4R chain and I have given lengths of these links; so I call one of them to be l min, another to be l max and the other two which are intermediate; I name them p and q.

So, what is Grashof criterion for a 4R chain? So this criterion is for determining whether a link exists in a chain, which can rotate completely with respect to all other links. So, what does this criterion say? It says this; so, as I have mentioned; l min is the length of the shortest link, l max is the length of the longest link and p and q are the lengths of the intermediate links.

Now the Grashof criterion tells us that; if l min plus l max is less than equal to p plus q which means the length of the shortest link plus the length of the longest link; if it is less than or equal to the sum of lengths of the other two links; then the shortest link can rotate completely with respect to all other links. I want you to note this statement; in its entirety shortest link, if this criterion is satisfied which you can easily check; given the link lengths, you can check this and if this is satisfied then the shortest link can rotate completely with respect to all other links.

So, it can rotate completely with respect to all other links. So, this is Grashof criterion for a 4R chain; so, the criterion is in terms of lengths of the links which can ensure complete rotatability of the shortest link. A 4R chain which satisfies the Grashof criterion is known as a Grashof 4R chain.

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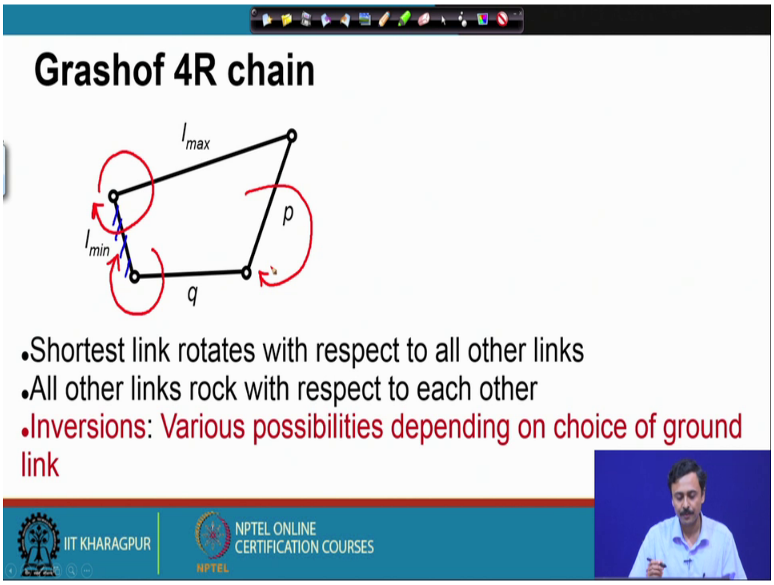

So, as I have mentioned the shortest link can rotate completely with respect to all other links. You can imagine; suppose this link rotates completely with respect to q; then it will rotate completely with respect to all other links. All other links rock with respect to each other; that means, this p and q for example, they can only rock; the longest link and p or the longest link and q, they rock with respect to each other.

So, if l min can rotate completely with respect to one link then it must rotate completely with respect to all other links. Now this leads to some interesting kinds of chains or mechanisms which are called inversions. So, what are inversions? Inversions of a chain are the various possibilities of motion depending on the choice of the ground link; that means, the link which I fix. So, to understand this; suppose this rotates completely and I have fixed this link.

So, therefore while this rotates; while l min rotates, this will rock because with respect to the ground; the link p will rock. Similarly, the longest link will also rock only link that can rotate completely is l min. But then suppose I fix this as ground then what happens? Since l min can rotate completely with respect to q, if I fix l min relatively q should be able to rotate completely with respect to l min and so must the longest link.

And so must the coupling link all of them can rotate completely with respect to l min; if I fix l min. So, these are inversions and you can have very different motion characteristics of these mechanisms, so that is what we are going to look at.

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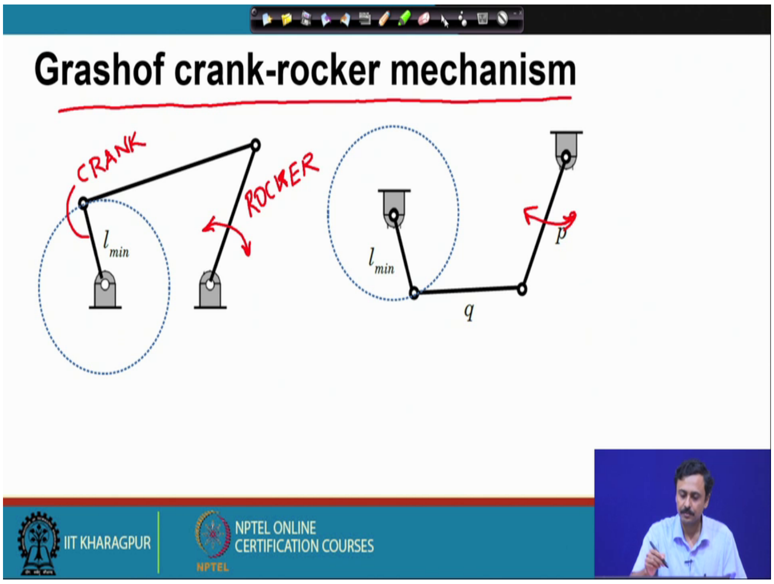

So, when I fix one of this links; it was q, so if I have fixed q here. So, l min can rotate completely as I have shown and this can only rock; this mechanism is called the Grashof crank-rocker mechanism, there is a crank and this is called the rocker. So, this is called a Grashof crank rocker mechanism or sometimes simply crank rocker mechanism because if there is a crank then it must be Grashof chain.

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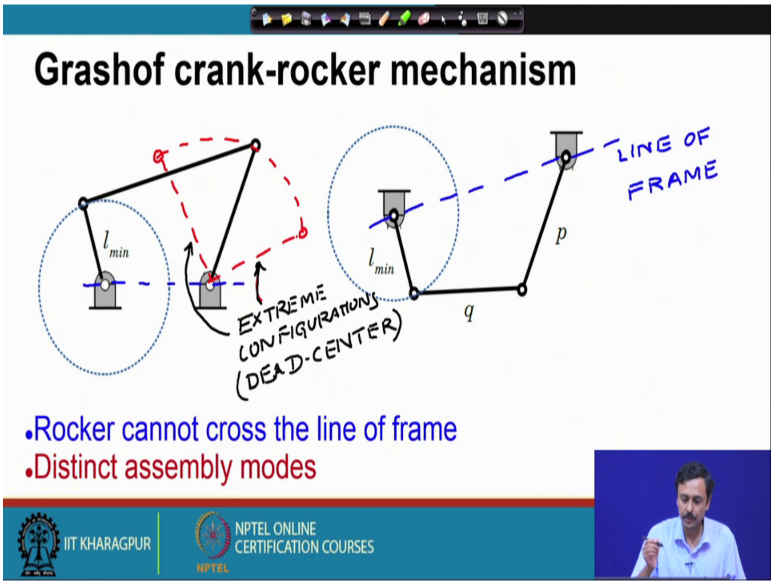

Now, it is interesting to note here that this rocker cannot cross the line of frame. So, what is line of frame? This is called the line of frame; essentially frame is the ground link. Now when this rocks; what this says is the rocker can possibly rock like this. So, these are the extreme configurations of the rocker and we have seen that these are the dead center configurations. So, these are the extreme configurations this and these are the extreme configurations or the dead center configurations.

So, the rocker cannot cross the line of frame in the case of a Grashofian crank rocker mechanism. There are distinct assembly modes; now we need to understand what are assembly modes?

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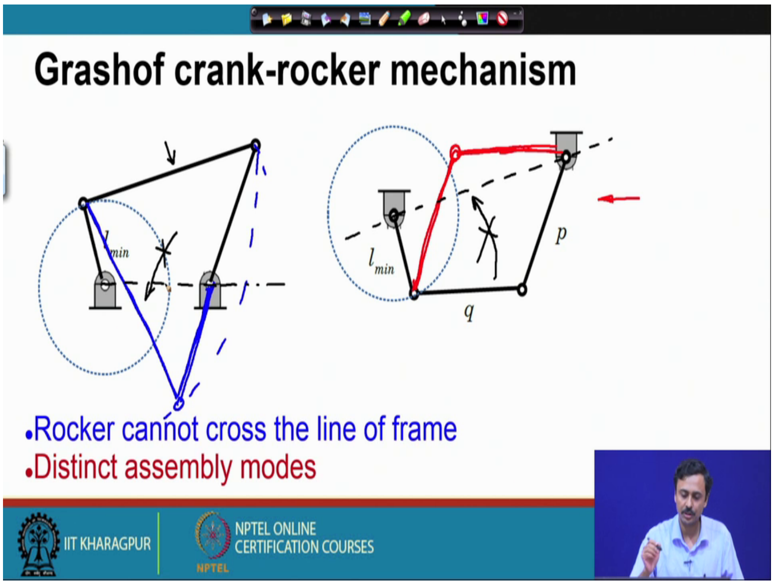

Assembly mode is a configuration in which I can assemble the mechanism; so if I am given 4 links; if I want to assemble it, how do I do it? I have two possibilities as I now show you. I can assemble it the way it is shown here or I can assemble it in this manner or I can assemble it in this manner. You see, I can assemble the mechanism in two ways either in this black configuration or in this blue configuration.

Similarly, in this example this is one configuration as shown, there can be another configuration in which I can assemble the mechanism. Now, interestingly as you can observe; the line of frame is this, this is the line of frame in the second example; this is the line of frame in this example. Now l min is the crank and this is the rocker, so the rocker can be assembled in this configuration; the configuration on top, it is a black configuration or the blue configuration.

Now, I have said that the rocker cannot cross the line of frame which means that starting from this configuration, I can never go to this configuration. From the black configuration I can never go to the blue configuration, here starting from the black configuration I can never go to the red configuration as the mechanism moves. So, these two are distinct assembly modes; one in which the rocker is on one side of the line of frame and the other in which the rocker is on the other side of the line of frame.

So, there are distinct assembly modes of a Grashof crank rocker mechanism.

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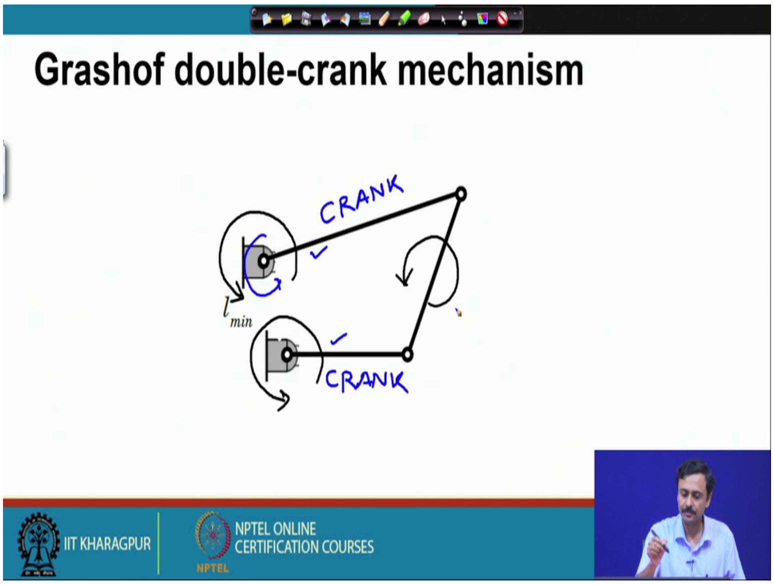

Now, if I fix l min as I have done here; I have told you that l min can rotate with respect to all other links. Therefore, now it should be easy to see that all other links therefore, should be able to rotate completely with respect to the ground here. Since l min can rotate completely with respect to all other links, so if I fix l min all other links should be able to rotate completely with respect to l min. Because rotational motion here is relative; this relative motion, this is called a double crank mechanism.

So, there are two links which are crank; this is also crank and this link is also a crank. So, I can have a motor connected to either of these two ground hinges. So, this is a double crank mechanism, there are two cranks. So, as I mentioned crank is a rotatable link which is hinge to the ground; we will not call this as crank, the coupling link as crank.

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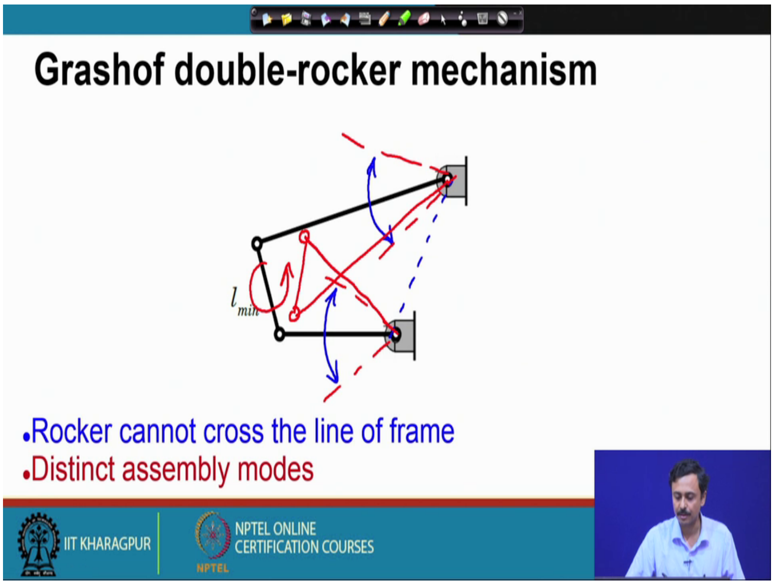

Now, if I fix one of the links which is not adjacent to l min; so the link opposite to l min. So, I have fixed the link opposite to l min; then what happens? Since other than l min, if you take any two links; they will rock therefore, this will also rock and this will also rock with respect to the ground link.; l min of course, will rotate completely, this can rotate completely.

So, this mechanism can achieve a configuration like this, which is a crossed configuration because l min can rotate completely, but other two links will rock. The rocker as we have discussed earlier cannot cross the line of frame. So, this rocker will rock within some limits which will never cross the line of frame and we have of course, distinct assembly modes.

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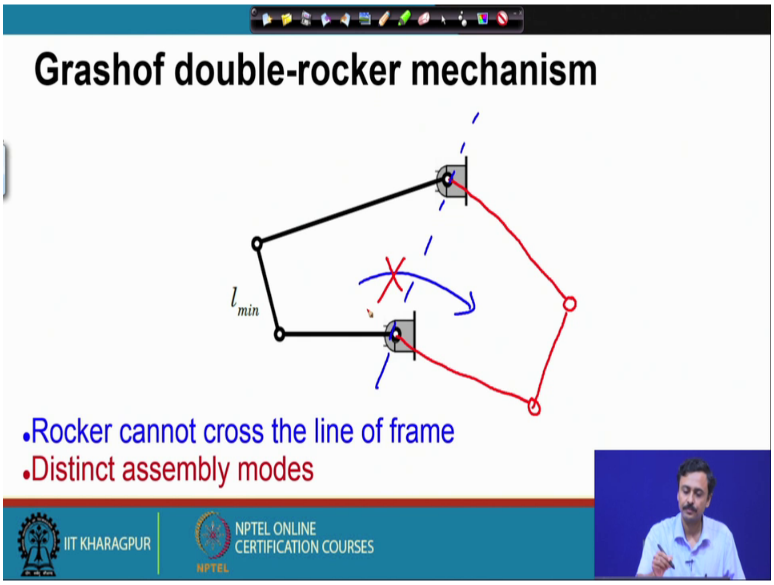

I can assemble the mechanism like this or I can assemble the mechanism like this on the other side; so this is mirror image about the line of frame. So, I can assemble either as in the black configuration or in the red configuration. So, it cannot go from the black configuration to the red configuration; this it cannot go, so there are distinct assembly modes.

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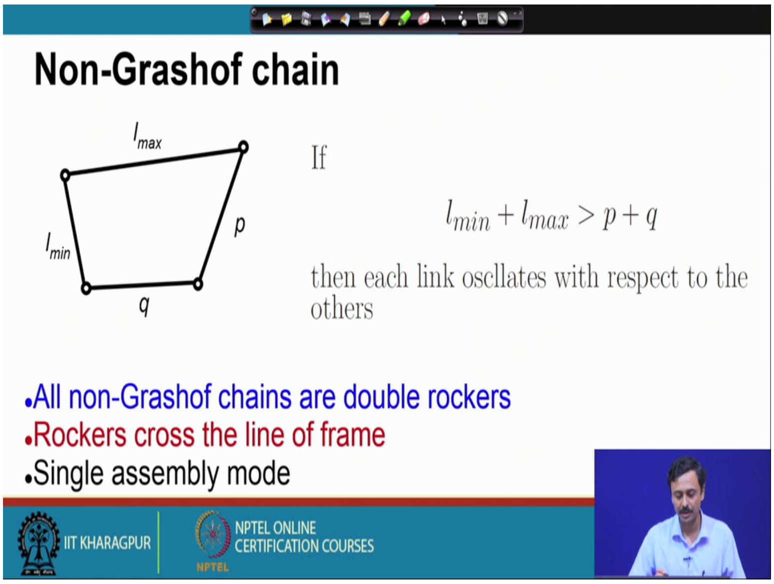

Now, what is a non Grashof chain? A non Grashof chain; obviously, is one which does not satisfy the Grashof criterion. So, to look at it quickly; if l min plus l max is greater than p plus q then each link oscillates with respect to the others.

So, all of them are oscillator link; there is no link that can rotate completely with respect to the other links. All non Grashofian chains are double rockers; obviously, because if I fix one of them, since all others rock; so, which means they will rock; there will be only double rockers. Rockers here can cross the line of frame; this is distinct feature of non Grashofian chains. And there is single assembly mode which you can now guess because rockers can cross the line of frame. So, whether you assemble the mechanism on one side of the line of frame or the other; since they can cross over; so, there is only a single assembly mode.

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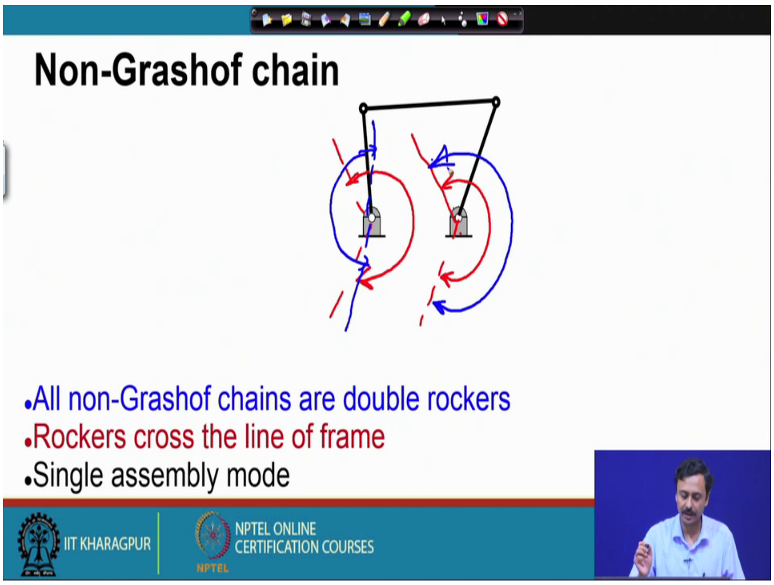

So, in a non Grashofian chain; as I have mentioned these are all double rockers. Now there are various possibilities; this can rock within certain limits and this can rock within another limit. Or it could be this way that; this rocks between certain limits and this rocks; so, one of them is call in out or out out. So, there are various possibilities and in in; so you can have three possibilities, but all of them are double rockers and they have a single assembly mode.

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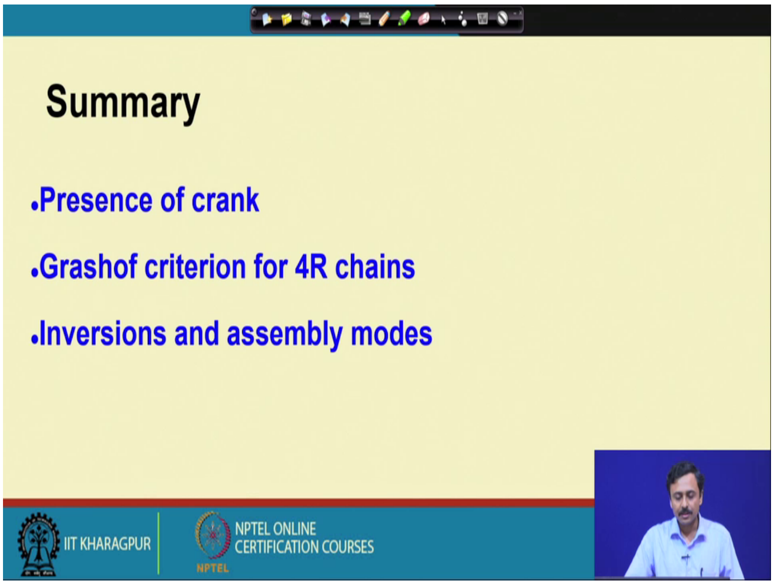

So, to summarize today we have looked at the condition for presence of a completely rotatable link with respect to the other links of a chain and this criterion is known as the Grashof criterion. We have looked at the Grashof criterion for 4R chains; we have also looked at inversions which are obtained by fixing one by one; the links of the chain, so you can have; as I have shown you we can have different possibilities of input output motion. In a Grashof chain you can have crank rocker, you can have double crank you can have double rocker.

But in a Grashofian chain; the rocker can never cross the line of frame, so you have distinct assembly modes. So, we have defined what are assembly modes? And we have also looked at the non Grashof chains, which are all double rockers; which have a single assembly mode, so with that I close this lecture.