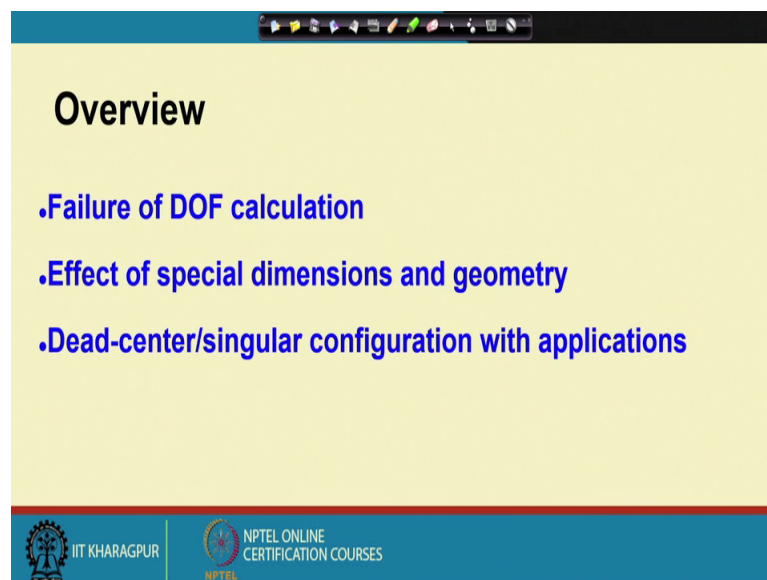


Mechanism and Robot Kinematics
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Lecture – 06
Degree of Freedom – Failure

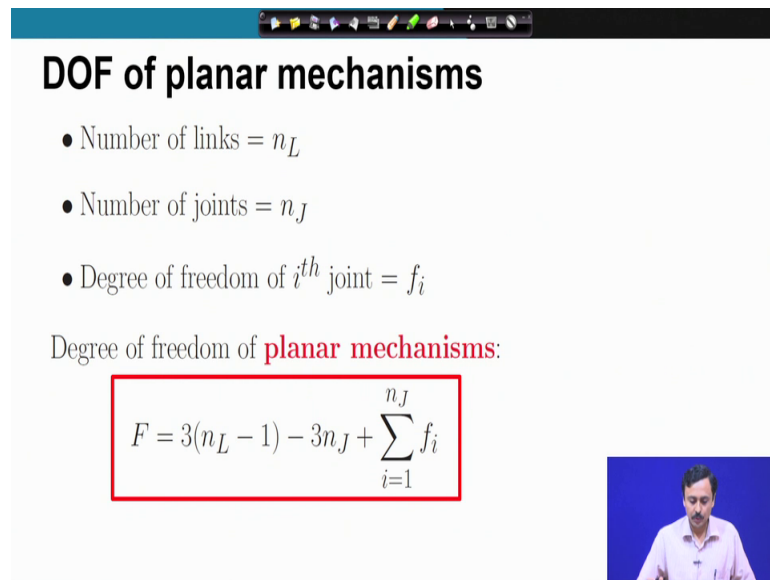
We had been looking at the calculation of degree of freedom using the degree of freedom formula. We have looked for spatial and planar mechanism with various examples, but this degree of freedom calculation formula, this fails under certain situations. So, in today's lecture we are going to look at these failure cases so that you know under what condition this might fail, will not give the correct answer.

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So, to give you an overview of what we are going to discuss today; we are going to discuss this failure of degree of freedom calculation. We are going to look at special dimensions and geometry, the effect of these special dimensions and geometry on this calculation of degree of freedom. And finally, we are going to look at a related concept of dead center or singular configuration with certain applications.


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DOF of planar mechanisms

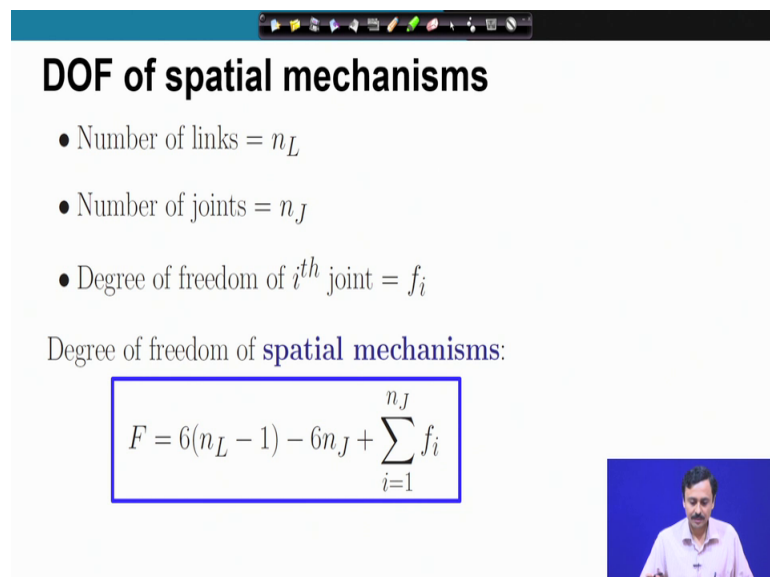
- Number of links = n_L
- Number of joints = n_J
- Degree of freedom of i^{th} joint = f_i

Degree of freedom of **planar mechanisms**:

$$F = 3(n_L - 1) - 3n_J + \sum_{i=1}^{n_J} f_i$$


So, just to give you a review of what we have discussed; this is the calculation of degree of freedom of planar mechanism.


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DOF of spatial mechanisms

- Number of links = n_L
- Number of joints = n_J
- Degree of freedom of i^{th} joint = f_i

Degree of freedom of **spatial mechanisms**:


$$F = 6(n_L - 1) - 6n_J + \sum_{i=1}^{n_J} f_i$$


Then this is the calculation of degree of freedom of a spatial mechanism, where we have this 6 in place of 3 in the case of planar mechanisms.

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Degree of freedom (DOF)

- A mechanism must have at least 1 degree of freedom
- A structure has 0 degree of freedom
- An over-constrained structure has negative degree of freedom

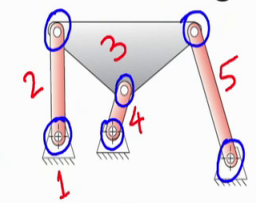


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And we have seen all these points that mechanism must have at least 1 degree of freedom, structure as 0 degree of freedom over constrained structure has negative degrees of freedom; you have looked at all these things.

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Failure of degree of freedom calculation



- $n_L = 5$
- $n_J = 6$
- $f_i = 1$ (for all i) $\Rightarrow \sum f_i = 6$

$$F = 3(5 - 1) - 3(6) + 6 = 0$$

$n_L = 5$
 $n_J = 6$
 $\sum f_i = 6$



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Now, we come to the discussion on failure of this degree of freedom calculation, we have just now seen. To start with let me take this example of the combination of links; let me count the number of links. So, ground is 1, 2 this turner is 3, 4, 5, so this has 5 links the number of joints; so 1, 2, 3, 4, 5, 6 and these are all revolute pairs. So, summation of

degree of freedom 6, so if I do the degree of freedom calculation; which I have done here, it turns out to be 0. So, this combination is a structure and it is indeed a structure, but what about this?

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Failure of degree of freedom calculation

$n_L = 5$
 $n_J = 6$
 $\Sigma f_i = 6$

} $F = 0$

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Here also you will find that this degree of freedom calculation remains the same. So, number of links is 5, number of joints is 6 and summation of degree of freedom is also 6. So, together if you calculate; that calculation remains the same, it turns out to be 0.

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Failure of degree of freedom calculation

•Extended parallelogram linkage: $F = 1$
 •DOF calculation remains same
 •Failure due to **special dimension**

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But this mechanism surprisingly or this combination surprisingly with this very special, you can see that this has got very special dimensions. This length is equal to this length, this length is equal to this length; is equal to this length, this length is equal to this length. So, all of them are parallelograms and when I try to move one of the links; this always remains a parallelogram.

So, the distances between these hinge points; they will remain fixed. So, I can join them by rigid bodies like the weight has been done, this is known as an extended parallelogram linkage and this can move; this has 1 degree of freedom. But our degree of freedom calculation says that this has 0 degree of freedom because the degree of freedom calculation does not know about the special dimensions I have used.

So, the degree of freedom calculation can fail on account of special geometry. So, though the degree of freedom calculation says 0 because of very special dimension of the mechanism, this has gained a degree of freedom. It has gained a degree of freedom because of very special dimensions, if I were to change; suppose this kinematic pair I bring here and join like this and remove this, immediately this becomes a structure. So, for very special dimensions; this is a mechanism.

A small deviation this way or that way, this makes this a structure. So, when you have to design such things you have to be very careful with the dimensional tolerances. So, if the degree of freedom calculation says that your system has 0 degree of freedom; yet it can move or yet you are thinking it will move or you are designing it to move, then you have to be very careful with the design and its manufacturing.

So, this property; or this very special dimensions is non generic; that means, any small deviation any small perturbation will make it the generic case of a structure; the structure is the generic case its robust, you make small deviations; it will still remain a structure. For very very special dimensions; this is a mechanism any small deviation from that it is going to become a structure.

So, that property of being a mechanism in this particular case is non generic; we will say this as non generic, generic thing is structure. So, degree of freedom calculation gives you a generic feature of the combination of interconnected links. So, degree of freedom calculation has remained the same as I have said, so the failure is due to special dimensions as we have discussed.

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Failure of degree of freedom calculation

Friction discs (pure rolling): $F=1$

$n_L=3$ $n_J=3$

$\sum f_i=3$

$F=3(3-1)-3(3)+3$
 $=0$

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Next is this example of two friction discs; by friction disc say I mean at this point you have very large friction; infinite friction like gears. So, these may be considered as the pitch circles of two gears.

Now, I know that this can roll I mean the gears can rotate. So, these two friction discs should be able to rotate and we know it will. But let us look at the degree of freedom calculation, so let me number the ground as 1, this body as 2 and this body as 3. So, number of links is 3; number of joints.

So, here I have revolute 2 revaluates and here it is like a lower pair contact, but this has only rolling because an infinite friction. So, because of friction this lower pair has 1 degree of freedom; this is one kinematic pair. So, there are three kinematic pairs and when I come to degree of freedom of individual kinematic pair. So, this revolute has 1 this revolute has 1; so, 2; 1 plus 1, 2 and this has only 1 degree of freedom because it can only allow rolling no slipping, so only rolling. So, summation of degree of freedom of the kinematic pairs is 3.

So, degree of freedom calculation says 3 times number of links minus 1, minus 3 times number of joints plus 3, so that turns out to be 0. So, the degree of freedom calculation says this is the structure, but we know that this is not a structure; this can rotate.

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Failure of degree of freedom calculation

Friction discs (pure rolling): $F=1$

- $n_L = 3$
- $n_J = 3$
- $f_i = 1$ (for all i) $\Rightarrow \sum f_i = 3$

$$F = 3(3 - 1) - 3(3) + 3 = 0$$

• Failure due to **special geometry**

The slide includes a diagram of two overlapping circles representing friction discs, a kinematic diagram with three links and three hinges, and logos for IIT KHARAGPUR and NPTEL ONLINE CERTIFICATION COURSES.

So, I have here once again that degree of freedom calculation and we are finding that; this is failing and this failing is because of the special geometry. What is the geometry? These are two circles; had they not been circles, then this would not have worked. Then this degree of freedom calculation what it is saying is correct, it becomes a structure because that this is these are two circles; so, very special geometry the degree of freedom calculation is failing. But if you deviate from even slightly from this circular geometry, immediately this will become a structure.

So, once again we say that the degree of freedom calculation gives you a generic feature whereas, very special dimensions under which this gains a degree of freedom is a non generic feature, any small deviation will take it to the generic feature of a structure. So, for this very special geometry; this degree of freedom calculation is failing. Now this degree of freedom calculation; the same thing happens for this combination of links.

So, suppose ground is 1, 2, 3; so 3 links; 3 kinematic pairs and they are all hinges, so summation of degree of freedom is 3; so 0 degree of freedom is 0 and we know that this is the structure. But if you bring this, suppose you have it combined like this; so this is a different combination there are two links and the hinge in between and they are straight.

Now, this corresponds to this case of the two friction discs; there are two ground hinges and 1 kinematic pair with 1 degree of freedom at the contact; so that is what I have drawn. So, this also actually gains a degree of freedom, but only slightly; if you even

slightly take it out of this configuration, immediately it cannot move but exactly at this configuration there is a possibility of moving.

Which you possibly know by experience also that if you put these two links and if you load it, if you give a load this has a tendency of moving. But only slightly, once it moves a small amount; it will not move any further because there it becomes a structure. The kinematic pair has gone out of this line; as long as the kinematic pair is on that line, there is the tendency to move, but as soon as it goes off from that line; it becomes a structure.

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Failure of degree of freedom calculation

The diagram shows a mechanism with three links: a triangular link, a trapezium link, and an L-shaped link. Three prismatic joints are indicated by red circles. Handwritten calculations below the diagram are:

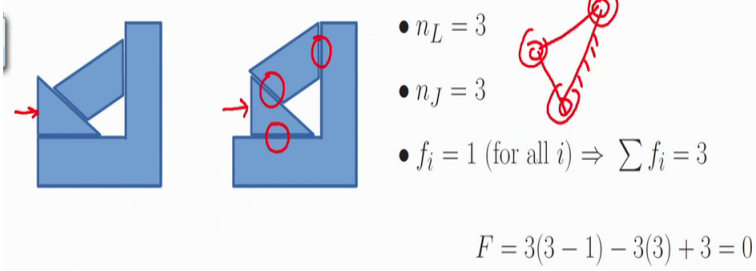
$$n_L = 3 \quad n_J = 3$$
$$\Sigma f_i = 3 \quad F = 0$$

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Now, we come to another case; as you can see I have made a figure with one L shaped link; one triangular link and trapezium. So, you have three links; number of joints, I have a sliding pair here, so prismatic pair, a prismatic pair here and a prismatic pair here. So, there are 3 prismatic pairs and summation of degree of freedom of individual kinematic pairs, so there are 3 prismatic pairs; each with 1 degree of freedom. So, I have 3; if you do this degree of freedom calculation, it will turn out to be 0, so let me show you.

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Failure of degree of freedom calculation




- $n_L = 3$
- $n_J = 3$
- $f_i = 1$ (for all i) $\Rightarrow \sum f_i = 3$

$$F = 3(3 - 1) - 3(3) + 3 = 0$$

• Failure due to **special geometry (3P loop)**

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This configuration; now you know, if I am to push this triangular link; it is going to come to this configuration and this trapezoidal link this will move up. So, this indeed has 1 degree of freedom, but our degree of freedom calculation says is a 0 degree of freedom.

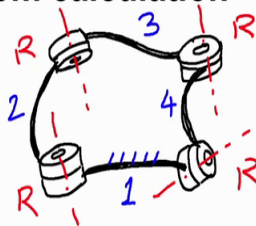
So, again our degree of freedom calculation has failed; so this our degree of freedom calculation, it says 0. But here I have shown you that it can move; so this failure is due to very special geometry; geometry of this 3P loop, so here P P P; so there is a 3P loop; 3 prismatic pairs in a loop. The same degree of freedom calculation also holds for the revolute joints, so if I have; just now we discussed.

Degree of freedom calculation does not change, but then this is very special 3P loops is special. You see, I have replaced individual revolute pairs that you see here by 3P; 3 pairs, that is all I have done and immediately it becomes a mechanism, it can move. So, the failure here is because of we say because special geometry of 3P loop.


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Failure of degree of freedom calculation

•DOF of 4R spatial mechanism:

$$n_L = 4 \quad n_J = 4 \quad \sum f_i = 4$$
$$F = 6(4-1) - 6(4) + 4$$
$$= -2$$


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Now, let us look at degree of freedom of a special mechanism and see the failure in that case. So, 4R spatial mechanism let me try to draw a spatial 4R mechanism. So, roughly this is how a general 4R spatial mechanism will look like, so these are all revolute pairs.

Let me count the number of links; so maybe I can ground this. So, 1, 2, 3, 4; so number of links is 4, number of joints is also 4 and summation of degree of freedom is also 4; is individually they have 1 degree of freedom; revolute pairs. So, therefore degree of freedom calculation says now this is a spatial kinematic chain, so 6 times number of links minus 1, minus 6 times number of joints, plus summation of degree of freedom of each joint.

So, this is minus 6; plus 4, this is minus 2; so this becomes an over constrained structure. So, it is very difficult to assemble this; if there is dimensional tolerance is not proper and this will be very difficult to assemble or it will assemble with internal stresses. So, this is an over constrained structure that is what our degree of freedom calculation tells us. So, a general 4R spatial mechanism; please note this word, spatial. So, this is a spatial mechanism is an over constrained structure.

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Failure of degree of freedom calculation





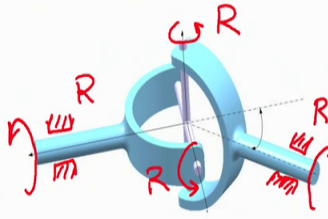
•DOF of 4R spatial mechanism:

- $n_L = 4$
- $n_J = 4$
- $f_i = 1$ (for all i) $\Rightarrow \sum f_i = 4$

$$F = 6(4 - 1) - 6(4) + 4 = -2$$

•A universal joint
4R-spatial K.C.

•Failure due to **special dimension and geometry**



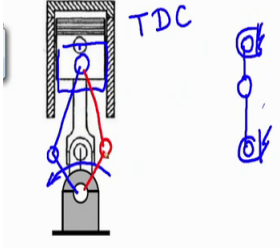
Let us look at this; so this is the degree of freedom calculation again for you, but you know this joint; this is called the universal joint or the hookes joint. This is used to connect two intersecting shafts; now what do we have here? We have bearings here and bearings here. So, therefore this shaft can rotate; this shaft being connected to there through this universal joint is also rotate.

So, let me show you what are the kinetic pairs involved, so this is a revolute pair; here there is a revolute pair. So, R here there is a revolute pair R and here there is a revolute pair; so R, R, R, R; so, this is a 4R spatial kinematic chain. So, 4R spatial kinetic chain and we know that this can rotate; so this has 1 degree of freedom.

So, once again we find that our degree of freedom calculation has failed, but then again remember; this is very special geometry and dimension. The 4R spatial mechanism that I had drawn in the previous slide; was a general situation, now here you can very well see that these angles and the dimensions, they are very special; only then this is a mechanism. So, this is a non generic feature, so you have to be careful with this universal joint; the dimensional accuracy, the angles. So, under special geometry and special dimensions; this is a mechanism. So, the failure in this case is due to special dimension and geometry.

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Dead-center/singular configurations



- Extreme configuration
- Arbitrary output speed
- Uncertainty of motion direction: gain in degree of freedom

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Now, we come to somewhat related thing of dead center configurations. Now what are these dead center; you must have heard these terms, top dead center of an engine; of a piston. So, what are these dead center configuration? So, here I have this example of an IC engine by which I motivate; this discussion. So, this is the top dead center of this engine; it is called TDC; Top Dead C enter of this piston.

Now what happens? This is an extreme configuration of the piston, while this crankshaft can still rotate and it can rotate on an arbitrary speed. At this configuration, it might be rotating at an arbitrary speed depending on the speed of the vehicle, but imagine just when it comes down a bit; the piston has come down a bit.

Now, the speed of the piston and the crankshaft are related; the only configuration where it breaks down is the top dead center as well as the bottom dead center. There the relation between the rotation speed of the crankshaft and the speed of the piston breaks down, the relation breaks down. So, that the crankshaft can have an arbitrary speed and the piston stays at rest; at the top dead center and the bottom dead center.

These are dead center configurations or we also call them a singular configurations. Why do we call them as singular configurations? These are of course, extreme configurations as I have shown the speed ratio is arbitrary, so output which is the crankshaft speed is arbitrary. And there is uncertainty in the motion direction, now this uncertainty means

there is a gain in degree of freedom. We just now we discussed this 3R combination of links; now piston is now stationary; so maybe I can think of like this.

So, here this has a tendency to move as you know; now as the piston comes down, I can move in this configuration, I can move to this configuration shown in blue or I can move to this configuration; as I shown in red. So, this is the uncertainty; the crankshaft can either go counterclockwise or can go clockwise. Of course, in an actual engine this is prevented by the flywheel, it has momentum and so the crankshaft moves only in one direction.

But otherwise imagine that this is a mechanism, just a mechanism; 3R1P mechanism then there is this uncertainty in motion; same thing here.

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Dead-center/singular configurations

Output Input

- Extreme configuration
- Arbitrary output speed
- Uncertainty of motion direction: gain in degree of fr

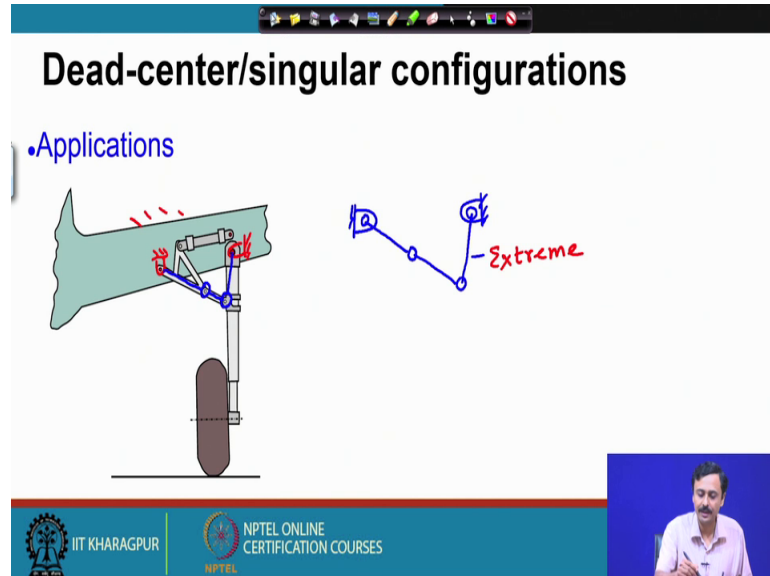
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Suppose I try to move this; I can have two possibilities this can go slightly like this or it can go slightly to this side; as I move the input. So, this was this black configuration was an extreme configuration; this input cannot move in this direction; in the clockwise direction, the input cannot move in the clockwise direction at the extreme configuration.

But this can have an arbitrary speed and again this uncertainty in motion; so we have a gain in degree of freedom. I can choose in which direction this will move; whether it is going to move in the counterclockwise direction or whether it is going to move in the

clockwise direction; I have this freedom of choice. So, there is a gain in the degree of freedom.

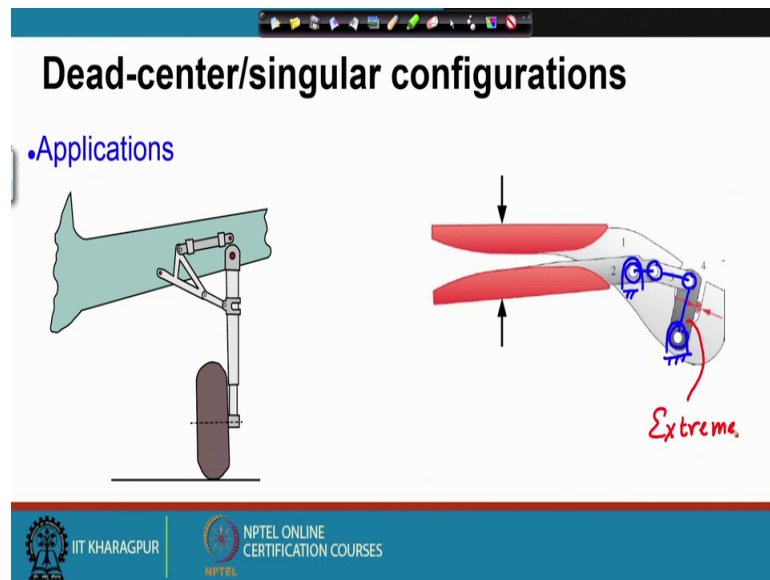
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Now, this is used in certain applications; one application is this landing gear mechanism. If you have been careful in noticing this; this is, so ground is there; aircraft wing. So, this is a ground hinge, this is a ground hinge we are interested in these two ground hinges and looking at the mechanism that is there. So, here I have only this is a kinematic pair, this is another kinematic pair. So, essentially what I have drawn here? I have repeated here another kinematic diagram in which I have shown.

So, here you see; this is an extreme configuration for this link, this cannot move in the counterclockwise since anymore, it can move back or it cannot move in the counterclockwise direction; this is an extreme configuration.

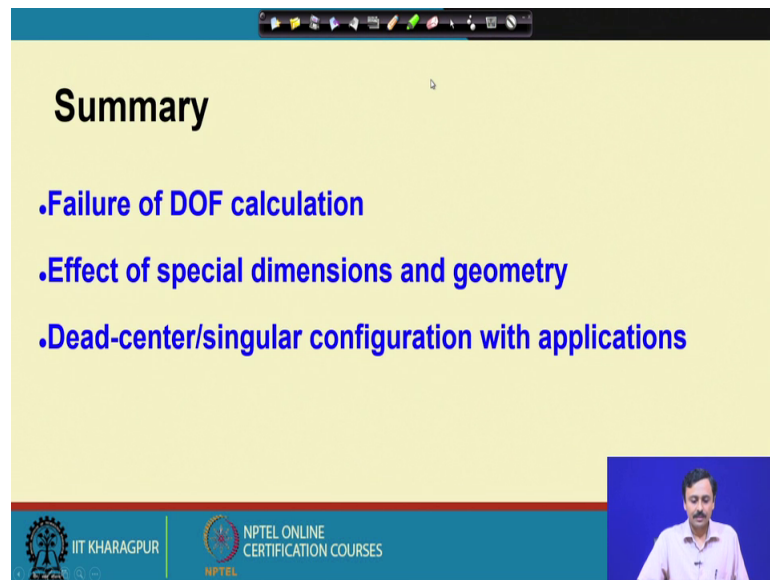
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So, we have this in the aircraft landing gear; another is this crimping tool. Once again you see, this is a ground; there is this link there is another link goes to a ground. Again this links 2 and 3; they are straightened out, so therefore link 4 is in almost an extreme configuration.

Now, these are used; now why these are used in these situations? Why this dead center configuration is used? In order to know this we have to proceed further in this course. And as we proceed, I will clarify how this dead center configuration is used in these two cases? Especially in this crimping tool, if you see this crimping tool; it can generate huge amount of force; it is much more than a plier, which uses a simple lever. So, that is the advantage you get at a dead center or singular configuration.

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Summary

- Failure of DOF calculation
- Effect of special dimensions and geometry
- Dead-center/singular configuration with applications

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So, with that I summarize today's lecture and close.