Mechanism and Robot Kinematics
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Lecture – 40
Coordinate Transformation – III

We have been looking at coordinate transformations and in the previous two lectures we have discussed about the rotational transformation and the homogenous coordinate transformation. In this lecture today we are going to look at an application of this homogenous transformation matrix to write out the kinematic relations for open chain robot manipulators.

So, to give you an overview of what we are going to discuss in this lecture, first we are going to look at the homogeneous transformation matrix and then we are going to apply the homogenous transformation matrix to find out the kinematic relations for a planar open chain robot. We are going to look at 2 examples rr and rp robot manipulators. As we know that a rigid body in a plane requires 3 coordinates 2 position and one orientation whereas, the rigid body in spatial motion requires 6 coordinates 3 position and 3 orientation.

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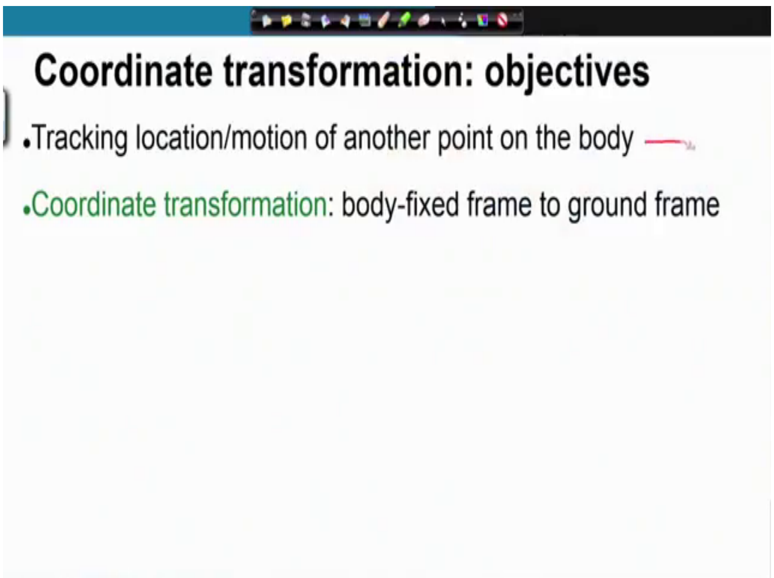

The objectives of coordinate transformation we have already discussed. So, we have this tracking of tracking of motion, of the point on a body which is in motion the coordinate transformation gives us a relation between the body fixed frame and the ground frame. So, if you want to represent a vector in a body which is under motion, we would like to find out the position of a point which is moving with respect to a body in the ground frame and we use this coordinate transformation.

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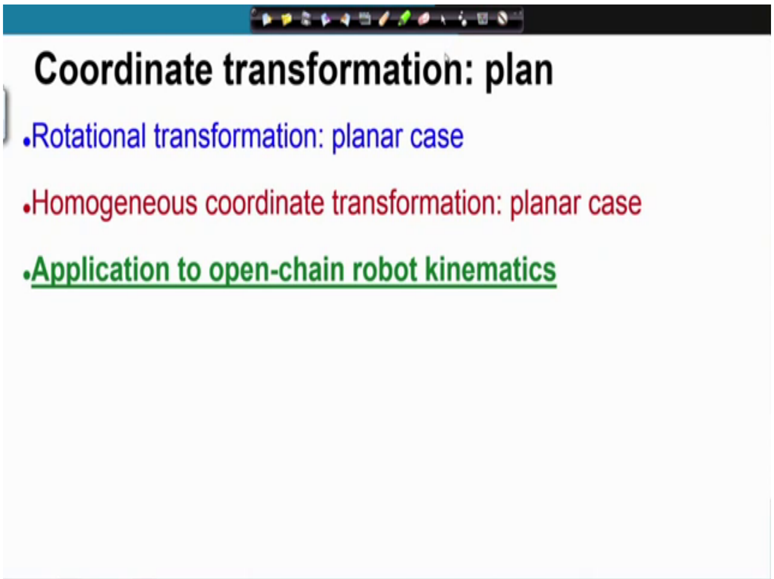

In this lecture, we are going to look at the application of the homogeneous coordinate transformation to the kinematics of open chain robot manipulators.

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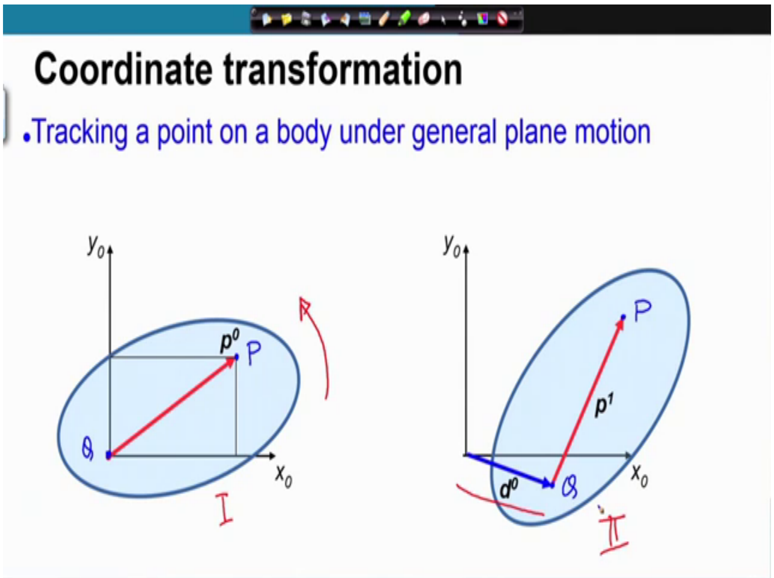

Let us review what we have discussed; for a body under general plane motion suppose this body goes from this configuration 1 to configuration 2. So, this is in rotation as well as translation.

So, if I consider this point as Q and this point as P. So, these points move respectively to P and Q in configuration 2. My objective is to determine this p 0. So, the objective is to determine p 0 in terms of p 1 and also of course, d 0.

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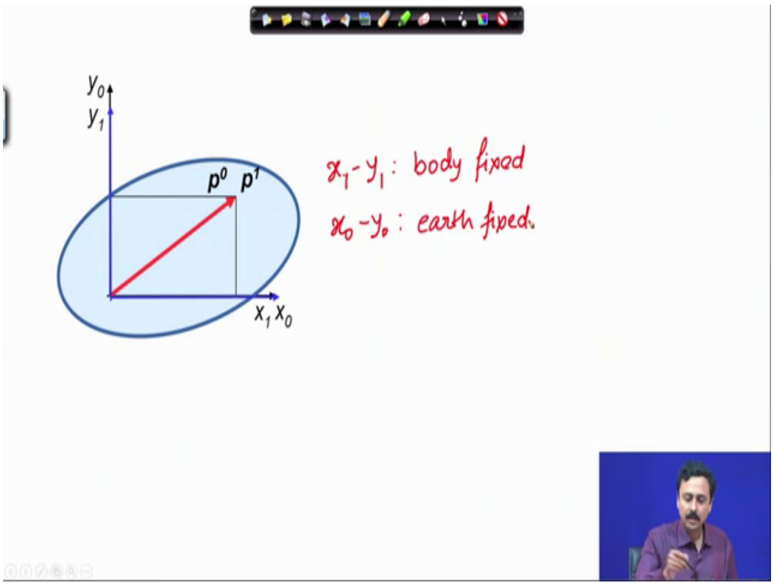

We consider the body again with the fixed frame x 1, y 1, x 1-y 1 is the body fixed frame and x 0-y 0 is theearth fixed frame.

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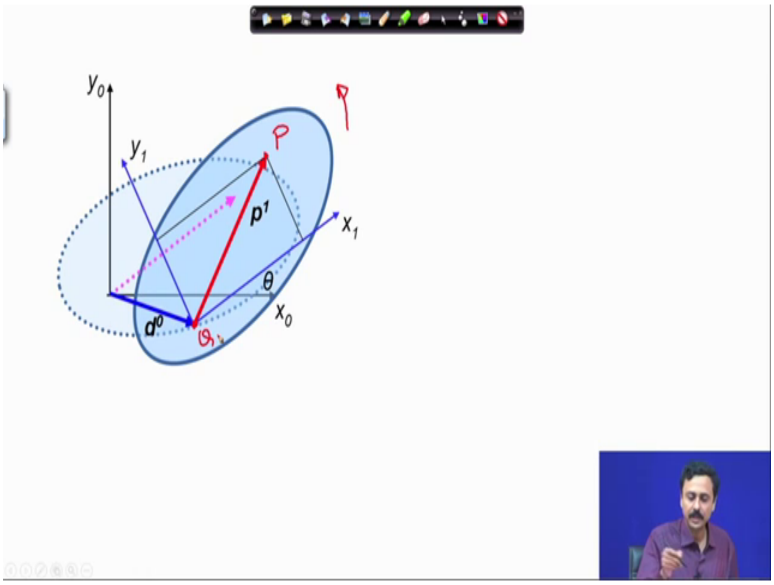

Now, as I rotate and move the body, the body is in general motion. So, this is rotating as well as translating the point P has moved here and Q has moved here.

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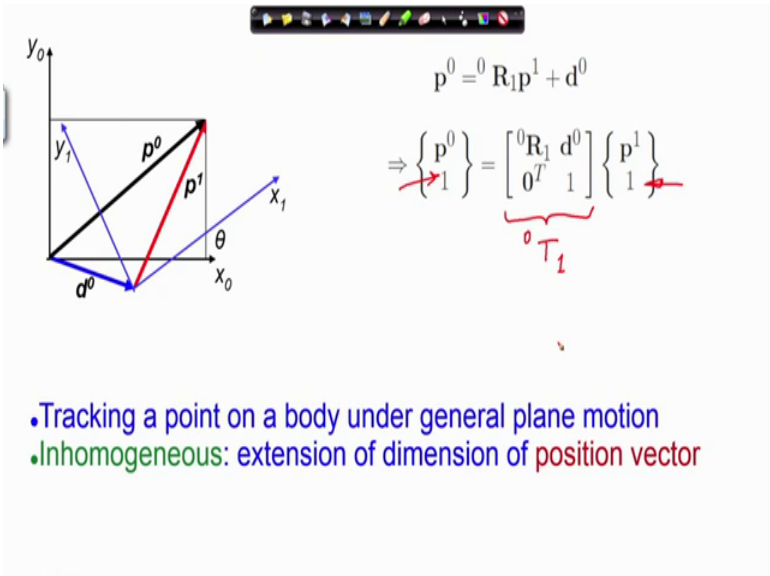

My objective is to find out p naught a relate p naught in terms of p 1 and d 0. To do that we had constructed the homogeneous transformation matrix now why did we construct the homogeneous transformation matrix is that, the reason is that here we have in homogeneity in the relation between p 0 and p 1, because of this additional term d 0.

We would like to have a multiplicative relation between p 1 and p 0. So, to do that what we did was we extended the position vector space by this additional term, we had this extra term 1 here and 1 here and we represented the transformation through this matrix, which we call the homogeneous transformation matrix. So, this is a multiplicative factor between p 1 and p 0. So, you have the relation between the vectors p 1 and p 0.

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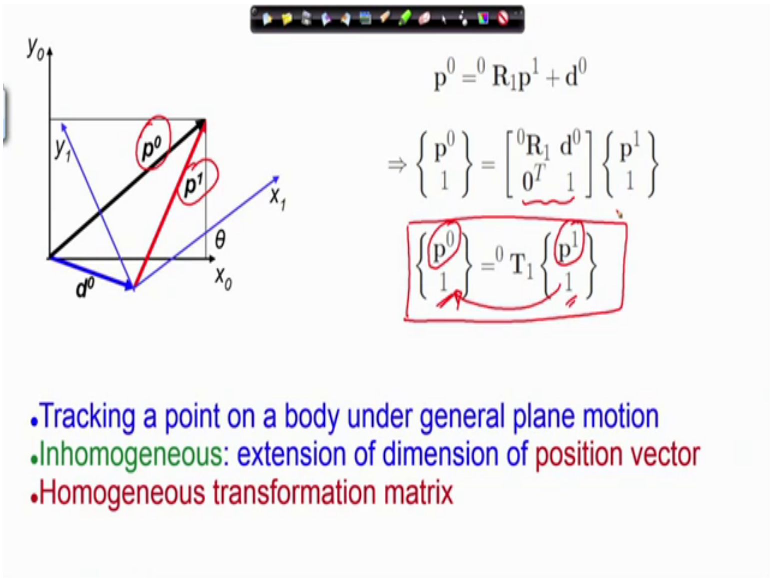

This is p 1 and p 0 we have additionally this entry 1 this element 1 in the in these respective vectors, which is just an extra term we only need to read out the first 2 elements of this vector, 1 will transform to 1 only that is the property of the homogeneous transformation matrix.

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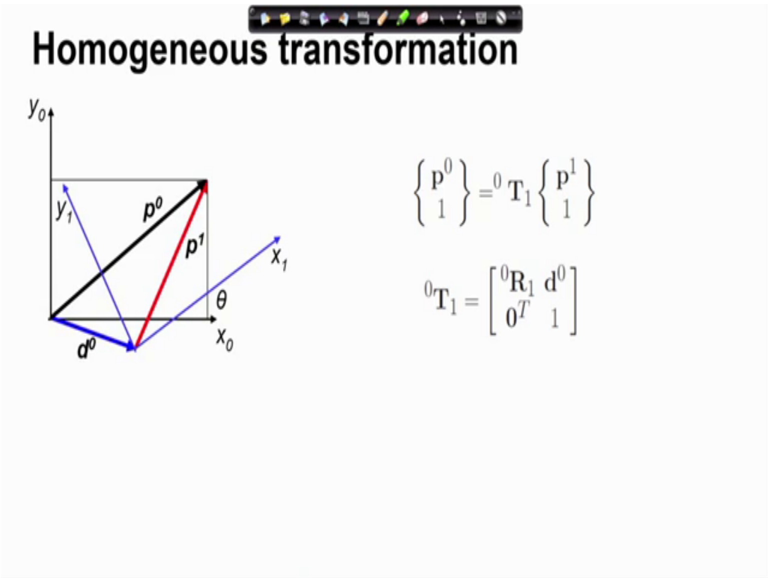

So, I have written out the homogeneous transformation matrix here.

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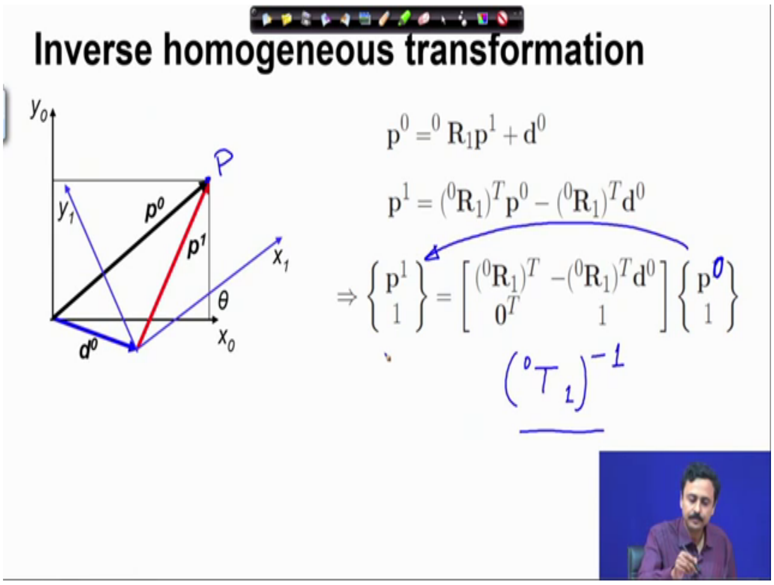

So, the inverse homogeneous transformation matrix was obtained by representing p 1 in terms of p 0, and again extending the position vector space by an additional term additional element 1, and writing out the homogeneous transformation matrix in the inverted form. So, this is T 1 to 0 inverse.

So, we will write it as T 1 to 0 inverse. So, this needs to be 0. So, this is the homogeneous transformation matrix inverse, that takes a vector in the zeroth frame to the first frame. So, it frames 0 representation of point p in frame 0. So, this is point p. So, this takes the representation in frame 0 to the representation in frame 1.

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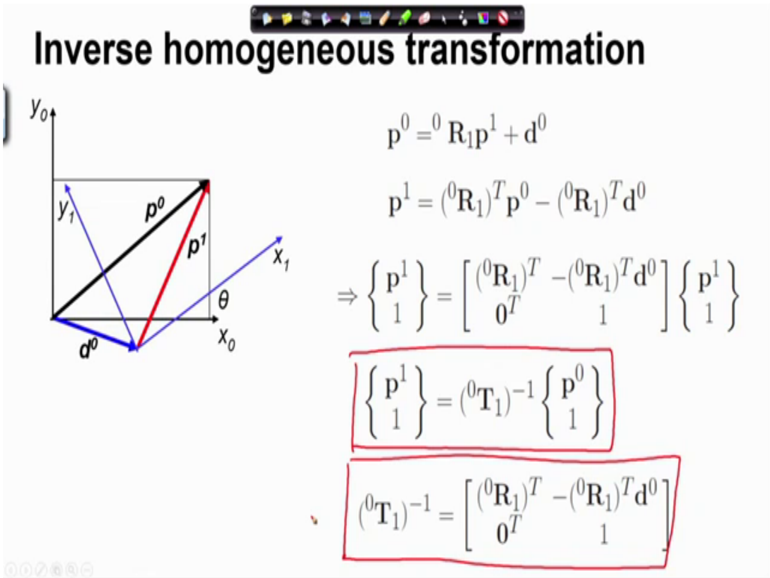

So, here I have the representation and the inverse homogeneous transformation matrix is a little more complicated but straightforward to write.

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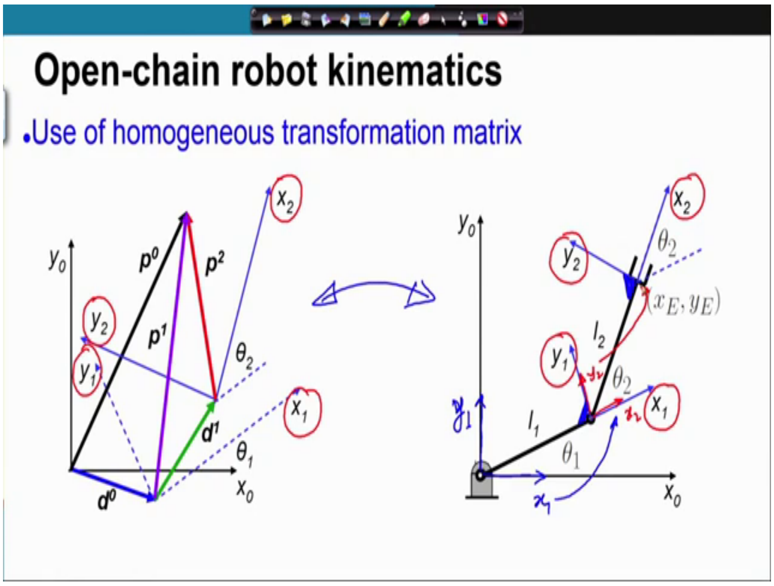

Then we had discussed about the sequence of homogeneous transformation matrix, and this is what we are going to use now to describe the kinematics of a planar open chain robot manipulator. So, how do we relate? You can see that this frame X 1 Y 1 is a frame

X 1 Y 1 for the manipulator. Now this X 1 Y 1 is fixed it is fixed to frame to the link l 1 this frame x 1 y 1 is fixed to link l 1.

Then we have this frame X 2 Y 2 correspondingly we have frame X 2 Y 2 which is a again a translation from X 1 Y 1 by this link length l 2 and this frame is fixed to the link l 2. So, X 1 Y 1 is fixed to link l 1 X 2 Y 2 is fixed to lengths link l 2. Now we see a correspondence between these 2. If you imagine that the frame X 1 Y 1 was rep was obtained by a general motion of the frame starting from X 0 Y 0.

So, we initially had x 1 y 1 like this then we translated and rotated it to obtain x 1 y 1 similarly we had a frame x 2 y 2 which was translated and rotated to obtain the frame x 2 y 2 as we see here.

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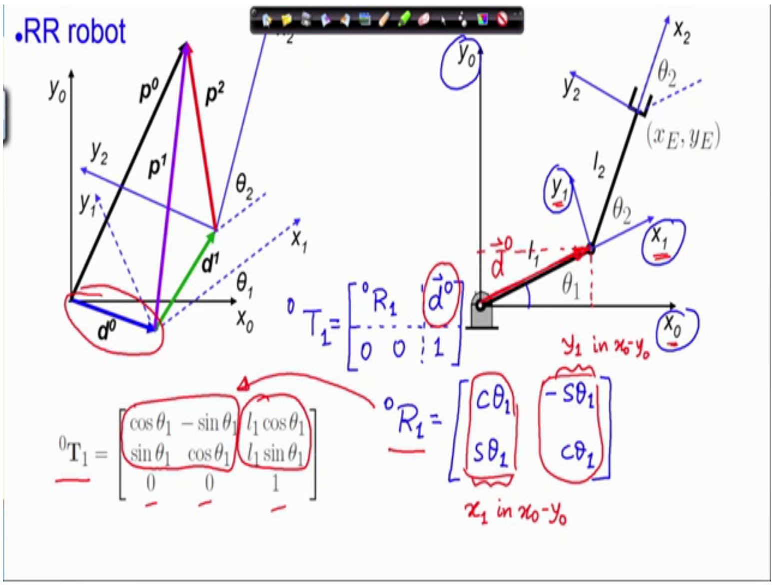

Now, we have written out the homogeneous transformation matrix, previously. So, how do you construct the homogeneous transformation matrix? Let us look into that.

Here I have already written out the homogenous transformation matrix T from 1 to 0, now how is this constructed? You remember that T 1 to 0 has four blocks one is R rotation from 1 to 0 here we have two zeros here we had the vector d 0 and finally, 1.

Now, this R 1 to 0 is a rotation matrix from frame 1 to frame 0 this is frame 1 and this is frame 0 for the robot. Now how do you represent R 1 to 0 you represent X 1 in X 0 Y 0 as the first column and Y 1 in X 0 Y 0 as the second column. Now X 1 in Y 0 X 0, Y 0 here is this angle theta 1. So, the unit vector along X 1 can be represented as cos theta 1, sin theta 1 a unit vector along y 1 can be represented as minus sin theta 1 cos theta 1.

So, remember that this is representation of x 1 in x 0 y 0 and this one this column. So, this whole column is a representation of y 1 in x 0, y 0. So, the first column is the representation of x 1 in x 0, y 0 the second column is the representation of y 1 in x 0 y 0. So, that gives us R from 1 to 0 and that is what I have written in the first block.

In the second block we have this d 0. So, this is this vector d 0 which is the translation of the origin of x 1, y 1 in x 0 y 0. So, it is the representation of translation of X 1 Y 1 in x z X 1 Y 1 in X 0 Y 0 this is the translation. So, this must be the vector d 0. So, what is this vector d 0? It is nothing, but l 1 cosine theta 1 and l 1 sin theta 1, as you can very easily see the length is l 1. So, l 1 cosine theta 1 and l 1 sin theta 1. So, that is d 0 and these terms are 0 0 1. So, that constructs for me the homogeneous transformation matrix T from 1 to 0.

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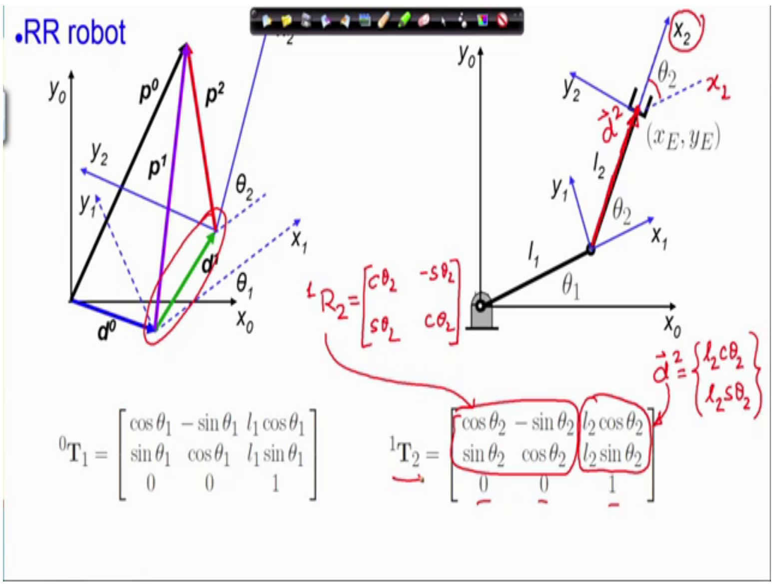

Similar procedure we can follow for T 2 to 1. So, we have to represent R from 2 to one. So, R from 2 to 1, the first column should be the representation of x 2 in x 1 y 1 now if this angle is theta 2 and this is x 1 direction. So, first column becomes cosine theta 2 sin theta 2 this is the representation of x 2 in x 1 y 1 the second column is the representation of y 2 in x 1 y 1 which is minus sin theta 2 cos theta 2. So, that is R 2 to 1 which goes in the first block.

Now, this second block this vector is nothing, but the representation of d 1. So, this is d 1. So, d 1 is the position vector of the origin of x 2 y 2 represented in x 1 y 1. So, this vector must be d 2 and that is d 2 vector is nothing, but l 2 this length is l 2 cosine theta 2 remember that we are representing in x 1 y 1 we are representing the origin of x 2 y 2 in x 1 y 1. So, it is l 2 cosine theta 2 and l 2 sin theta 2 and that is what we have here as a second block.

And the other terms are 0 0 1. So, that constructs for us the homogeneous transformation matrix T from 2 to 1.

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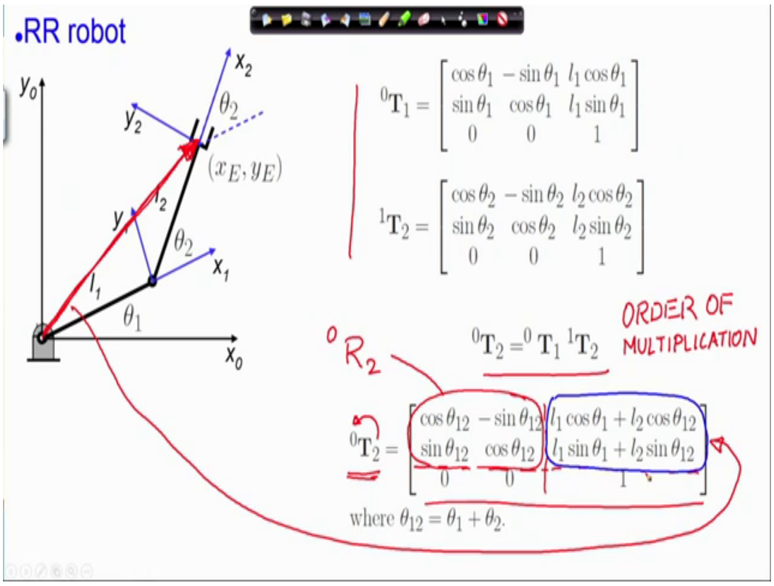

Now, we have these 2 homogeneous transformation matrices. So, therefore, the combined transformation from 2 to 0, the combined transformation from 2 to 0 is given by the transformation of 1 to 0 times the transformation of 2 to 1. So, this gives us the transformation from 2 to 0.

Now, if you do that you will get this transformation matrix, if you multiply these 2 in this order. Please remember that this order is very important order of multiplication of matrices. This is extremely important, if you multiply these 2 matrices in this order then you will get the homogeneous transformation matrix from 2 to 0, which is of this form now this block this block is nothing, but the transformation from 2 to 0 the rotation made rotational transformation from 2 to 0 which you can check very easily.

The second block this vector is nothing, but the vector from the origin of x 0 y 0 to the origin of x 2 y 2 this vector is the entry of the second block. This vector is the entry of the second block and this is what we have done as the forward kinematics relation we have discussed this, this relation is the forward kinematics relation for this to our manipulator.

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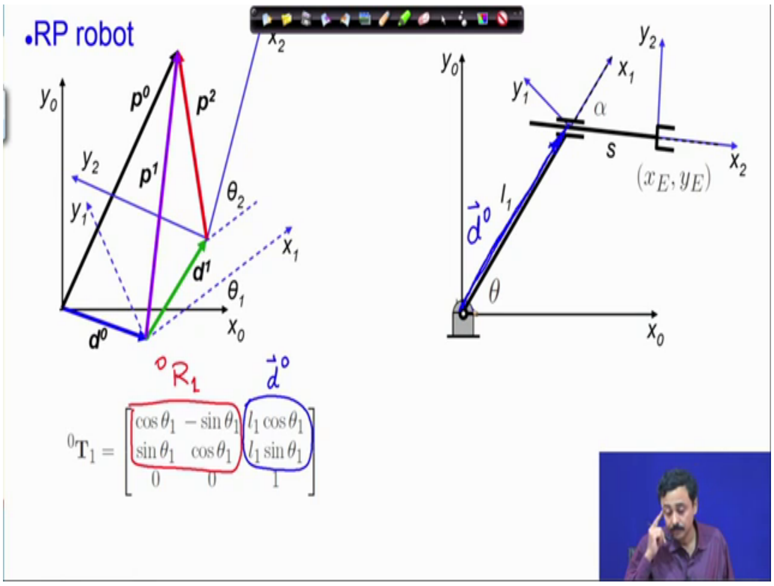

Let us move on to the R p manipulator kinematics here we have the frame X 1 Y 1 in this schematic representation as the frame X 1 Y 1 here the ground fix frame is of course, X 0 Y 0. So, this is ground fixed X 1 Y 1 is at the end of link l 1 and it is welded to it. So, X 1 Y 1 is fixed to link l 1 x 2 y 2 is fixed to link l 2.

Now, we can see the correspondence between these 2, let us start writing the transformation matrices. Once again the homogeneous transformation matrix from frame 1 to frame 0 this remains same as that of the 2 r manipulator let me mark out the blocks this is nothing, but R from 1 to 0 the rotation matrix from 1 to 0, and the second entry this which is d 0 is this vector.

So, this vector is d 0 essentially this is the translation of frame 1 from the origin of frame 0, which is d 0.

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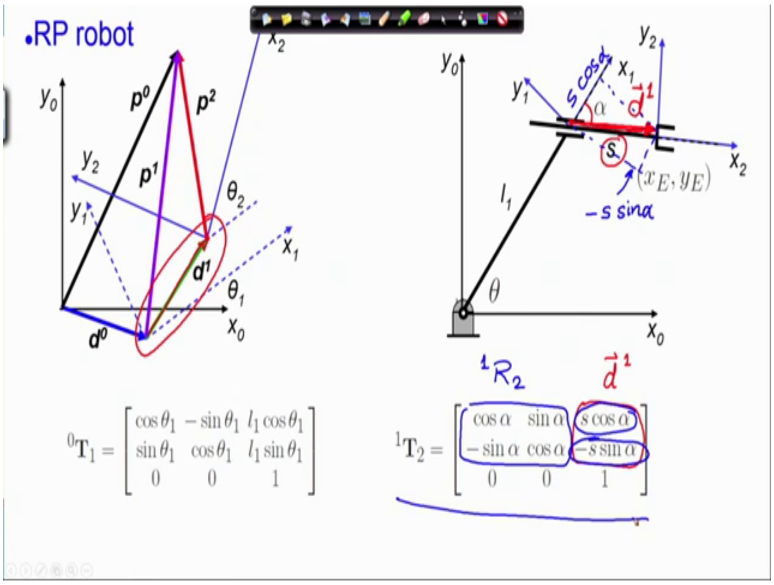

Similarly you can write out T 2 to 1 the homogeneous transformation matrix from 2 to 1 this block is the rotation matrix from 2 to 1, and this column is nothing, but d 1.

Now, what is d 1? D 1 is this vector this is d 1 essentially d 1 is the representation of the origin of frame 2 in x 1 y 1 remember that d 1. So, this is d 1 in the schematic; d 1 is the location of the origin of frame X 2 Y 2 origin of the frame X 2 Y 2 in X 1 Y 1 that is d 1 d 1.

Therefore is nothing, but S times cosine of alpha. Now if I look at this is S cosine of alpha and the other entry is minus S sine of alpha as you can see very easily here. So, we are representing the vector the origin of X 2 Y 2 in X 1 Y 1. So, the 2 components are S cosine alpha and minus S sin alpha. So, that is what I have. So, the first component is s cosine alpha the second component is minus S sin alpha. So, that is the representation of d 1 in X 1 Y 1. So, that is that completes the homogeneous transformation matrix T from 2 to 1.

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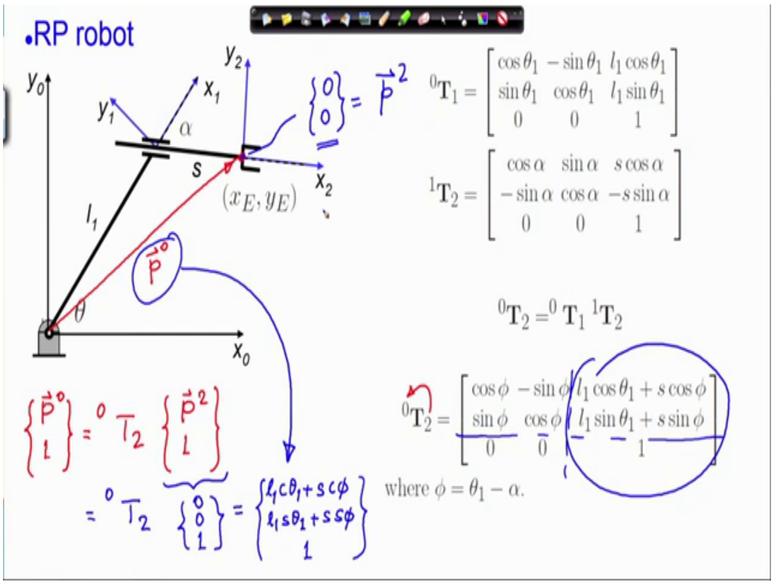

Now, let us collect these two matrices and to obtain T from 2 to 0, I need to multiply T from 1 to 0 and T from 2 to 1 to obtain T from 2 to 0 the homogeneous transformation matrix T from frame 2 to frame 0. Now if I want to if I carry out this product, if I multiply these two matrices I obtain this transformation from frame 2 to frame 0.

So, as we have noted the idea is to have a vector in frame 2 and if I multiply with the homogeneous transformation matrix from 2 to 0, I will get T 0 1. Now if I want to really find out the position vector this, I need to consider the origin which means the representation now what is this point in X 2 Y 2? This point in X 2 Y 2 is nothing, but 0 0, this is the origin of the frame X 2 Y 2 this is p 2.

So, if p 2 is 0 0, then this becomes 0 0 1 this becomes 0 0 1 because I have considered the point to be the origin of the frame X 2 Y 2. So, the origin of the frame X 2 Y 2 is nothing, but 0 0 in X 2 Y 2 the representation is 0 0, that is the origin of X 2 Y 2. So, therefore, this extended vector becomes 0 0 1 and when I multiply this with T 2 to 0 you can very easily see and when if I multiply this with 0 0 1, I only filter out this column.

So, therefore, I have l 1 cosine theta 1 plus s cosine phi l 1 sin theta 1 plus s sin phi and 1. So, this is what I will have. So, essentially this p 0 is this vector. So, that is the origin of X 2 Y 2 represented in X 0 Y 0. Now if I want to take another point. So, this is for the origin if I want to take another point let us take another point let us take a point q along X 2.

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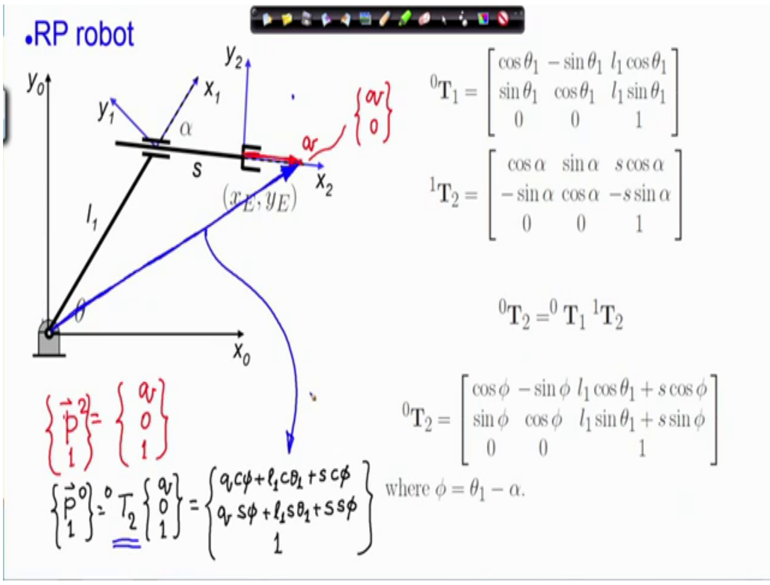

Now, the representation of this point is nothing, but q 0; the x coordinate is q and the y coordinate is 0 therefore, the extended vector p 2 becomes q 0 1, this is the extended vector. I have to now if I want to find out p 0; P 0 1 is nothing, but T from 2 to 0 q 0 1 and if you do that you will get q cosine phi plus l 1 cos theta 1 plus s cos phi q sin phi plus l 1 sin theta 1 plus S sin phi and 1.

Now, what is this vector? This vector is this vector it is this vector that we have now obtained. So, this way I can represent any point in the x 2 y 2 frames, now this is of advantage because when the manipulator moves we will know the point that it has to grasp or reach in the local frame in the end effector frame. So, if you know the point in the end effector frame you exactly know the coordinates of that point in the ground frame.

So, if you are if the manipulator is holding something some extended body and if you want to find out the coordinates of this point let us say in the ground reference frame your ground fixed frame then you need to find out this vector in the local end effector frame and then pre multiply it with the transformation homogeneous transformation matrix T from 2 to 0 and you will get the representation of that point in the ground fixed frame.

So, that is how we are we can very easily, find out the coordinates in the ground fixed frame using the concept of homogeneous transformation matrix.

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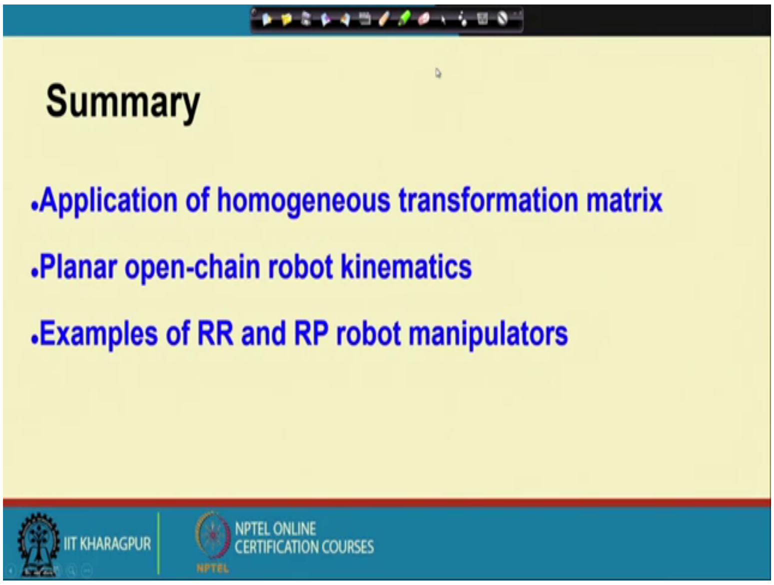

So, to summarize we have looked at the application of homogeneous transformation matrix to robot manipulators, to write out their kinematic relations, we have looked at 2 examples of planar open chain manipulators the RR and RP planar manipulators and we have determined their homogenous transformation matrices.

Now, this is very useful in representing the kinematics of such manipulators. Now this is 1 step towards understanding the coordinate transformation in case of spatial manipulators or in 3 dimensions. So, in this lecture we have taken that 1 step, and we have understood how to use the homogeneous transformation matrix in the to write out the kinematic relations for robot manipulators, and with that I will conclude this lecture.