Mechanism and Robot Kinematics

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Lecture - 04
Degree of Freedom - I

We define a mechanism as a combination of interconnected links or rigid bodies, which can move. So, this is the basic definition of a mechanism. So, the natural question that arises is can any combination of rigid bodies are interconnection of rigid bodies give rise to a mechanism; that means, a combination that can move. Well, to know that we need to calculate what is known as the degree of freedom of kinematic chain.

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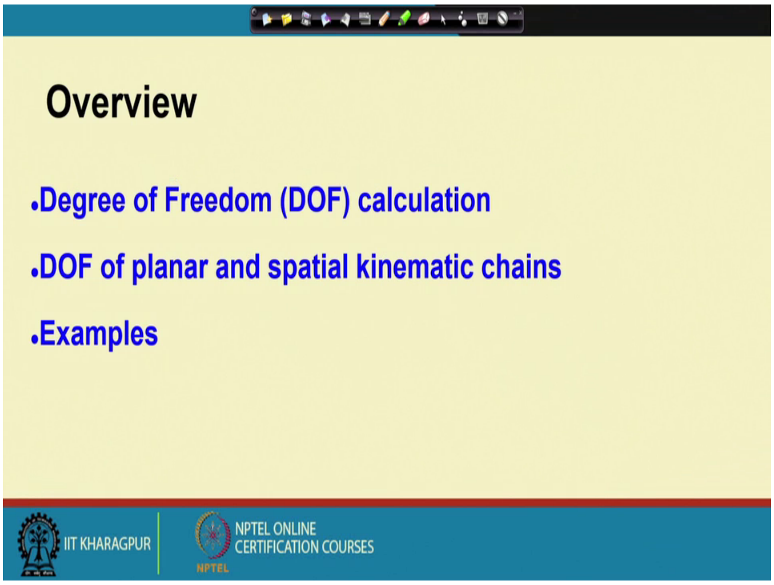

So, in today's lecture we will start with the calculation of degree of freedom or DOF. We look into in the course of the lectures. We look into the calculation of degree of freedom of planar and spatial kinematic chains with examples. So, the question first that arises is how do we define degree of freedom.

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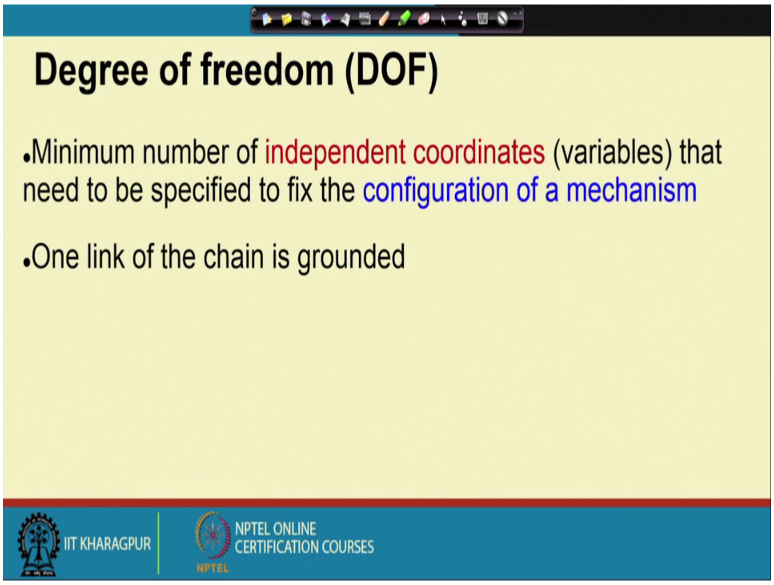

So, here I have written out the formal definition, is the minimum number of independent coordinates or variables that need to be specified to fix the configuration of a mechanism. So, minimum number of independent coordinates, and configuration of mechanism these are the keywords.

So, configuration by configuration I mean, how does the mechanism look like when it is fixed. Now when we do this calculation, one link of the chain must be grounded; now why this requirement? Suppose I have a plier, I can take this plier from here to another place, but that does not constitute the degree of freedom of that plier, the degree of freedom of the plier is what the relative motion of the links of the plier can have, but relative motion they can have.

So, I must hold one of the links and move the other links with respect to that. So, we must ground one link. So, that is the requirement. Now let us look into how we calculate degree of freedom.

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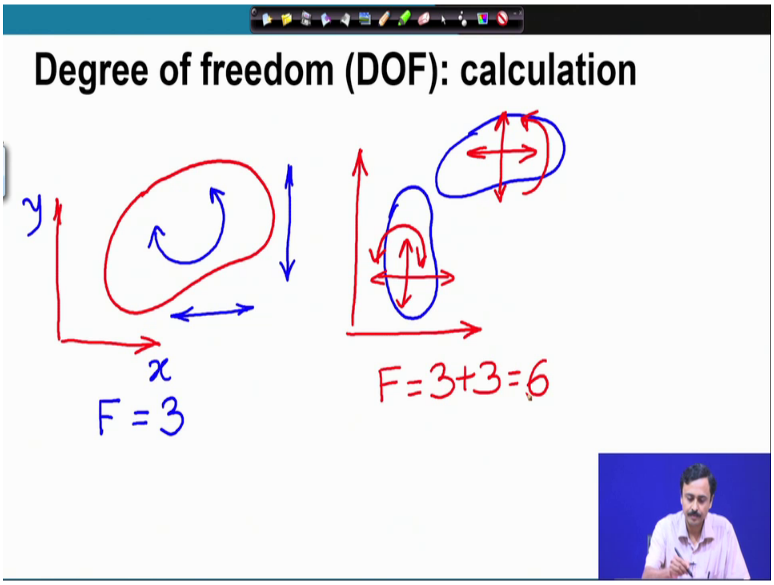

Let me start with a simple thing a planar body. So, a body on a plane; as you know this link this body has 3 degrees of freedom, what are the 3 degrees of freedom? It can move along this coordinate let me call that as x, it can move along y and it can rotate in the plane.

So, these are the 3 degrees of freedom. So, we will say that the degree of freedom of a rigid body in a plane is 3. Let us look at another example. Suppose there are two bodies now again in a plane, now this body can have this kind of motion, this body can also have this kind of motion. So, individually they have 3 plus 3. So, the total system of these two rigid bodies is 6.

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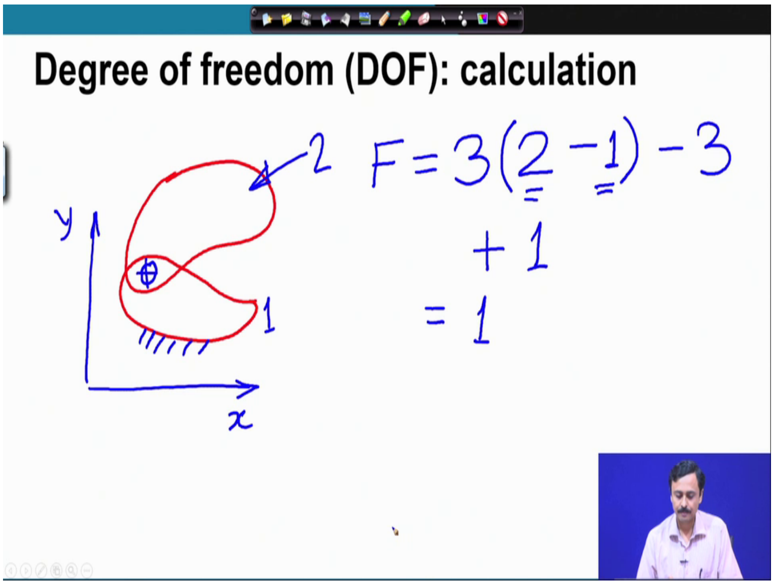

Now, we look at this example, we have two links now connected by a kinematic pair. So, this forms a chain as we had discussed, I will ground one body. If I ground one body immediately it loses all its degrees of freedom. So, right now I have this rigid body this link which can move. So, without this kinematic pair, let us see without this kinematic pair this body has 3 degrees of freedom. This body had 3 degrees of freedom, but I have grounded it.

So, what I will do is, I will say that there were two rigid bodies, I will subtract one from it because I have grounded it and multiply it with 3; because in a plane now there is one rigid body which can actually move out of these two rigid bodies, which I have one is grounded. So, I subtract this one from the total number of rigid bodies and multiply it with 3;s o that is the degree of freedom that this rigid this link let me call this as 1 and this as 2. So, link two can have, but no I have a kinematic pair here let me say that this kinematic pair takes away or every kinematic pair takes away 3 degrees of freedom; that means, it is completely immobilized it completely immobilizes link 2.

But again it does not completely immobilize, what does it do it allows the degree of freedom afforded by that kinematic pair. For example, if this is a hinge then as you know a hinge has one pair variable. So, it has one degree of freedom. So, I add that one to it. Therefore, finally, I have degree of freedom of this system as 1, let me take another example.

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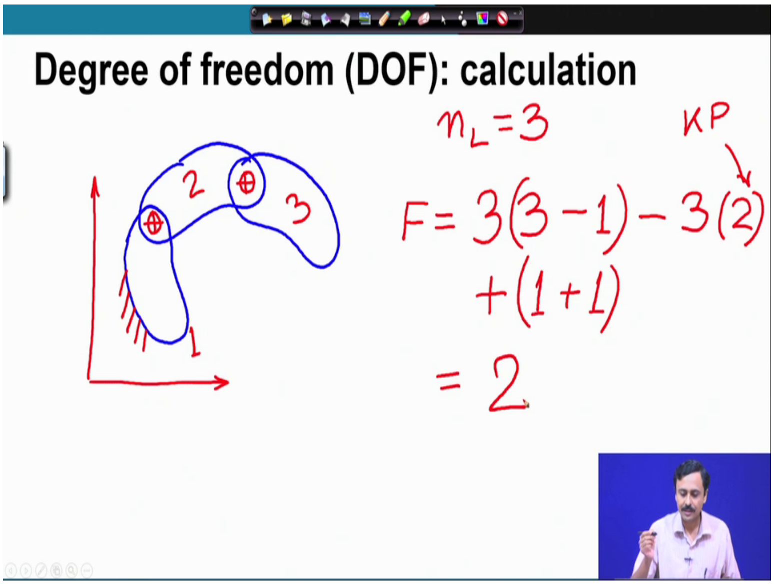

This is a kinematic chain with 3 links. So, let me write number of links is equals to 3, as discussed we are going to ground 1.

Now, we have degree of freedom as. So, I had initially 3 links minus 1 is grounded. So, it has lost all degrees of freedom. So, now, if I multiply this with 3, I get. So, this is 1 2 3 I essentially get the degrees of freedom of links 2 and 3, but there are two kinematic pairs let me say that each kinematic pair takes away 3 degrees of freedom; that means, it completely mobilizes.

So, I must subtract because it is taking of a degree of freedom, it is taking away there are two kinematic pairs; it is taking away 3 degrees of each kinematic pair is taking away 3 degrees of freedom. So, 2 will take away 6 degrees of freedom, but then these kinematic pairs must have their pair variables. So, it must have some degree of freedom. So, I will add to it the degrees of freedom of the individual kinematic pairs.

So, suppose again these are hinges. So, there are two hinges. So, one each hinge has one degree of freedom each hinge has one degree of freedom; so 1 plus 1. So, therefore, the degree of freedom is 2 in this case. Now you can keep increasing the number of links change their interconnections and do this calculation, now if you keep doing it you can then generalize this. So, that is what I am going to show you now.

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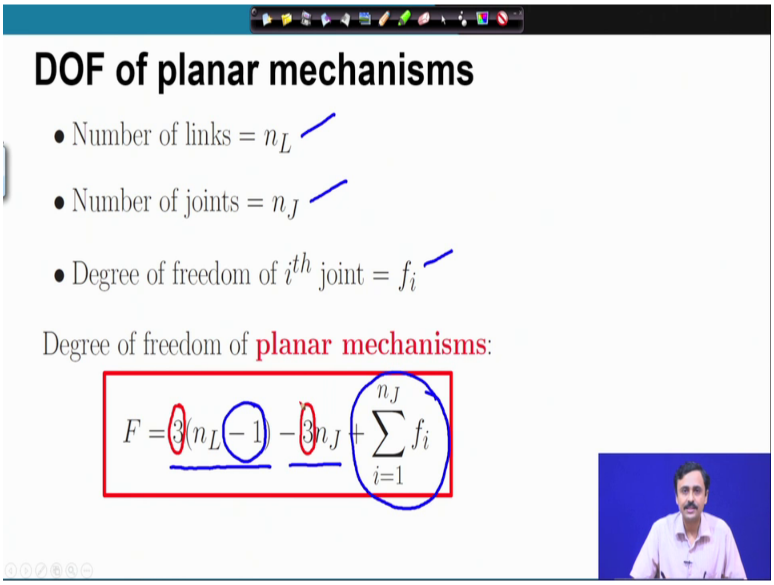

So, this is the generalization. So, if the number of links is n L number of kinematic pairs or joints is nJ and degree of freedom of the i-th joint is f i, then the degree of freedom of a planar mechanism can be given as f is equal to 3 times n L minus 1. So, this term gives you the degree of freedom of the movable links. So, because I have grounded one, I subtract this one. So, total numbers of links minus 1 are the movable links. So, n L minus 1 are the number of movable links, multiplied by 3 because on a plane they individually can have 3 degrees of freedom. Then I subtract from that 3 times the number of joints why because first I say that each joint takes away 3 degrees of freedom, it completely immobilizes, but it does not.

So, I add the sum of degrees of freedom of individual kinematic pair or individual joints that gives me the degree of freedom of a planar mechanism. Now I would like you to look at this 3 why did we have 3; because in a plane a rigid body can have 3 degrees of freedom. So, from there comes this 3. Now if it is a body in space then what happens? For a body in space it has got 6 degrees of freedom as you know a rigid body in space has 6 degrees of freedom 3 translation and 3 rotation about these 3 axis.

So, 3 translational motions say about a along x along y and along z and 3 rotational motions along x along y and along z. So, you have 6 degrees of freedom for a rigid body.

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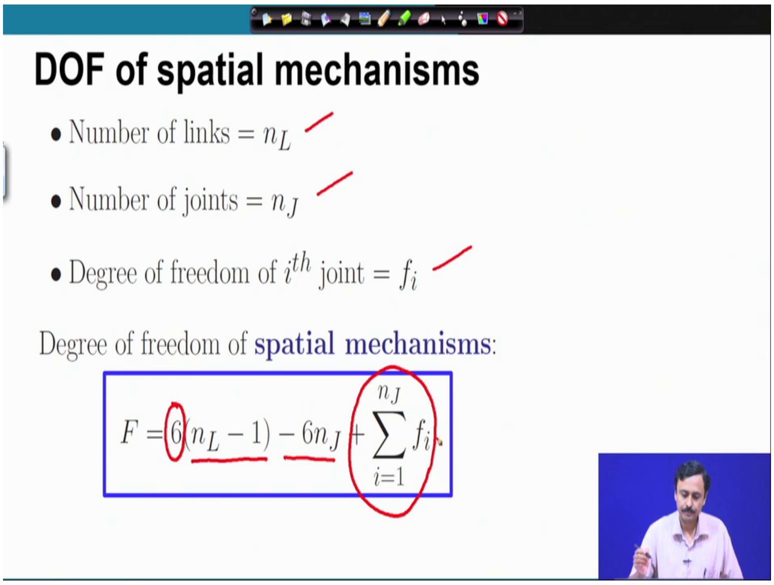

So, therefore, for a spatial mechanism, again the definitions remain the same, number of links is n L, number of joints is n J, degree of freedom of the i-th joint is f i. N L minus 1 because 1 will be grounded. So, n L minus 1 will be the movable links, now we multiply this with 6 because they will have 6 degree individually, they will have 6 degrees of freedom. So, number of movable links into 6 will be the total degrees of freedom minus we subtract what the did the number of joints time 6; why we say that each joint or each kinematic pair takes away all the 6 degrees of freedom, but they do not. So, I have added this third term, which is a summation of degrees of freedom of individual joints.

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Now, let us see what a mechanism must be in terms of its degrees of freedom. As I have mentioned right at the outset that a mechanism is something that can move. So, it must be able to move, it must have at least 1 degree of freedom. So, from this calculation we can now determine the degree of freedom of a combination or an assortment of links connected by certain kinematic pairs. Now if this assortment or this combination has to move then it must have at least one degree of freedom.

So, a mechanism must have at least 1 degree of freedom, now what is the structure? A structure is something that cannot move. So, a structure must have zero degrees of freedom. Now if you calculate the degree of freedom and find it to be negative it can happen in certain cases, we will see very soon that we calculate the degree of freedom and you find it to be negative, it means that it is an over constrained structure.

Now, will take some examples and do some calculations of degree of freedom.

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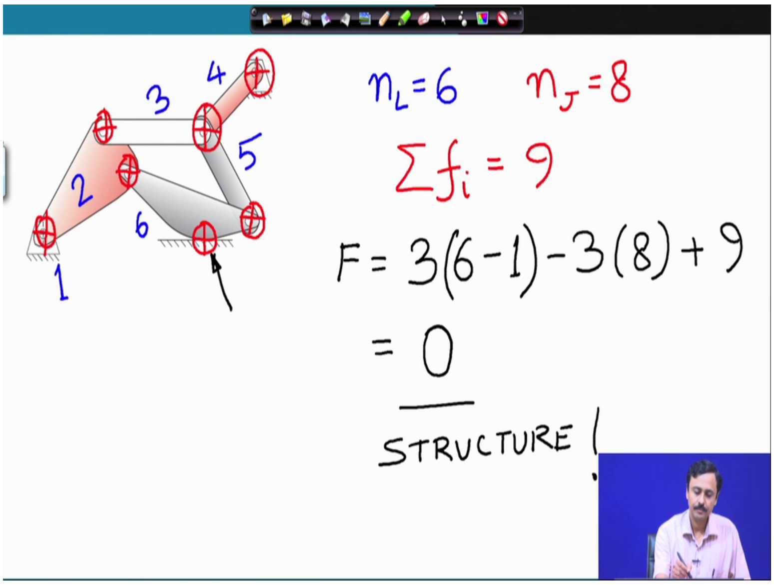

The first example that I have here is like this. So, let me first count the number of links that is what, we need in the degree of freedom calculation, the number of links the number of joints and the degree of freedom of individual joints. So, let me start this counting, as I mentioned the ground is always 1, 2, 3, 4, 5, 6. So, we have number of links as 6 the number of joints 1 2 3 4 5. Now here you see there are 3 rigid bodies connected at a hinge, as we have discussed these this is a special case of two hinges because there are 3 links connected at this. So, there are two kinematic pairs here; so 5 6 7 8. So, number of joints is 8.

Now let us look at the degree of freedom of individual joints and sum them up because we require the summation of degrees of freedom of individual joints. Now this has one degree of freedom these a hinge, 2 3 this is a higher pair lower pairs, this is a lower pair contact as you can see this at this contact, there can be both sliding and rolling of body 6. This is a lower pair contact with point contact. So, this has two degrees of freedom as we have discussed previously.

So, 3 4 5 6 and here there are two kinematic pairs; so 7 8 9. So, summation of degrees of freedom of individual joints is 9. So, therefore, the degree of freedom calculation is 3 times 6 minus 1 minus 3 times 8 plus 9. So, if you do this you will get the answer to be 0. So, 15 plus 9 is 24 and this is minus 24. So, the answer is 0 what does this say about this assortment of links? It says that this is a structure.

So, just a combination of link need not move like this one it cannot move. So, this is a structure of course, we assume that this lower pair contact is maintained; otherwise if this lifts off then this kinematic pair goes out of our calculation. So, maintaining this contact this combination of links cannot move, it is a structure.

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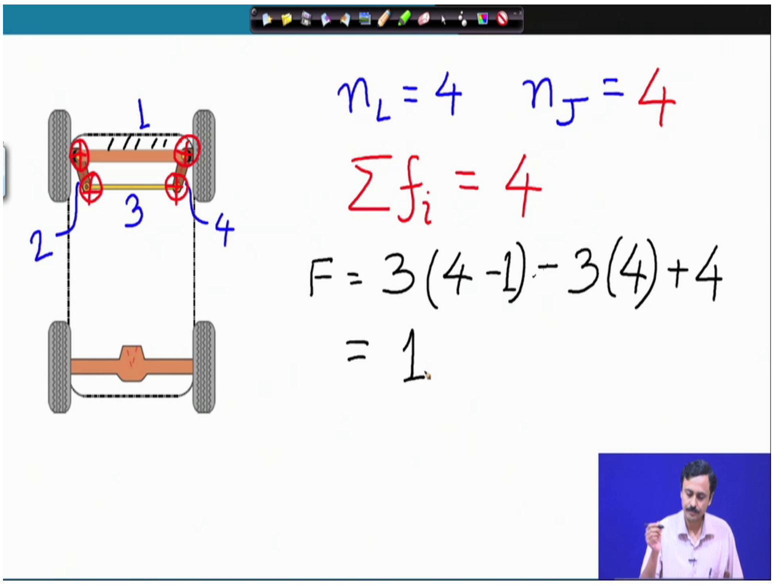

Next we look at this steering wheel mechanism of a car. So, let me again start counting this I will ground this link is connected to the body of the car, and number this as 1, this as 2, the coupling link as 3 this as 4. So, 2 and 4 are connected directly to the wheels.

So, number of links is equals to 4, number of joints as you can see here there is one kinematic pair 2 3 4; summation of degrees of freedom of individual joints. So, each is a hinge therefore, 1 2 3 4. So, the summation is 4. So, therefore, degree of freedom 3 times number of links minus 1 3 times number of links minus 1 minus 3 times the number of joints plus summation of degree of freedom of individual joints.

So, this has. So, this is 9 plus 4 13 minus 12. So, this is 1. So, degree of freedom of this mechanism is 1, which is the steering wheel mechanism and as we expect it to be we operate the steering wheel just by rotating that our steering wheel in the car. So, and it has got only one rotational degree of freedom so, but just by one degree of freedom we are able to steer the car.

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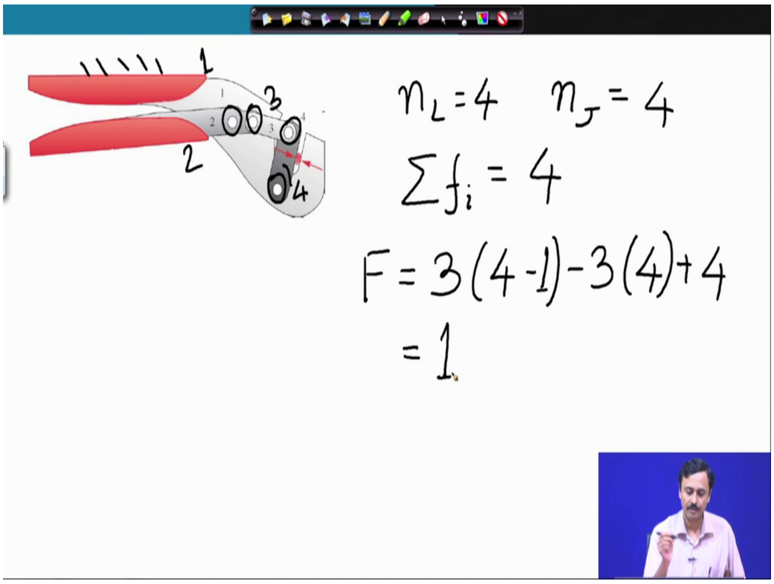

Next we look at this crimping tool let me start by counting the things I will ground this one and number as 1. So, it is already numbered. So, this is two this is 3 and this one is 4. So, number of links is 4 number of joints is 1 2 3 4. Summation of degree of freedom of each joint they are all hinges, there 4 hinges each has one degree of freedom. So, this is 4. So, degree of freedom is 3 time number of joints plus summation of degrees of freedom of individual joint. So, that turns out to be 1. So, this has again one degree of freedom.

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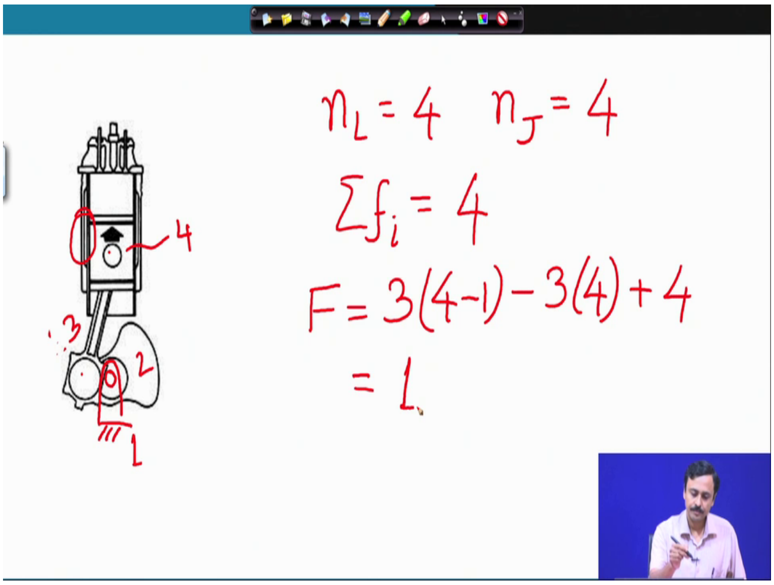

This is an ice engine remember that this is the crankshaft.

So, will start counting 1 2. So, I will count this as 2, 3 is the connecting rod and 4 is the piston. So, number of links is 4 number of joints is 1 2 3 and there is a prismatic pair here; so 4; summation of degree of freedom of individual joints. So, they are all one degree of freedom is a hinge or a prismatic pair they are all one degree of freedom kinematic pairs. So, degree of freedom is 3 times number of links minus 1 minus 3 times number of joints plus 4 that happens to be 1 again.

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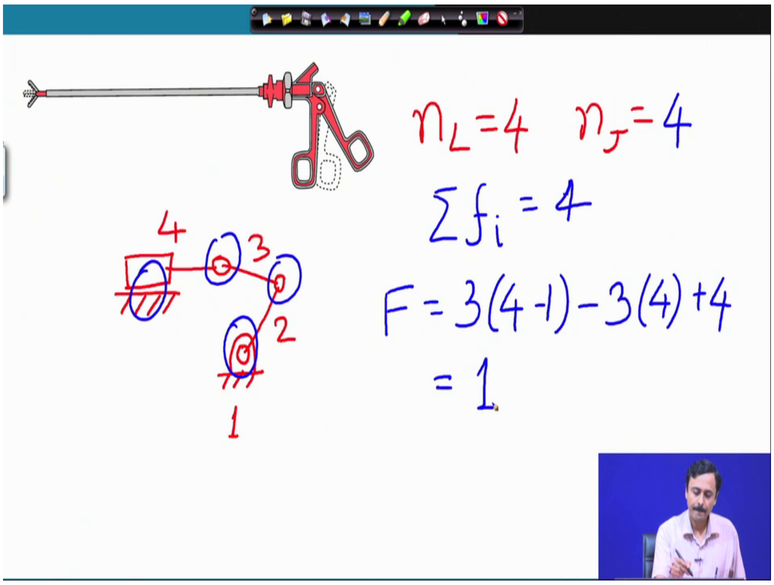

This is the surgeon's tool. First I will quickly draw the kinematic diagram as we have seen.

So, 1 2 3 and 4; number of joints as you can see 1 2 3 4, summation of degree of freedom of each joint they are all one degree of freedom joints: so 4. So, degree of freedom turns out to be 1.

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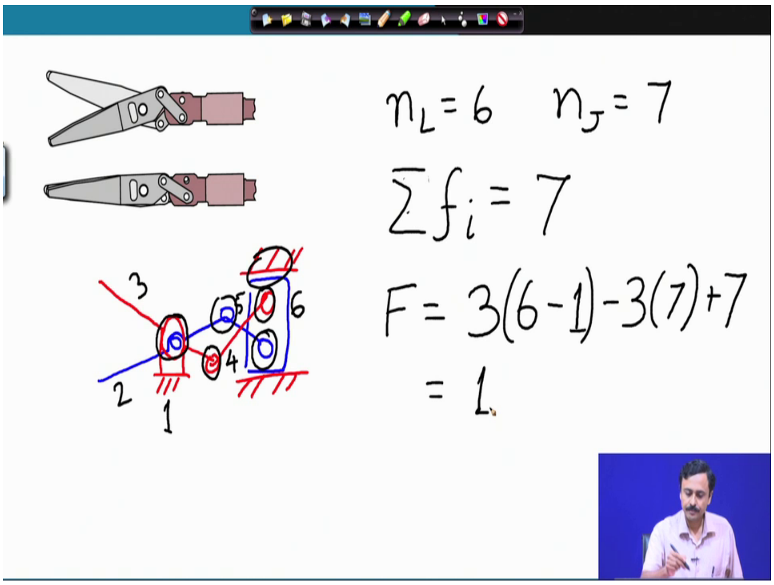

Let us look at this scissors, again I need to draw the kinematic diagram. So, this was the kinematic diagram this is a ground hinge. So, ground is 1 2 3 4 5 6 is the slider.

So, number of links is 6, number of joints. So, here there are two kinematic pairs 2 3 4 5 6 and 7 is the prismatic pair, then we have the summation of degree of freedom of individual joints. So, here there are two kinematic pairs of one degree of freedoms each; so 2 3 4 5 6 7 degrees of freedom. So, that turns out to be 15 plus 7 22 minus 21 is 1. So, this has one degree of freedom.

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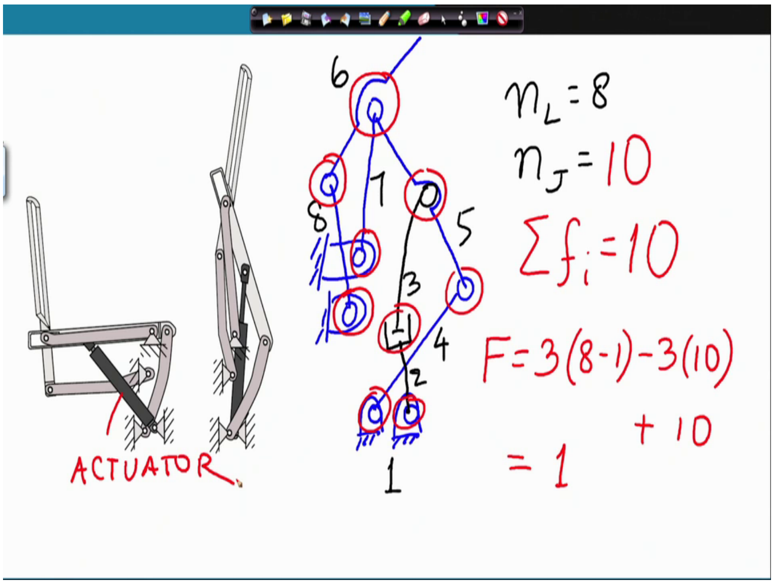

Let us move over to this transfer device, let me draw the kinematic diagram first, here you can see there are two ground hinges. Here there is a an actuator, here there are two more ground hinges this one is connected like this, and from here it is connected like that. So, this is the kinematic diagram. So, let me count the number of links. So, ground is 1 2 3 is the two links which have the prismatic pair, 4 the ternary link has 5 6 7 and 8.

So, number of links is 8, number of kinematic pairs 1 2 3 4 5 now here again 2. So, 6 7 8 9 10; summation of degree of freedom of individual kinetic kinematic pair they are all one degree of freedom except. So, here 1 2 3 4 5 6 7 there are two kinematic pairs 8 9 10. So, degree of freedom is. So, that 31 minus 30, this also has one degree freedom and that is actuated by this actuator. So, I have discussed the kinematic diagram as well as the degree of freedom calculation of this transfer aid.

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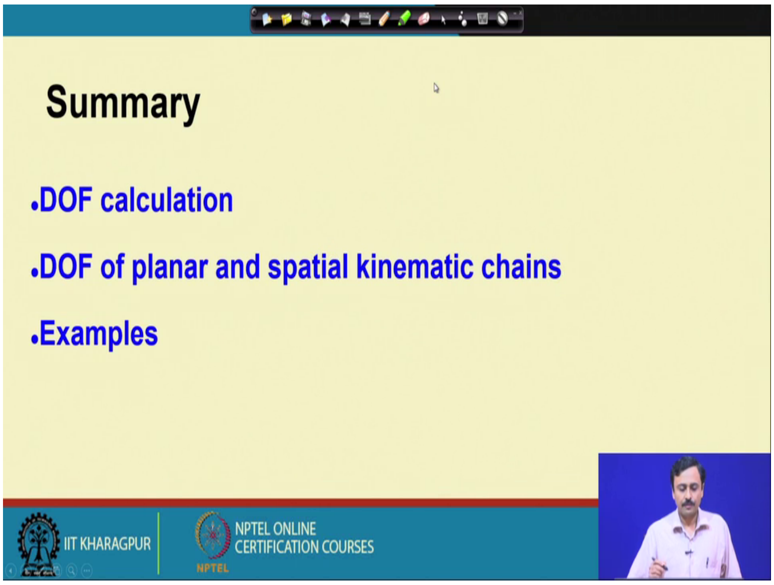

Finally, I will leave you with the summary of what we have discussed today and close this lecture.