Mechanism and Robot Kinematics
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Lecture – 39
Coordinate Transformation – II

We have been discussing about coordinate transformations and representation of a point in a coordinate system and what happens when this coordinate system rotates, how do we relate the coordinates of the point in the local frame in the rotating frame with the coordinates in the ground fixed frame?

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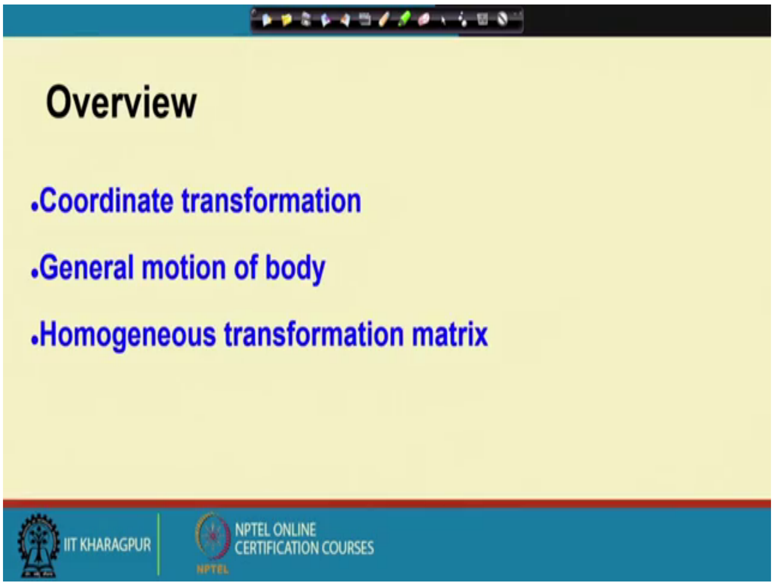

We are going to continue with this discussion, in this lecture I will be discussing about a general motion of a body, a general plane motion of a body and I will also discuss the homogenous transformation matrix.

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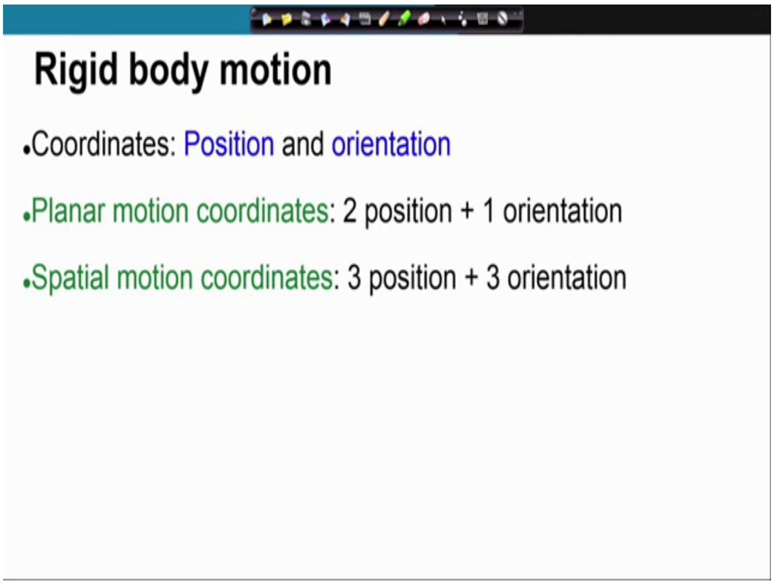

As we have discussed earlier, a rigid body motion requires position and orientational coordinates of the body to be tracked. For planar motion we require two position coordinates and one orientation coordinate whereas, in spatial coordinate a spatial motion of the rigid body we require three position coordinates and three orientation coordinates.

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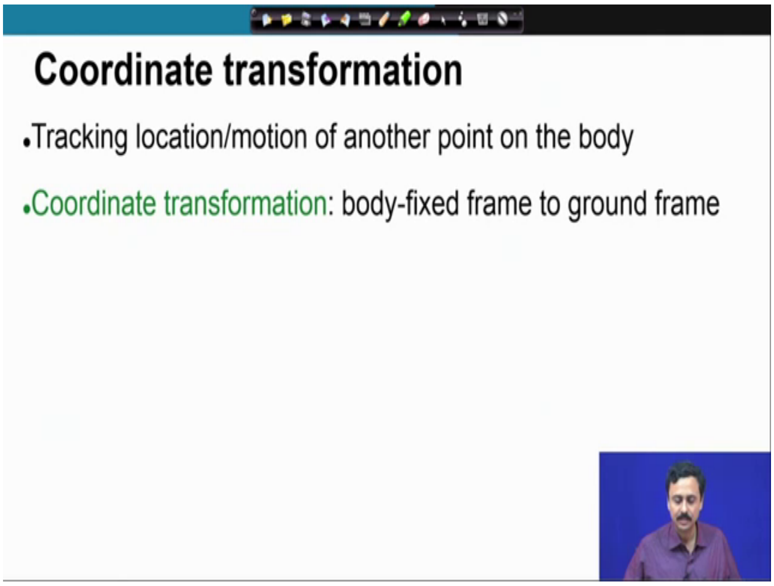

A coordinate transformation helps us in tracking or locating points on a body, which is in motion. So, our coordinate transformation will give us the coordinates of a point in a body, which is moving it will give us the coordinates of the point in the ground fixed frame; so, that is the purpose of coordinate transformation. So, it transforms from a local body fixed frame to a ground fixed frame.

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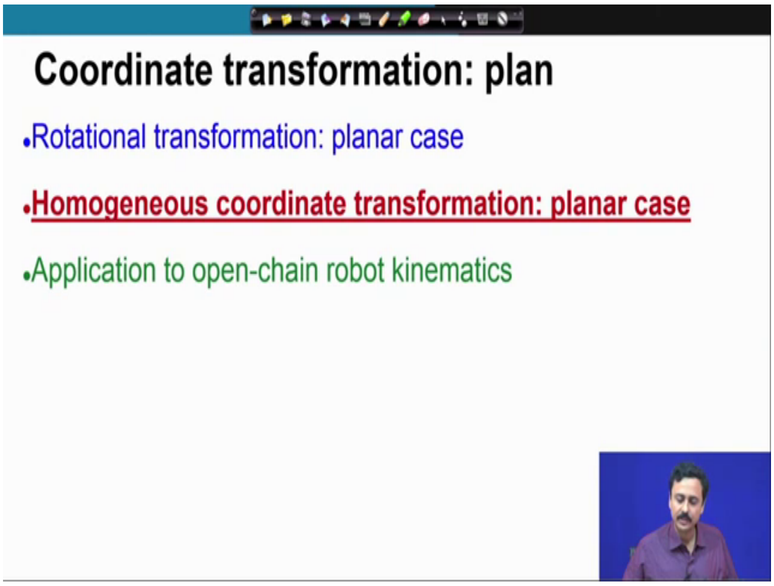

We have discussed that rotational transformation for the planar case, in this lecture we are going to continue and this discussion and look at the homogeneous coordinate transformation for the planar case.

And subsequently we are going to look at applications of this coordinate homogeneous coordinate transformation for open chain planar robot manipulators.

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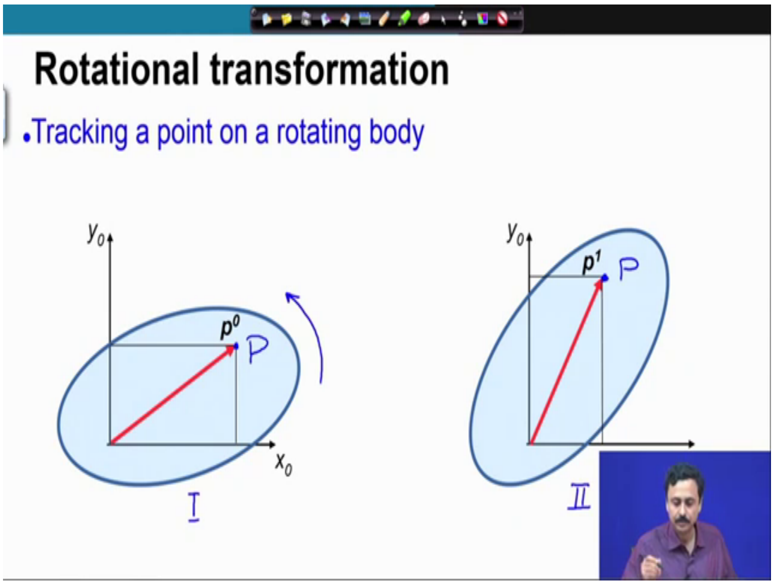

So, we have discussed this, we will review this once again the rotational transformation. So, we need to track the position of a point let us say P in a body as it rotates and comes to this configuration II from configuration I; So, the point p has moved to this new location, I would like to find out the coordinates of point p in the fixed frame x 0 y 1 0. So, that was the objective of the rotational transformation.

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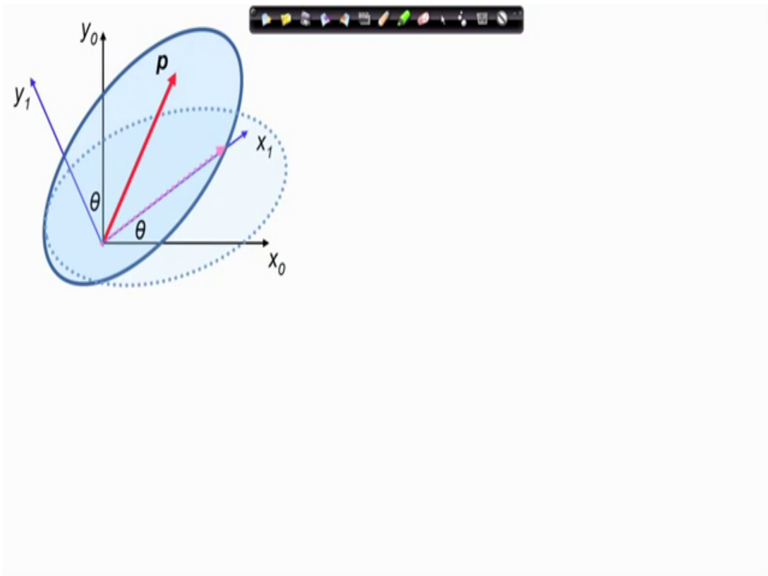

Here I have shown you x 0 y 1 0 as the ground fixed frame and x 1 y 1 as the body fixed frame as the body rotates; the body fixed frame also rotates with that and the point P takes up a new location.

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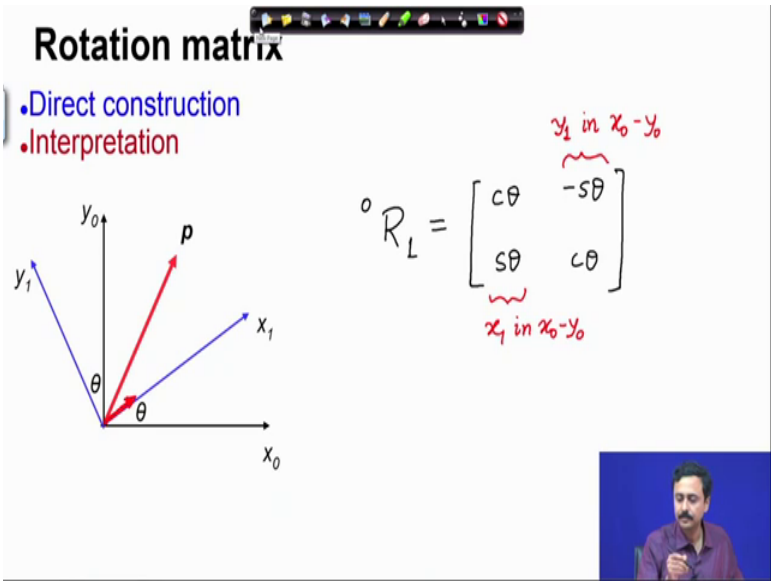

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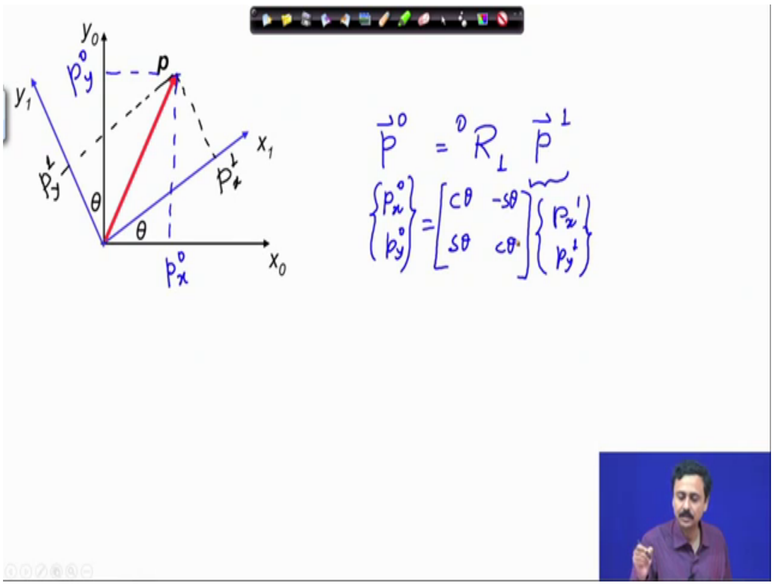

So, we have looked at the construction of the rotation matrix, which relates the body fixed coordinates p x 1 p y 1. So, from p x 1 p y 1we can obtain p x 0 p y 0.

So, this was obtained through the rotational transformation this is nothing, but p x 1 p y 1 and p 0 vector is px 0; py 0. And the rotation matrix we have already discussed this the first column of the rotation matrix is nothing, but the representation of x 1 in x 0 y 0. So, this is cos theta sin theta and the second column of this rotation matrix R 1 to 0 is the representation of y 1 in x 0 y 0.

So, this was minus sin theta cosine theta. So, this was the direct construction. So, let us look at the direct construction once again the rotation matrix when I want to represent frame 1 in frame 0. So, that is R from 1 to 0. So, this rotation matrix takes me from frame 1 to frame 0. So, I must represent the axis of frame 1 in frame 0; the first column is the representation of x 1 axis and frame 0, which is cosine theta sin theta. So, this is the x 1 unit vector along x 1 that is being represented as the first column and the second column is a representation of y 1 in x 0; y 0 which is minus sin theta cosine theta.

So, this is nothing but this column is nothing, but representation of y 1 in x 0; y 0. So, that is the rotation matrix and we have directly constructed the rotation matrix.

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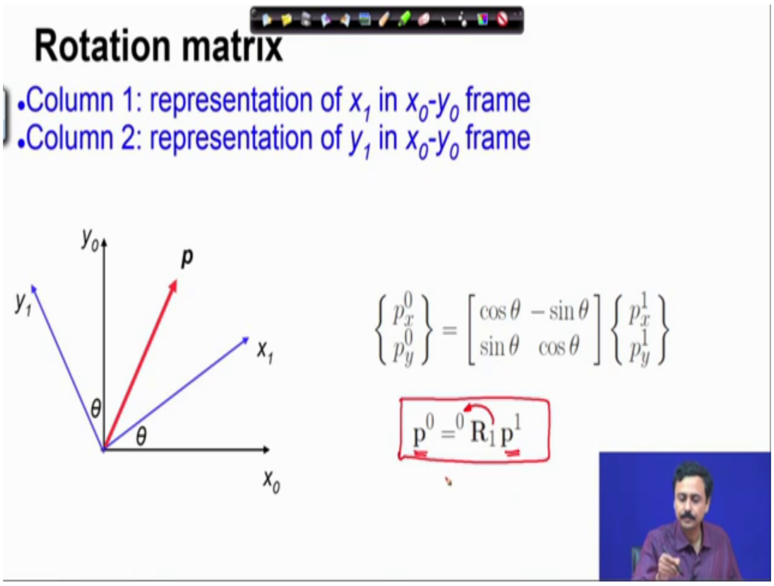

So, this is how we finally, represent the transformation. So, a vector in frame 1 is being taken to a vector in frame 0 and that is how we are representing the rotation matrix.

So, on the right we have one and on the left we have 0. So, which means this rotation matrix is taking a vector in frame 1 to a vector in frame 0 and that is what we have. So, on the there on the right we have the vector p 1 in frame 1 and on the left of this equation we have the same vector being represented in frame 0.

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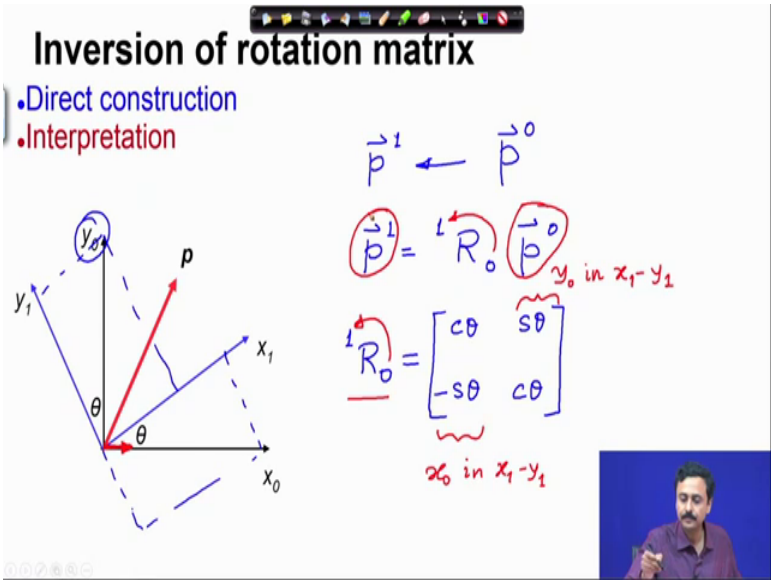

Now, the inverse rotation matrix which is to say that I have the representation of p in frame 0, I would like to take it. So, I have the representation in frame 0, I want to take it to frame 1, I want to represent the same vector p in frame 1. So, which means I will write the rotation matrix as from 0 to 1, p in frame 0 this rotation matrix R 0 to 1.

Since it is taking the frame 0 to frame 1, I must represent the vectors of frame 0 in frame 1. Now let us look at the frame vectors in frame of frame 0. So, this is the x 0 this is a unit vector along x 0 that I must first represent in frame 1. So, the first column should be the representation of x 0 in frame 1. So, this must be cosine theta, and this must be minus sin theta that is a representation of this unit vector along x 0 in x 1 y 1.

So, this column is a representation of x 0 in x 1, y 1 and the second column must be the representation of y 0 in x 1 y 1. So, y 0 in x 1 y 1; so, this is nothing, but sin theta cosine theta is you can very easily check this is the this column is the representation of y 0 in x 1 y 1 that gives me the rotation matrix from 0 frame to frame 1. So, from frame 0 to frame 1; so, on the right we have a vector in frame 0 and we get a vector in frame 1. So, that is the inverse of the rotation matrix here I have written out these relation in a formal manner.

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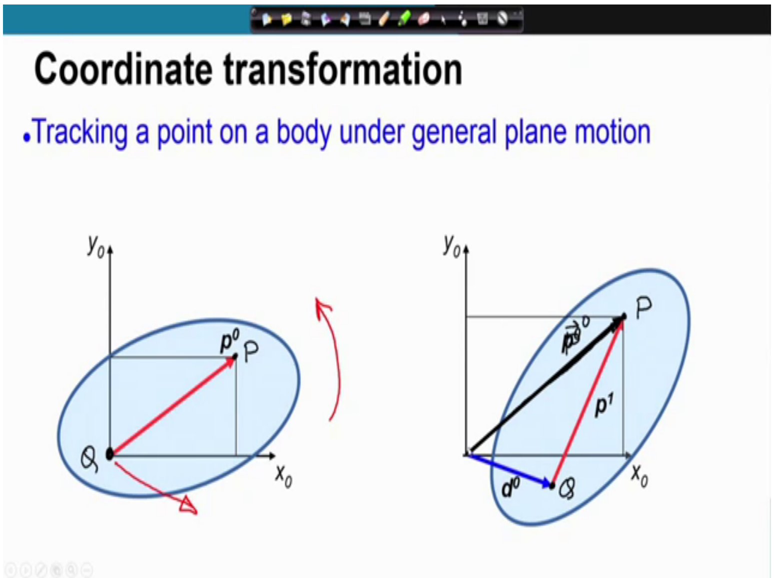

Now, let us go to the general plane motion. So, a general plane motion; a body can not only rotate, but it can also translate. Here I have shown you a body which is not only rotating, but also translating. The point which was at the origin of the frame here I have this point P on the body and I have a point let us say Q sitting at the origin. Now this point Q has moved to this point under this general motion and P has moved here; our objective is to determine this vector p in frame 0.

Our objective is to determine this p vector in frame 0. So, the position vector of the point p, which I have shown here p 0 is the position vector of this point p 0 is a position vector of point p in x 0 y 0 coordinate system, that is what we want to determine.

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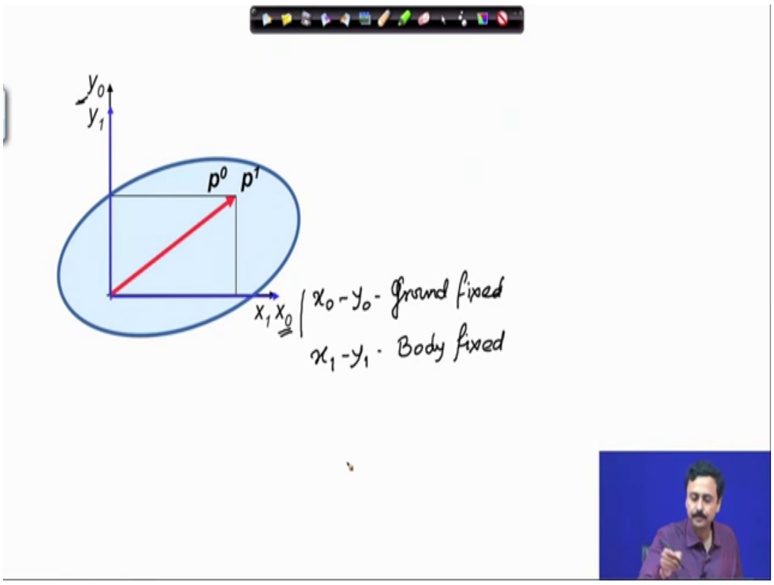

Here I have shown you the situation once again right from the beginning, I have a frame x 0 y 0 which is fixed ground fixed and x 1 y 1 is body fixed. So, x 0 y 0 is ground fixed x 1 y 1 is body fixed.

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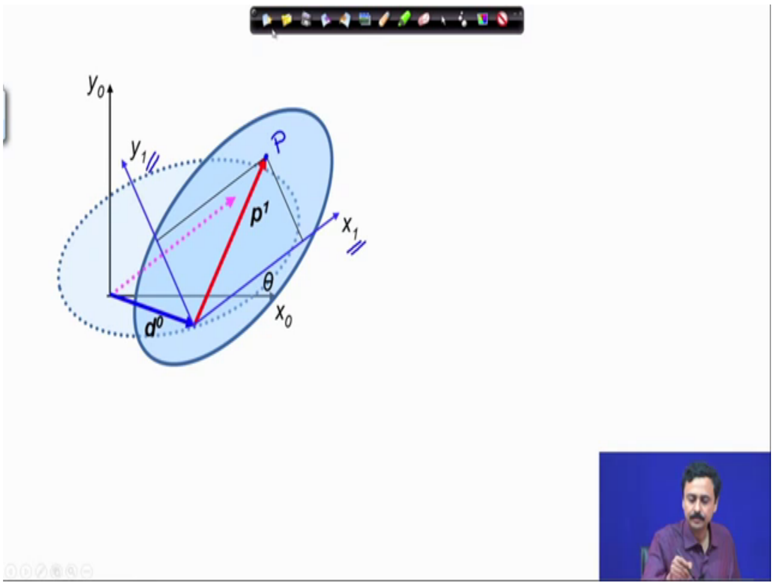

Under general motion the body moves like this and the frame also moves in this manner. So, I have the frame x 1 y 1 here which is fixed with the body and the point p is currently here.

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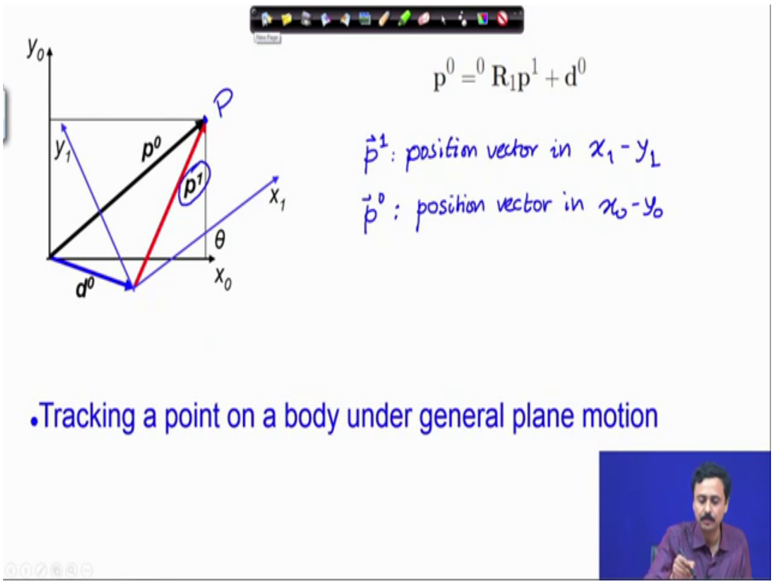

So, our objective is to determine the position vector p naught in frame x 0 y 0 of the point p. So, this is our objective to represent p naught the position vector of p in x 0 y 0 under a general motion of the body. Here I have shown the process schematically, we are tracking this point p under general motion. In the body fixed frame the vector representation is p 1, p 1 is a representation of p in x 1 y 1.

So, this is the position vector in x 1 y 1, p 0 is the position vector in x 0 y 0 you would like to represent these we all like to find a relation between these two representations of the same of the of the position vector of p. Now the relation is like this p 0 is R 1 to 0 times p 1 plus d 0.

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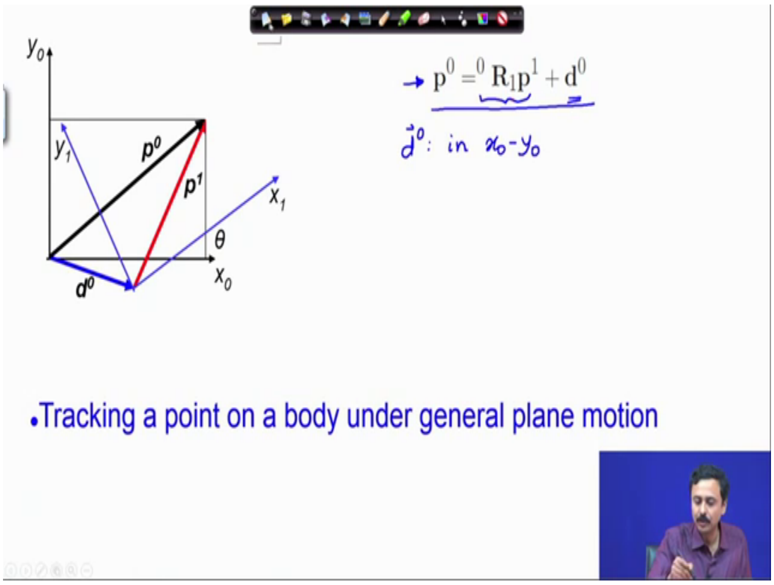

d 0 is represented in frame 0 d 0 has been represented in frame x 0 y 0.

Now, p 1 as I have mentioned is represented in x 1 y 1. So, therefore, in order to be able to add these two vectors to find out p 0, I must first transform p 1 in the frame x 0 y 0 this term therefore, this term is a representation of p 1 in x 0 y 0.

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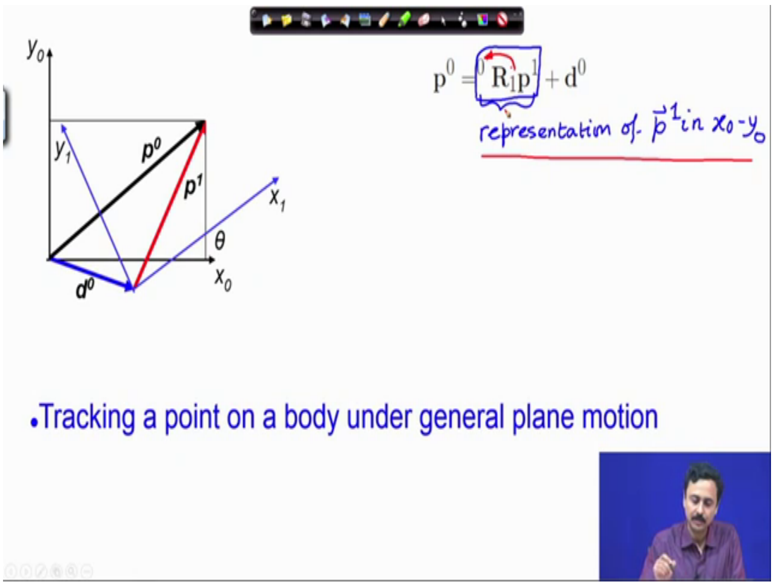

Through the rotation matrix I have brought p 1 in frame 0, through the rotation matrix R 1 to 0, I have found the representation of p 1 which is in x 1 y 1 as a vector in x 0 y 0.

Therefore this whole term is nothing, but the representation of p 1 in x 0 y 0, to that I have added d naught. So, that is how I get p naught.

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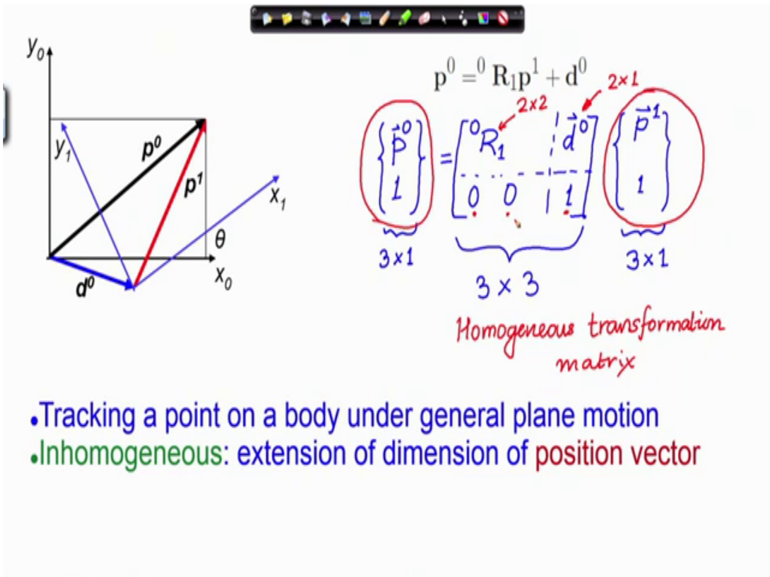

Now you see that this relation is inhomogeneous between p 1 and p 0 this is inhomogeneous. We have set out to relate p 1 and p 0, but now we have a relation which is inhomogeneous, in homogeneity turns out to be because of this additional term d 0.

What do we do to make it homogeneous? Now why do we want to make it homogeneous why do we want to have just a multiplicative factor to relate p 1 and p 0? The reason will get cleared as we move into the application. Here I have p 0 to that let me add an additional term which is one and enlarge this vector remember this p 0 vector is 2 cross 1 and now since I have added this additional element, this becomes three cross one I want to relate this with p 1 and 1.

So, I am enlarging the space in which I am representing p 0 and p 1 by one additional term. So, that both these vectors both these position vectors become three cross one this one is just a factor that I have introduced then you can very easily see, that I can write here R from 1 to 0 which is the 2 cross 2 matrix, put here 2 zeroes put here a one and put here the vector d 0 by this very special manipulation what I have achieved is I have been able to represent the relation between p 1 and p 0 by a just multiplicative factor of this matrix which is 3 cross 3.

Remember that this R is 2 cross 2 d 0 vector is 2 cross 1, here I have added two zeros and a one and I made this whole matrix as 3 cross 3 this matrix is known as the homogenous transformation matrix this is known as the homogeneous transformation matrix. So, what it achieves is that in an enlarged space, I have only a multiplicative matrix which takes p 1 to p 0.

Of course you must remember that when I say p 1 in this enlarge space it is p 1 and one and when I say p 0 it is p 0 and 1. 1 is the next row factor which I need not worry about that one remains one under this transformation as you can very easily check.

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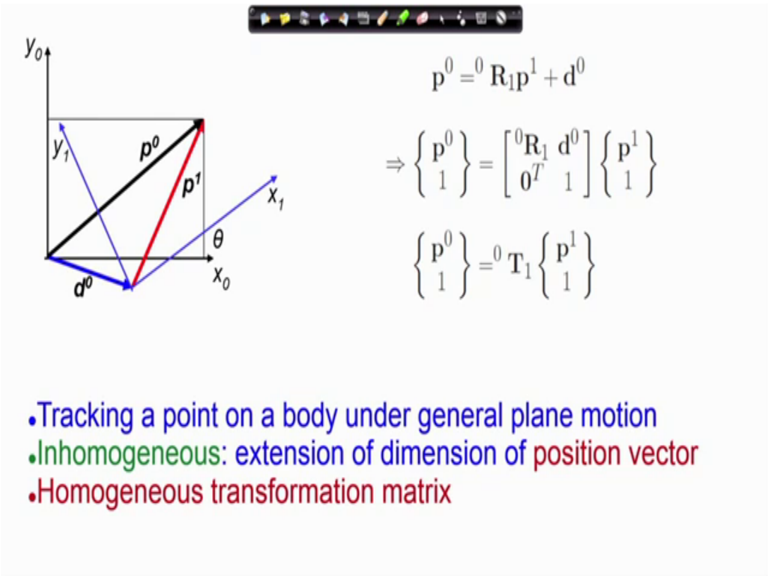

So, by this way we have been able to get to what is known as the homogeneous transformation matrix and which is represented as T from 1 to 0 which takes me takes a vector p 1 1 to p 0 1 this one is an additional element which is of no consequence at present.

Here I have written this as a 0 transpose which is 0 0 as a row, here we have the rotation matrix here is the translational vector d 0. So, here we have on this figure the vector d 0 which is represented in x 0, y 0. Here we have 0 transpose these two zeroes as a row and we have this scalar one. So, we can apportion this homogeneous transformation matrix as this rotation matrix in the top left entry as a top left entry the rotation matrix at the top left entry, the translational vector at the top right entry as a column vector.

And the bottom left entry is a row vector of zeros, and the bottom right entry is a scalar 1. So, that gives us the homogeneous transformation matrix, there is a structure of the homogeneous transformation matrix and we represented as p from 1 to 0.

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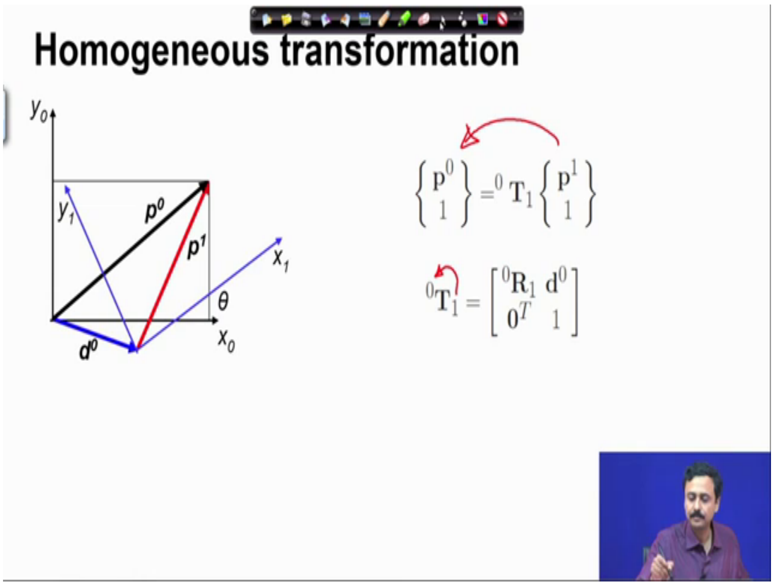

So, this T takes us from 1 to 0 now the inverse homogeneous transformation matrix, how do you invert. So, in the previous case we went from we went from frame 1 to frame 0 as you can see we went from frame 1 to frame 0.

So, p 1 was in frame 1 p 0 was in frame 0. So, you went from frame 1 to frame 0, now what if I want to go from frame 0 to frame 1.

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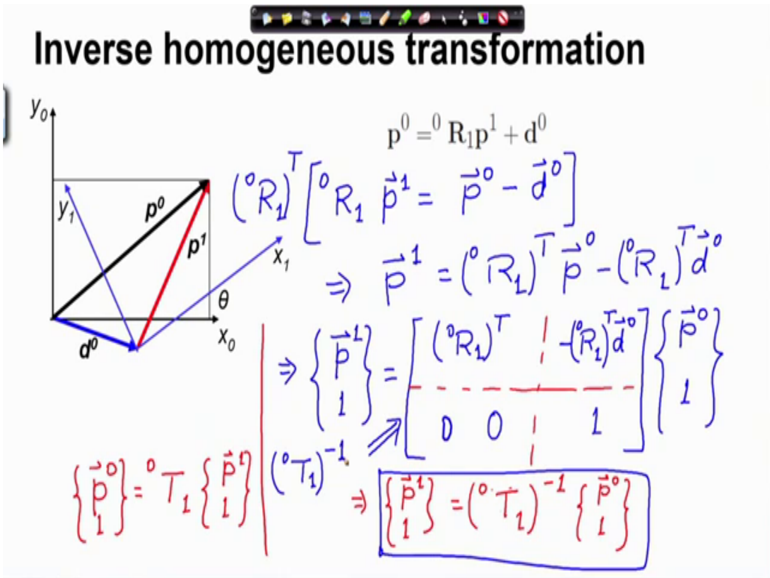

So, we will start off with the same relation as we had. So, this is the forward relation takes us from one to 0 I want to go from 0 to 1. So, essentially I want to represent p 1 and that you can see is nothing, but p 0 minus d 0 if I multiply this pre multiply this with the transpose of R 1 to 0.

If I multiply pre multiply this equation by transpose of R 1 to 0, since transpose of R 1 to 0 is the inverse of R 1 to 0. So, this implies p 1 is equal to R from 1 to 0 transpose, p 0 minus R 1 to 0 transpose d 0. Now we do the same trick again, we extend the space here by an additional element which is 1 in a similar manner on the right we have extended p 0 by an additional element 1.

So, now we look at the entries R 1 to 0 transpose, here we have minus of R 1 to 0 transpose times d 0, here we will have two zeros and a 1. So, this is how we decompose the transformation matrix. So, this homogeneous transformation matrix takes us from 0 to 1 we will write it as. So, we have written we have this notation p 0 1 as transformation from 1 to 0 T 11.

And here we have p 1 1 is equal to T 1 to 0 inverse of p 0 1. So, this is the inverse of the homogeneous transformation matrix T 1 to 0. So, this is how we are going to represent it we are going to say that this is inverse of T 1 to 0 this matrix is the inverse of T 1 to 0.

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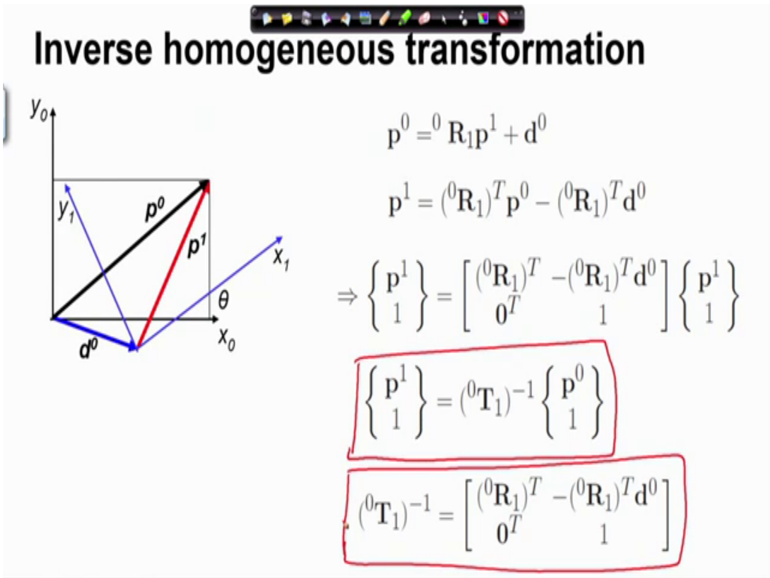

So, here I am formally showing you the steps this is what we finally, obtained this is our representation where the inverse homogeneous transformation matrix is given here.

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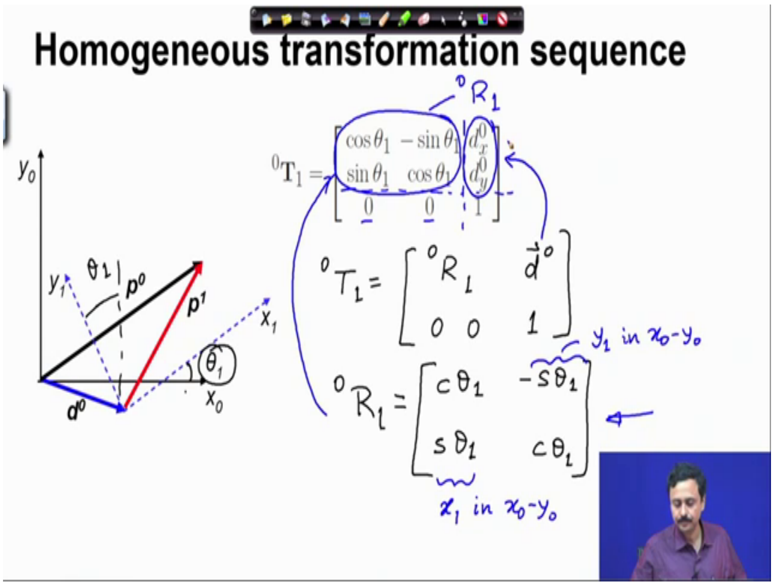

Now, if you have a sequence of rotations I have first x 1 y 1. So, a body moves from x 0 y 1 0 to x 1 y 1 and this x 1, y 1 to x 2, y 2 in the frame x 2 y 2 we have the vector p 2, I would like to find out relation between p 0 which is in frame x 0 y 1 0 and p two which is in x 2 y 2.

Now, to do that we write out the homogeneous transformation matrix from frame 1 to frame 0 which I have done here already now how did I do it? Let me just go through this. So, the homogeneous transformation matrix is nothing, but R from 1 to 0 d in 0 0 0 and 1. Now R from 1 to 0 is nothing, but the representation of frame x 1 y 1 in x 0 y 0.

So, the first column will be the representation of x 1 in x 0, y 0 if this angle is theta one then this must be cosine theta 1 and the y coordinate will be sin theta 1. So, this represents x 1 in x 0 y 0 the second column should represent y 1 in x 0, y 0 now if this angle is theta 1. So, this angle is theta one therefore, this must be minus sin theta one cosine theta 1.

So, this second column is nothing, but representation of y 1 in x 0 y 0 and that is what I have written in this. So, this part is nothing, but R from 1 to 0 which I have obtained here, and then we have two zeros and here we have the vector d 0 being represented as the column. So, that is how I have constructed T 1 to 0.

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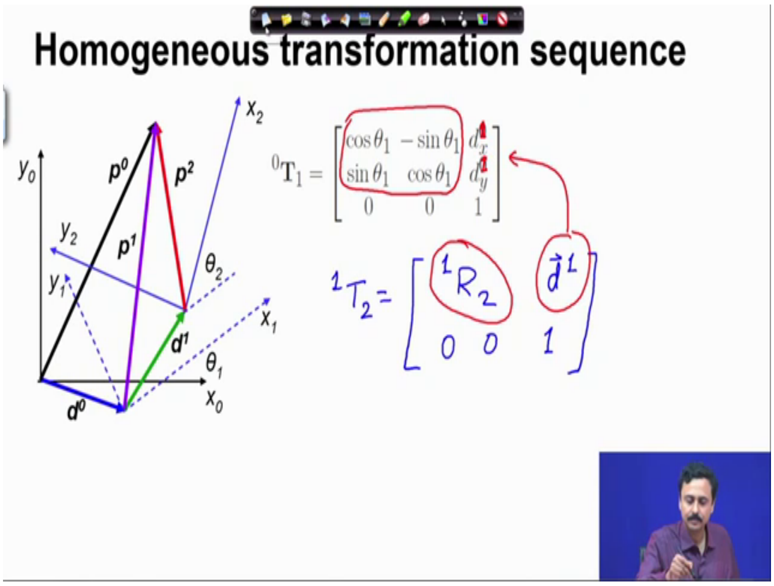

Now let us move to the second sequence the second motion, when it goes from x 1 y 1 to x 2 y 2, there is the same thing here the matrix the homogenous transformation matrix T from 2 to 1 is nothing, but R from 2 to 1 0 0 and here I have d represented in frame 1 and finally, here we have a 1.

You can very easily check that R 2 to 1 is given by this matrix and the vector this should be 1.

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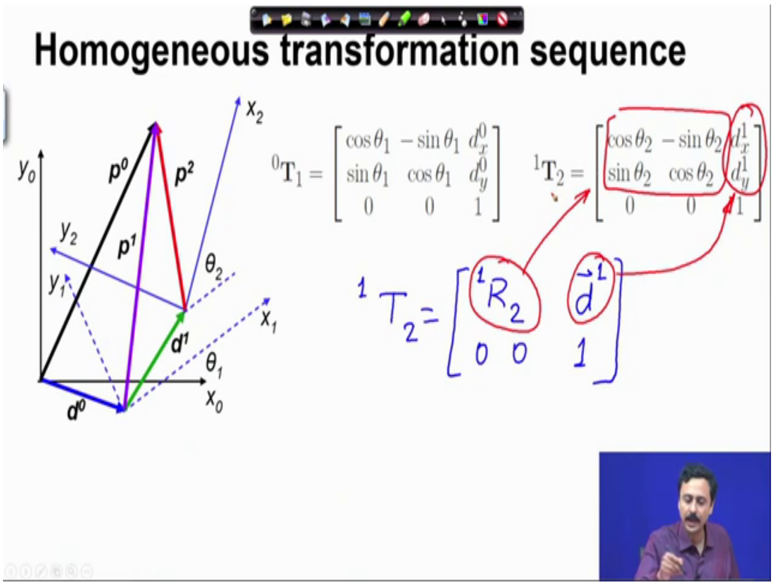

So, here I have the homogeneous transformation matrix T 2 1, which is R from 2 to 1 d 1 0 0 1. So, this R from 2 to 1 is this matrix and d 1 is this vector. So, that is the homogeneous transformation matrix 2 to 1.

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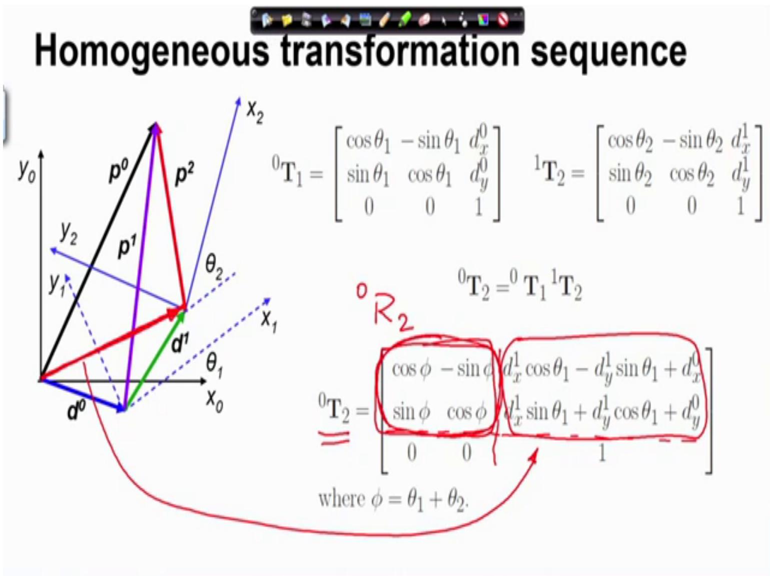

Now, if I want to find out the transformation from 2 to 0 then I just multiplied. So, transformation from 2 to 0 is transformation from1 to 0 times the transformation from 2 to 1. So, 1 2 0 to 2, one gives me two to 0. So, this gives me 2 to 0. So, in order to obtain that homogeneous transformation matrix 2 to 0, I just multiply this in a sequence T 1 to 0 and T 2 to 1 in a sequence that gives me transformation from two to 0 frame 2 to frame 0.

And if you work out the final transformation matrix, this is what you will get I have written it out now this part is nothing, but the rotation matrix 2 to 0 and this vector is nothing, but that eddie the vector that I have represented here in red. So, this vector is this vector. So, it is a location of the origin of frame 2 in frame 0.

So, this vector is nothing, but the location of the origin of frame 2 in x 0, y 0 and this transformation matrix R 2 to 0 is given by this entry of the homogeneous transformation matrix T from 2 to 0.

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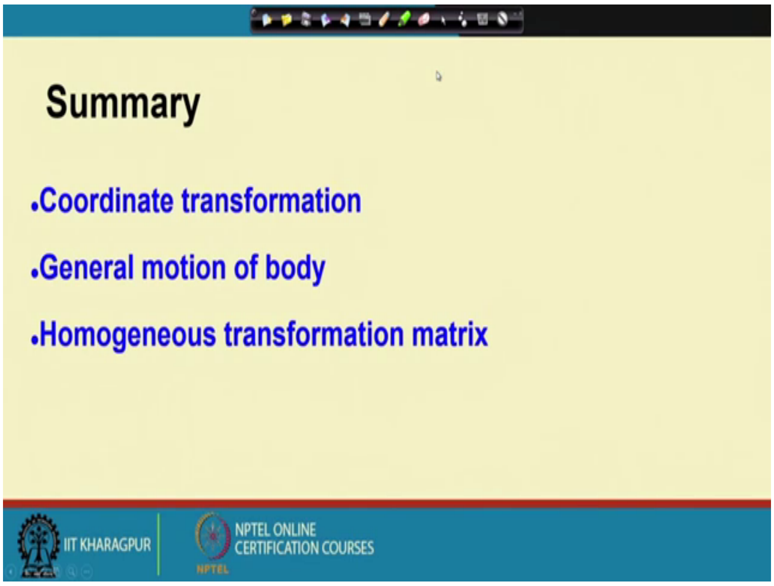

So, let me summarize we have looked at the coordinate transformation for a general motion of a body in plane in a plane, we have discussed the rotation matrix and the homogenous transformation matrix which is nothing, but a multiplicative factor, which you multiply with the extended position vector in the local frame to get the position vector in the world in the body in the global frame or the earth fixed frame.

We have looked at the construction of the homogenous transformation matrix and we have also looked at sequence of transformations general transformations and constructed the homogenous transformation matrix corresponding to a sequence of transformation. So, with that I will close this lecture.