Mechanism and Robot Kinematics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 37
Force Analysis – II

We have been discussing force analysis of mechanisms. So, as I mentioned in the previous lecture that this force analysis is not about dynamic forces, but equilibrating forces and we have used the principle of virtual work to analyze force transmission by mechanisms.

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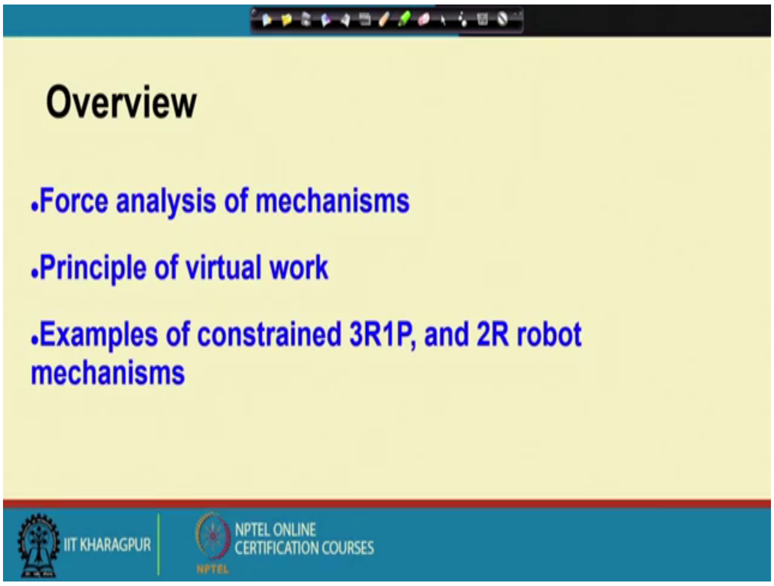

So, in today's lecture the overview of this lecture is as follows, we will look into the problem of force analysis of mechanisms using the principle of virtual work and consider examples of the constraint 3R1P chains and 2R planar robot mechanisms.

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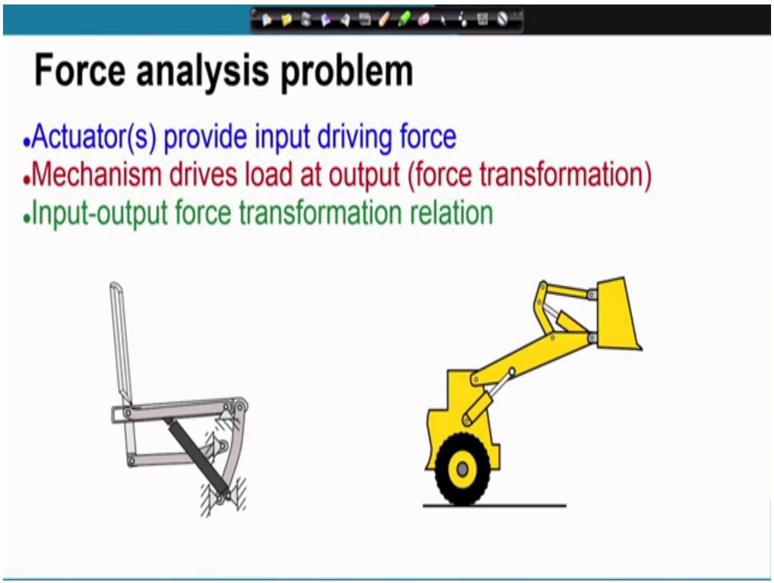

So, let us recapitulate what we have discussed. The force analysis problems the problem is about finding the input output force relation for a mechanism. In a mechanism or in a robot the actuators provide the input dry driving force and the mechanism drives certain load at the output. So, the mechanism is responsible for the force transmission and transformation. So, the objective of force analysis is to find this input output force relation.

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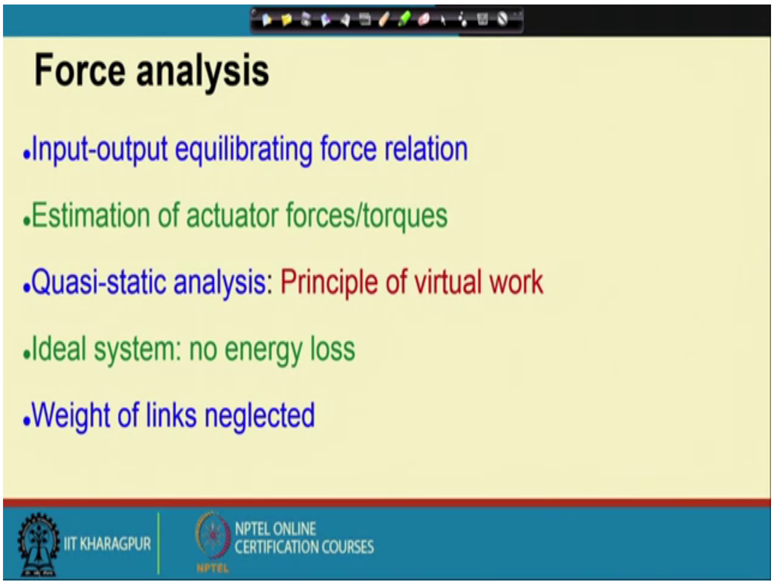

We have discussed that the input output equilibrating forces is what we are interested in; this helps them in us in estimating the actuator forces or torques required. We will use the quasi static analysis using the principle of virtual work; that means, there is no dynamic force in our analysis. We will consider ideal systems; which means that, there is no energy loss there is no dissipation, the weight of the links are neglected in this analysis.

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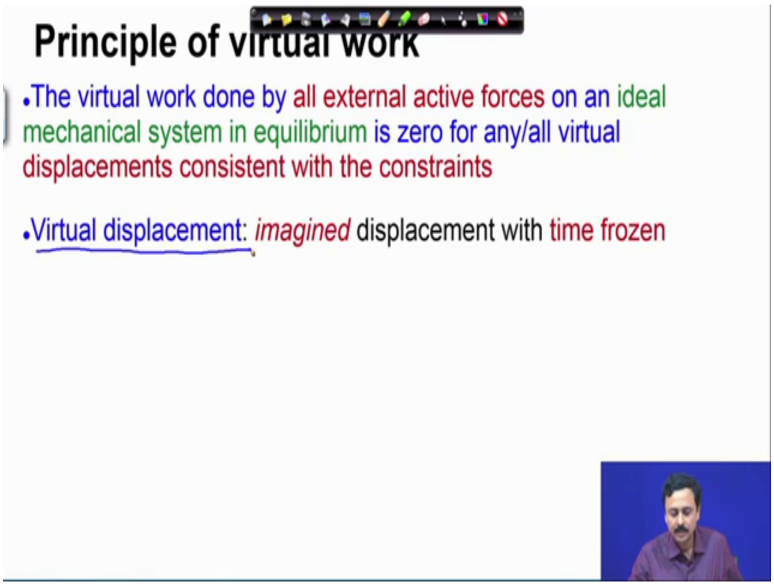

Now, let us review the principle of virtual work. The statement of the principle of virtual work states; that the virtual work done by all external active forces on an ideal mechanical system in equilibrium is 0, for any or all virtual displacements consistent with the constraints. So, here a very important term is this virtual displacement.

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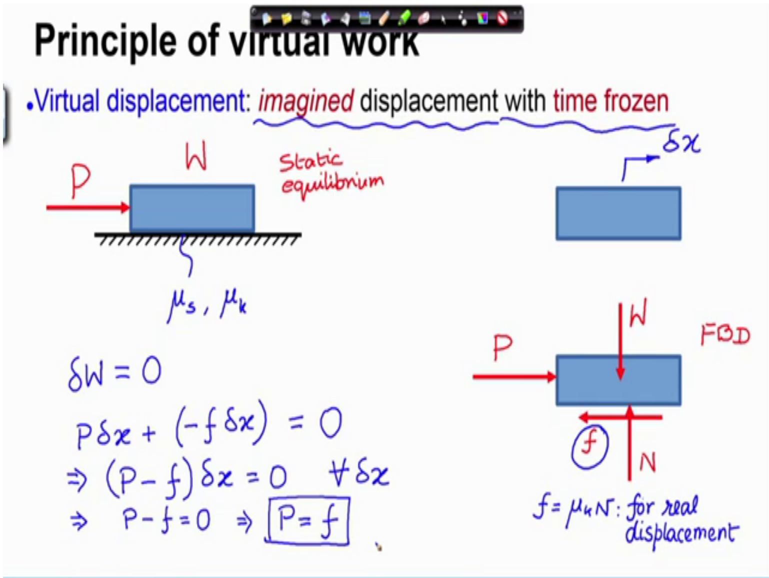

So, we look into this notion of virtual displacement through an example, we will review this.

Consider a block of weight W, on a rough surface being pushed by a force P, but we will assume that this system is in equilibrium it is in static equilibrium. So, this block remains in static equilibrium. Here, I have the free body diagram of the block. So, you have the weight W the applied force P the reaction force normal reaction force from the ground N and the friction force f.

Through this example I would demonstrate what is meant by virtual displacement. Since, this surface is rough we will assume the static friction coefficient and the kinetic friction coefficient mu s and mu k. Now virtual displacement is an imagined displacement with time frozen. So, suppose this block is given a virtual displacement delta x, now what is this virtual displacement? What is the difference between virtual displacement and real displacement?

In a real displacement, if I give a real displacement to this block this f the friction force will immediately go to the value mu k times N so, this is under real displacement. So, for real displacement; so, if I give a real displacement to the block then the because of sliding; the force of friction will immediately become mu k times N as we know; however, I am interested in finding out f under static equilibrium and that is definitely not mu k times N.

So, under virtual displacement f will still remain f as unknown. So, under virtual displacement the force of friction will remain f unknown force f. So, that is the difference between real displacement and virtual displacement. So, this is an imagined displacement with time frozen. So, if there are time varying forces then the force values will be frozen at the instant of time we are trying to find out the equilibrium force relation. So, that is the significance of time being frozen.

So, in time varying systems in systems where the forces may be functions of time we will freeze the time; that means, the value of the force, the vectors the force vector will get frozen and then we will give an imagined displacement, which is consistent with the constraints. For example, here the constraint is that the block will never leave the table. Ok then the statement of the principle of virtual work states that the work done the virtual, work done must be 0. The net virtual work done by all active forces must be 0 under virtual displacements.

Now, if I have a virtual displacement delta x in the direction I have shown; then the virtual work done by the force P by the applied force P is P times delta x. The virtual work done by f is minus f times delta x, the reason being that f and delta x are in a position. Delta x is to the right, but f acts to the left. W and N which are which are the weights and the normal reaction from the ground, they do not do any work under the displacement delta x as you can see.

So, therefore, there is no other active force which is doing virtual work so this must be 0. So, from here it follows that P minus f delta x must be equal to 0. Now this should hold for all arbitrary virtual displacements delta x. This statement must hold for arbitrary displacements delta x. Then it immediately implies that P minus f must be equal to 0; which means that P must be equal to f, but this is a statement which you could have also easily derived by taking writing out the equilibrium equation in the horizontal direction.

Then why do we use virtual work the reason is that we need to only consider the input and output forces, we do not need to consider the internal forces and this advantage will get cleared as we look into apply this principle to mechanisms.

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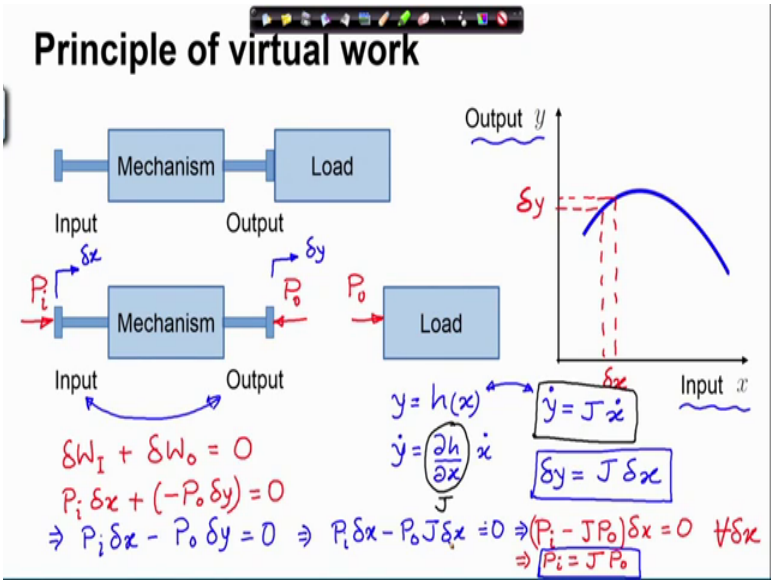

So, let us look at this once again through a schematic figure, as I have shown here. Here I have a mechanism an imagine mechanism shown as a block, which has an input and an output. At the input let us say we have applied the force Pi, which is being transformed and transmitted to the load as Po, which is doing certain work on the load.

Therefore, the mechanism feels a negative force Po so Po in the opposite direction. So, that is a reaction force on the mechanism. So, we have Pi at the input and Po in this direction on the output. Now, if I have to apply the principle of virtual work then I must know this input output relation. The input output relation, suppose the input output relation is known in this form so that at the particular configuration x. If I give a virtual displacement input displacement delta x. If I obtain a virtual output displacement delta y and if I am able to find the relation between delta x and delta y, then I can proceed to apply the principle of virtual work.

So, the principle of virtual work says that the work done at the virtual work done at the input does the virtual work done at the output must vanish for this mechanism. So, this is the network shall work done work done at the input and some work being done at the output. Now, work done at the input you can easily see if I give a displacement delta x and if I have the virtual displacement delta y at the output.

So, these are corresponding to one another through the input output relation. In that case, the virtual work done at the input is Pi times delta x, you can see that Pi and delta x are in the same direction plus, now Po and delta y are in oppositions so therefore and this must vanish. So, that implies Pi delta x minus Po delta y must be equal to 0.

Now, let us look at the input output relation. We have studied this input output relation under displacement analysis. So, we can have y as a function of x. We have also studied input output velocity relations; y dot is some Jacobean times x dot. Now you can very easily relate this input output relations by differentiating this displacement relation with respect to time and that is what we have done.

And therefore, this term is nothing but the Jacobean. So, we have the velocity relation input output velocity relation for mechanism. And from there I can directly find out the virtual input and virtual output relation. So, if I consider the time is being frozen so then I can write delta y as J times delta x. So, I am imagining that time is being frozen. So, that any input displacement delta x is related to the output displacement delta y through the Jacobean.

So, therefore, using this I can rewrite this as Pi delta x minus Po and J times delta x and that and that is equal to 0, which implies Pi minus J times Po delta x must be equal to 0. So, therefore, and this should be satisfied for all virtual displacements at the input delta x. This immediately implies that Pi is equal to J times Po. And this is our force relation, this is our force relation. So, that is we have related the input output forces for the mechanism.

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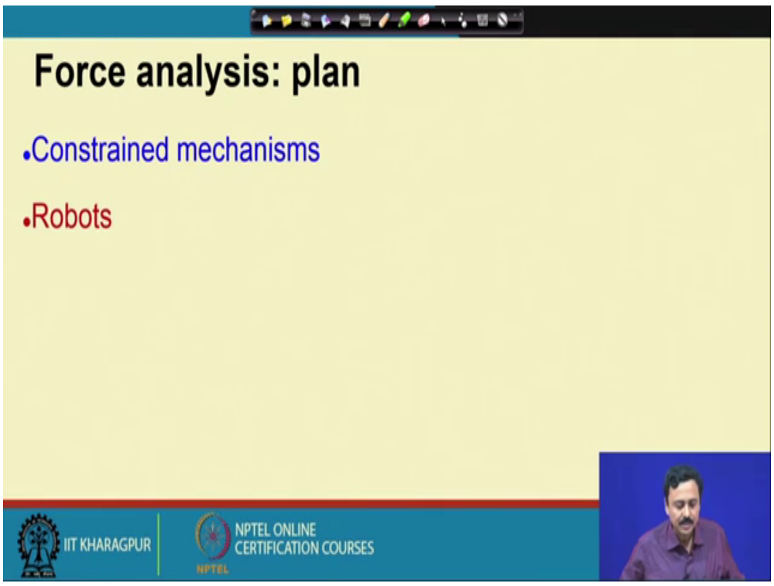

So, our plan is to study constraint mechanisms and robots.

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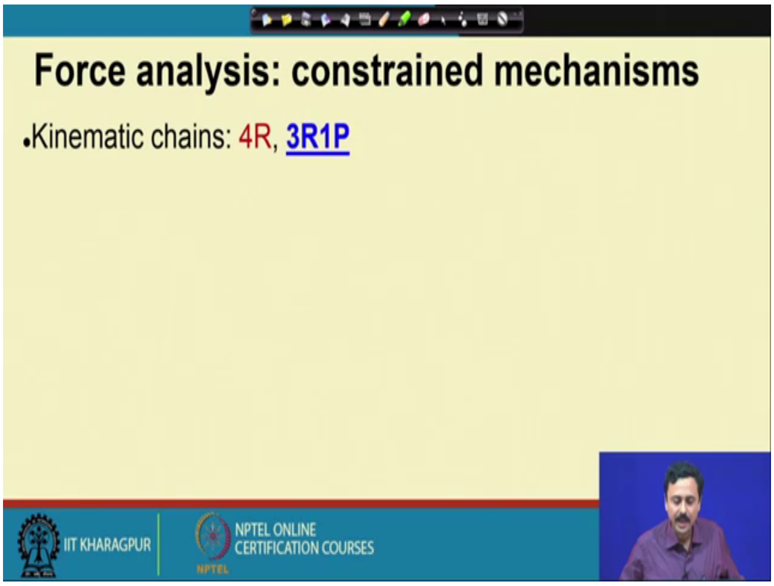

In the constraint mechanisms we are going to look at 3R1P chains.

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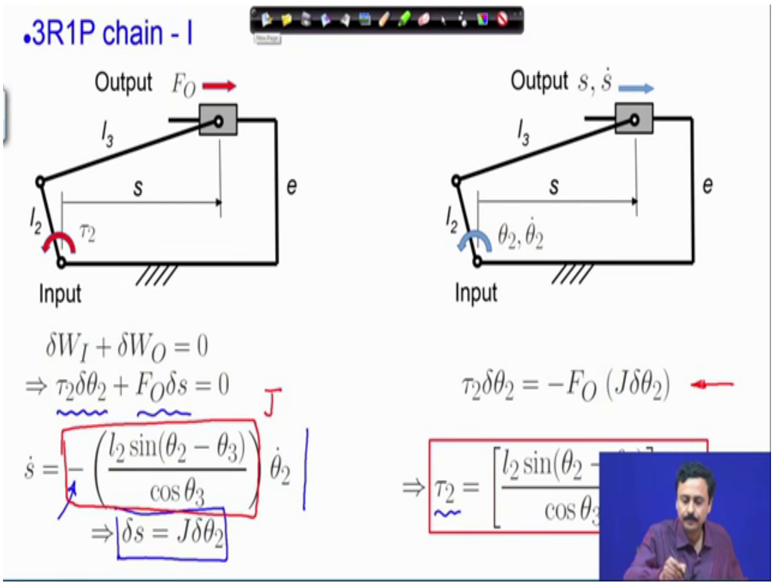

So, here I have shown you a 3R1P chain of type 1, and as you can see I have marked the input and the output. So, at the input I have a torque tau 2 and at the output I have a force Fo, the force at the output and we had to find a relation between tau 2 and Fo.

Now, from the kinematics from velocity relations, we had already related theta 2 dot and s dot. So, these 2 have been related through velocity analysis which we have discussed previously at a specific configuration. So, then if we proceed further, the statement of principle virtual work says that delta Wi plus delta W equal to 0; where W delta Wi is the virtual work at the input.

Now, virtual work at the input you can see is tau 2 times delta theta 2 and virtual work at the output is Fo times delta s. You can very easily come to this statement by noticing that tau 2 and theta 2 dots they are the same direction, and Fo and s dot have been considered in the same direction. Now from the previously discussed velocity relation we have already obtained this velocity relation, where this negative sign takes care of the velocity direction at the output because of a counterclockwise theta 2 dot.

So, therefore, the relation between the virtual displacements at the input and output are obtained as delta s is equal to the Jacobean. So, here the Jacobean; so this is the Jacobean so, again from the principle of virtual work by replacing delta s using the virtual displacement relation we have obtained this and therefore, because delta theta 2 must be arbitrary therefore, I obtain finally, this relation between the input torque tau 2 and output force F naught.

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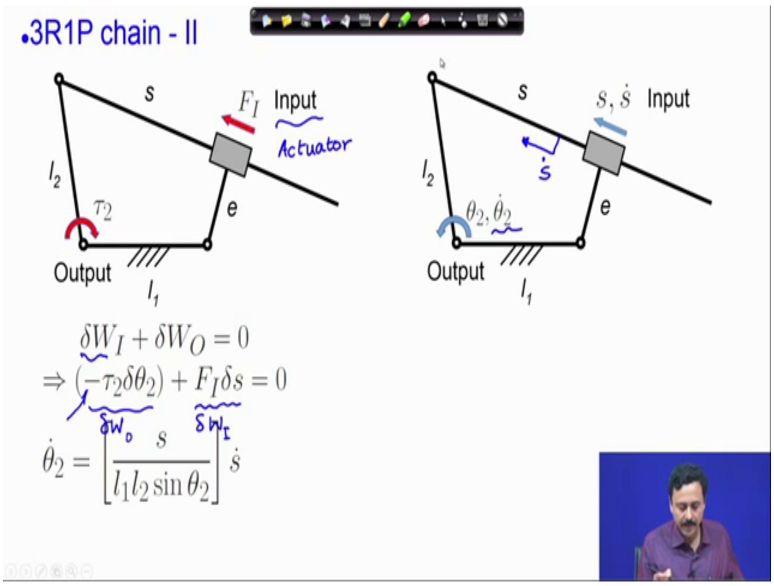

Now, if you look at the 3R1P chain of type 2. We have the input here so we consider maybe a hydraulic or a pneumatic actuator. And the output is the torque tau 2 so to find out a relation between the input force at the actuator and the output torque tau 2. Now the motion relation, so at the input because of the actuator force we consider that this link moves out at a rate s dot.

So, that is the expansion rate of the actuator and theta 2 dot which is the rotation of this link is counterclockwise. So, the principle of virtual work states that; the network must vanish. Now at the input at the input we have Fi as the force, and delta s as the displacement virtual displacement. And at the output so this is delta Wi, at the output we have tau 2 as the output torque and delta theta 2 as the virtual displacement, at the output now they are in opposition as you can see. So, that is why; you have this negative sign sitting here the work done being negative. Now we have the velocity relations which we have derived earlier.

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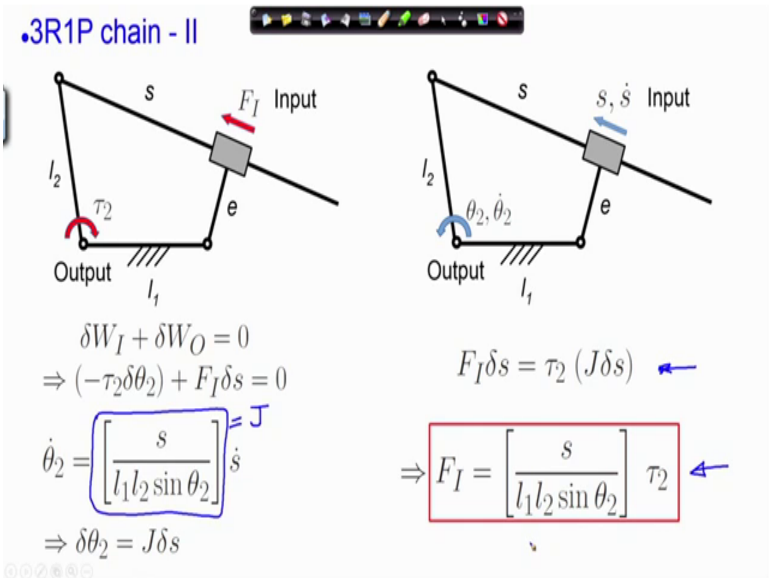

And from here follows the virtual displacement relations delta theta 2 is a Jacobean times delta s here we have the Jacobean.

So, therefore, from the statement of principles virtual work by replacing this delta theta 2 I have obtained this relation and finally, since this relation must hold for arbitrary delta s we must have Fi related to the torque through this relation. Now you can see that the Jacobean has come into the relation.

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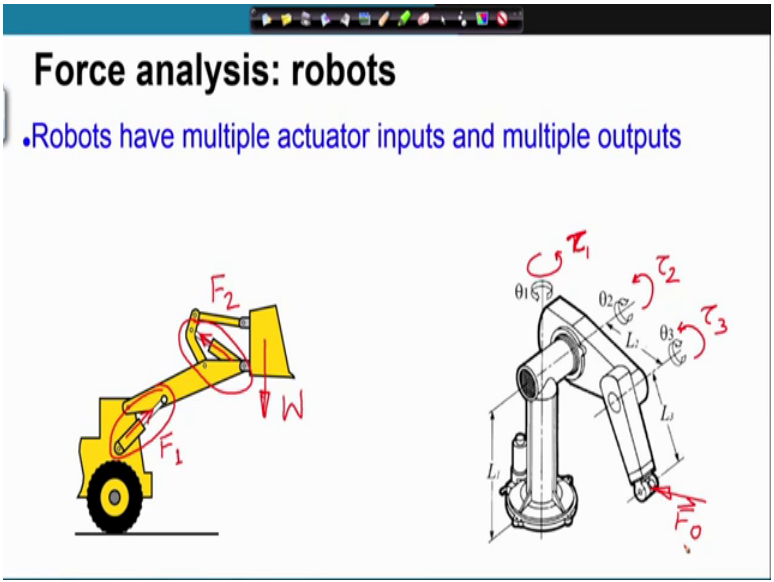

Now, we look at the force analysis for robots. As you know robots have multiple actuators as inputs and there can be multiple outputs. For example, here we have this excavator, we have a load maybe W, and we have 2 actuators which are applying forces F1 and F2 and that is responsible for equilibrating the load W.

In this example of the puma robot, suppose there is some force at the end effector. That must be equilibrated by input torques tau 1, tau 2 and tau 3. Now this force is a vector here so therefore, I have multiple input torques and force as a vector. So, there are 3 components so there are multiple outputs.

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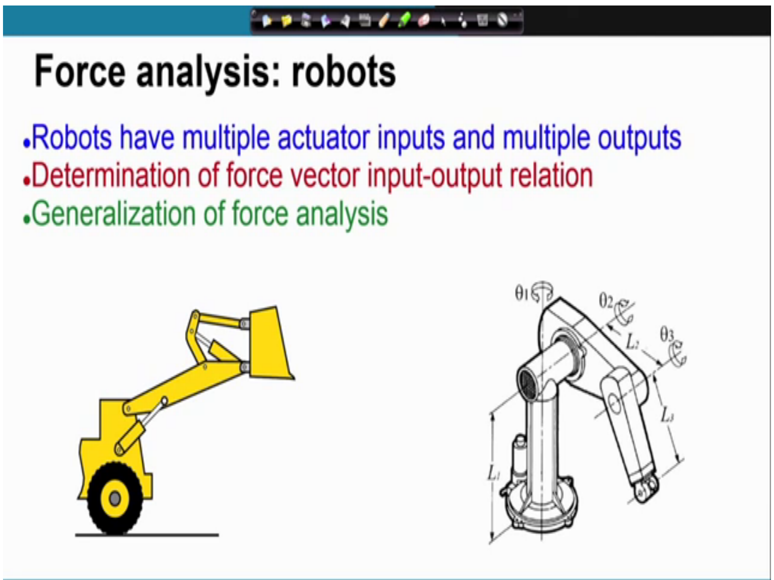

So, we have to find out this force vector input output relation. So, the joint forces and the output forces these are both our vectors.

So, we need to generalize our force analysis that we have been looking at. So, till now we had considered only scalar forces in one dimension, but now when we have multi input multi output system, then the forces the joint forces and the output forces both are vectors. So, we need force relations between vectors so we need to generalize our force analysis.

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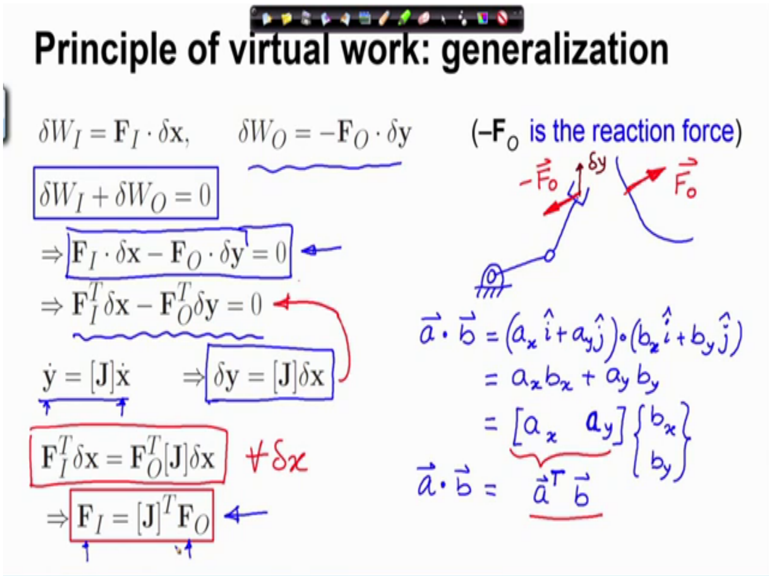

So, in order to generalize our force analysis, I have written out the virtual work done at the input delta Wi is Fi dot delta x.

Now, delta x is the virtual displacement at the inputs. And Fi is the force vector at the input. At the output we have F naught as the force being applied. So, if I consider for example, a manipulator the 2R manipulateor, let us say which we are going to look at very soon suppose it is applying a certain force Fo. So, on the manipulator there must be a reaction force now since this is a vector I must write it as minus Fo.

So, Fo is the force being applied by the manipulator on the external world through the end effector and minus Fo is the reaction force that the manipulator feels. So, therefore, if delta y is the virtual displacement at the output, if delta y is the virtual displacement at the output therefore, delta Wo which is the work done virtual work done at the output is given by minus Fo dot delta y. So, minus Fo being the force experienced by the robot manipulator at the output.

So, Fo is the force being given to the external world by the robot. So, minus Fo is the reaction force. Now the principle of virtual work states that delta Wi plus delta Wo must vanish. So, if I substitute these expressions, I obtain this relation. Now, I need the input output relations for the manipulator. Here I have written out this statement which is a dot product, which uses dot product of vectors as matrix product for our further simplification.

So, to understand this let us consider 2 vectors, in dot product so a dot b so is nothing but ax i cap plus a y j cap dot b x i cap plus b y j cap. And, as you know that this is ax bx plus ay b y now, I can write this as a matrix product like this, which is a vector transpose into b vector. So, the transposition converts a column vector to a row vector as I have written here. So, the transposition of the; a vector which is considered to be a column vector so, the transposition makes it a, row vector so as a matrix product this becomes a transpose b. So, a dot b can be written as a transpose b and that is what I have used here.

Now, let us look at the input output relations. Suppose, the output velocity is related to the input velocity or the joint velocities through the Jacobean like this; therefore, when we consider freezing time and giving virtual displacements at the input and looking at the virtual displacement the output they are related to the Jacobean as delta y is J times delta x where j is the Jacobean matrix.

So, using this relation in here rather in here so using this Jacobean relation in here, I obtained this relation now this should be true for all virtual input displacements delta x. So, therefore, I must have this as the relation between the input and output forces. So, you should note that the input output forces are related through the Jacobean transpose. So, on the left we have the input force on the right we have the output force.

While the velocity in the velocity relation, it is the output velocity at the left and input or the joint velocities and on the right. Here we have input forces on the left and the output forces on the right. So, this now generalizes our force analysis.

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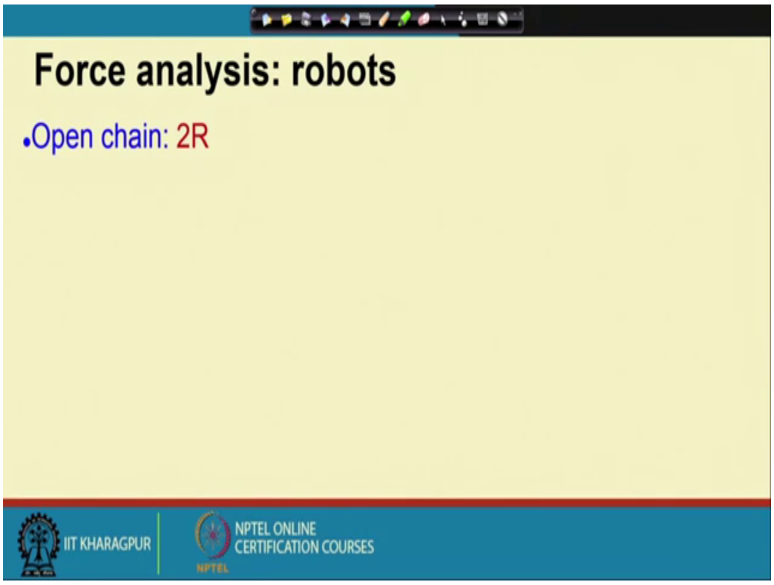

So, let us proceed further, we will look at open chain 2R manipulators.

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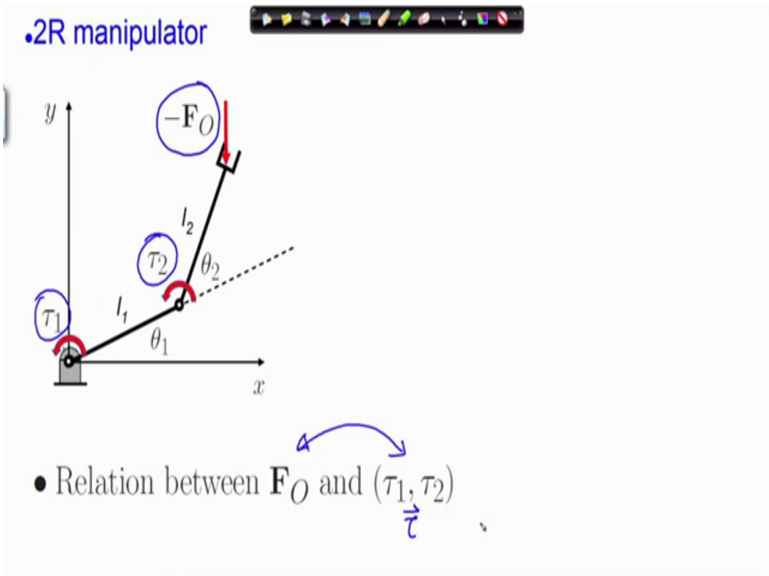

So, here I have a schematic for the 2R manipulator. We have these input forces which are the joint dots tau 1 and tau 2. And the reaction force that is experienced by the manufacturer that the end effector which is minus Fo I have to find a relation between this Fo and Fo vector and the joint torque vector tau.

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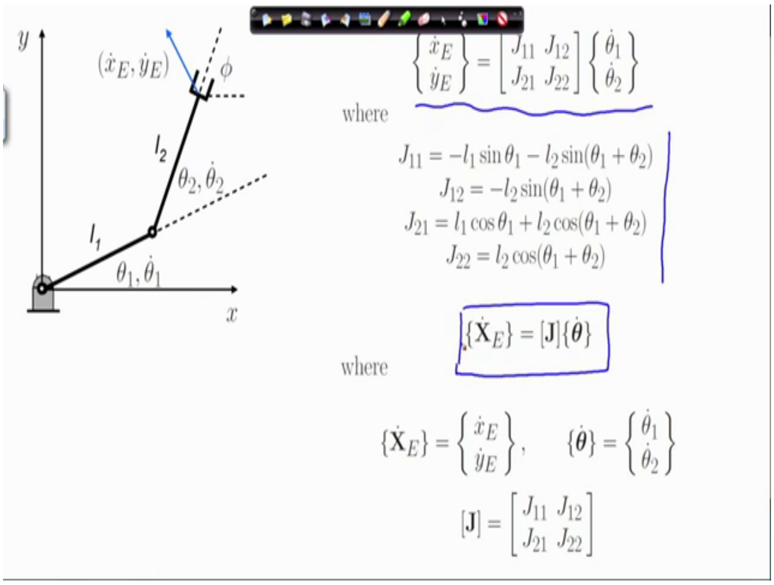

Now, we go through the velocity analysis that we have already discussed. So, we know that the velocity vector at the output is related to the joint velocities at the input through this Jacobean relation, whether Jacobean elements are given here. So, this we wrote in a compact form as XE dot vector dot is J times theta dot.

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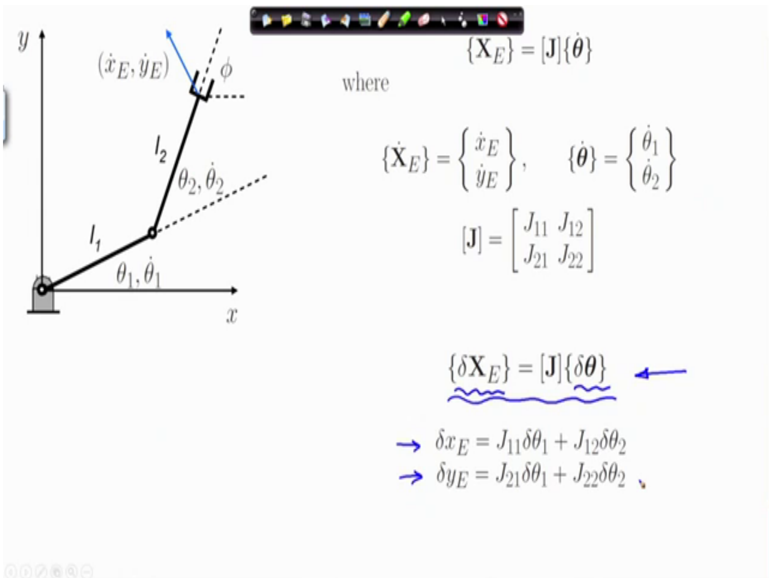

Now, starting from there we can now write the virtual displacement relations when we freeze time and provide an arbitrary input virtual displacement delta theta, and obtain the corresponding output virtual displacement delta XE. And they are related through the Jacobean as we observe here. So, the implication is like this that delta x E is j 1 times delta theta 1 plus j 1 2 times delta theta 2 and delta yE is j 2 1 times delta theta 1 plus j 2 times delta theta 2.

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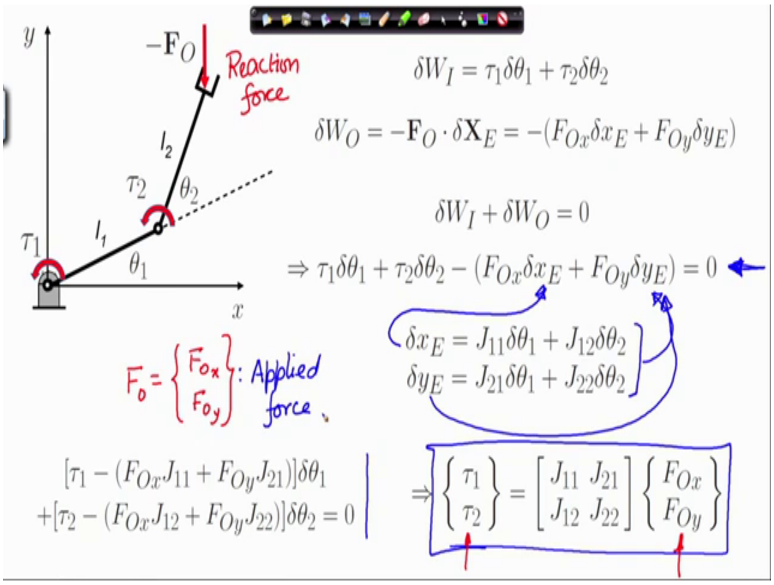

Now, we compute the virtual work done at the input, which is tau 1 times delta theta 1 plus tau 2 times delta theta 2. The virtual work done at the output because of this reaction force minus Fo is minus Fo dot delta x E. So, which is obtained here now the statement of principle virtual work states that the net virtual work must vanish. So, therefore, I have this relation following from the statement of principle of virtual work.

Now, since delta xE and delta yE we have already found the expressions. We will just plug in these expressions in here. And collect terms of delta theta 1 and delta theta 2. So, when you plug in delta xE here and delta yE here. We have the principal virtual work in terms of only delta theta 1 and delta theta 2. Then we collect terms of delta theta 1 and delta theta 2 to obtain this relation which can be easily done.

And finally, when we assemble we obtain this relation between the joint torques on one hand and the output forces on the other. So, these are the joint torques and these are the forces at the end effector. So, remember that minus Fo is the reaction force. So, therefore, this Fo is the applied force. So, this is the applied force. Applied force on the external world by the manipulator.

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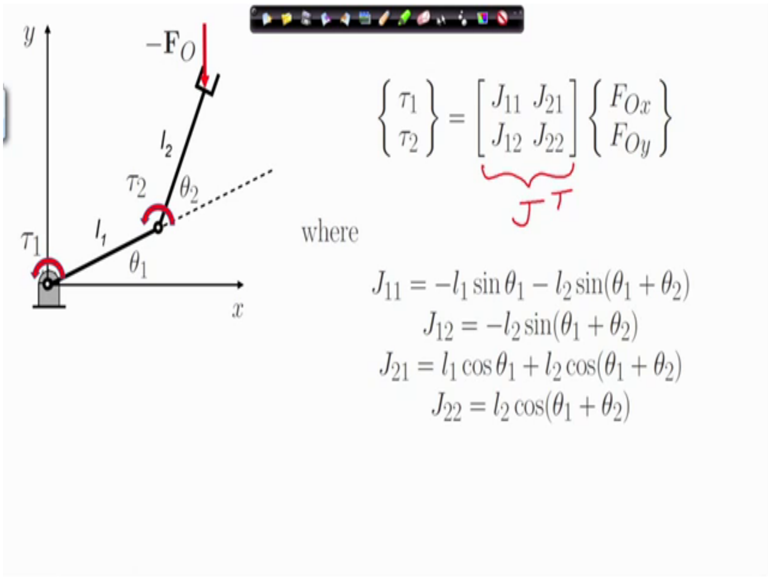

So, when you write out the elements of the Jacobean j 11 j 2 1 j 1 to j 2, here you note one thing that this is nothing but the Jacobean transpose.

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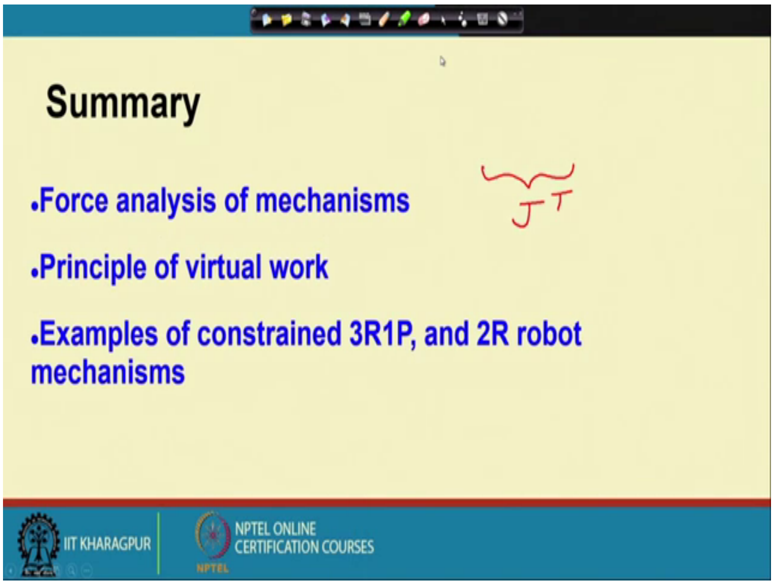

So, let me summarize what we have studied in this lecture, we have looked at the force analysis problem for mechanisms and robots using the principle of virtual work. We have considered the example of 3R1P constraint chains of 2 types: type 1 and type 2. And we have also considered the planar open chain manipulator 2R manipulator and discussed the force analysis for this manipulator. So, with that I close this lecture.