Mechanism and Robot Kinematics
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Lecture – 36
Force Analysis – I

We have been discussing about the motion, which is the primary thing in kinematics. The motion of various links; which are the rigid bodies in relation to one another; so, we have inputs, outputs and we have been discussing about the relation between the input motion and output motion. Motion as we know is characterized by displacement, velocity, and acceleration. And, we have discussed these 3 relations 4 mechanisms as well as 4 Robots.

There is another concept or another reason, why we use mechanisms and that is to transform or transmit forces. So, a mechanism also transforms forces from the actuator on one hand and the output link on the other. So, the actuator forces are converted or transformed to the output forces, that this mechanism applies or it may drive certain load. For example, in the case of screw pair let us say screw pair, we have revolute we have a rotary input and the motion of the nut is a linear motion.

So, I can convert torque to force. Similarly, in 4R mechanism let say for our chain. We have a torque input that gets again converted to another torque output, but through amplification. So, a mechanism transforms and transmits force. So, in this lecture, we are going to look at these force relations at input and output. Now, when I bring in the concept of force in the study of kinematics it has to be understood that this is not the dynamic force; that means, the force that produces the motion.

But, the force that gets transmitted so; it is the equilibrium forces that we are talking about. At a certain configuration, if I have a certain input torque what is the output torque at that configuration? So, it is not about producing acceleration, but about converting force or amplifying force or diminishing force or transmitting force from the actuator to the output.

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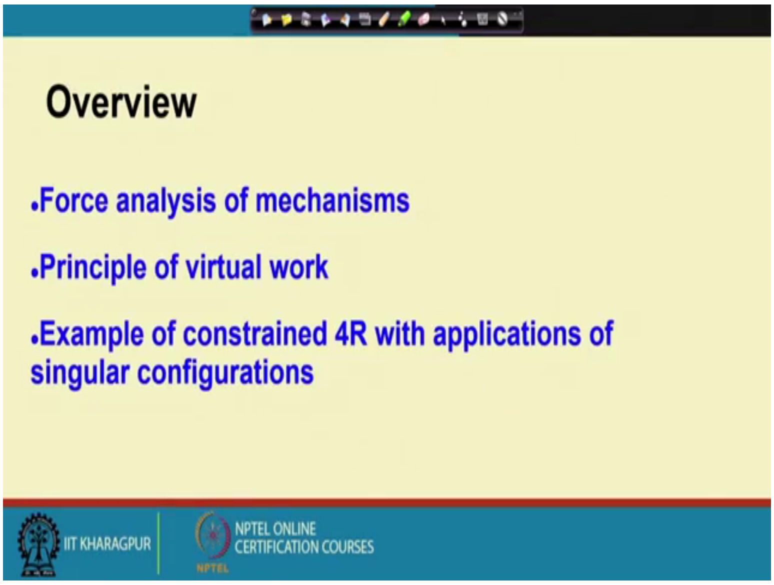

So, to give you the overview of what we are going to discuss today. So, we will start with the force analysis problem. I will introduce this problem and discuss the principle of virtual work and I will explain the concept through examples of 4R chain and subsequently I will also discuss examples of robots.

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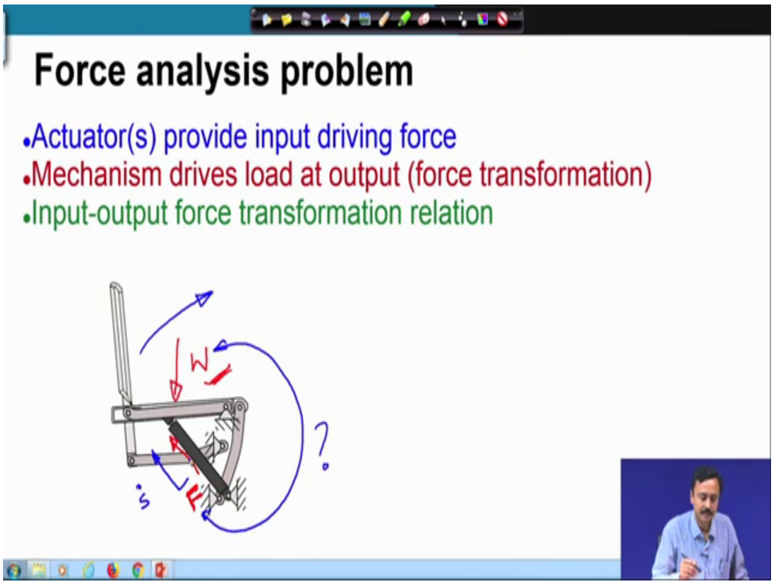

So, what is the force analysis problem? So, as I have mentioned the actuators provide input driving force, the mechanism drives certain load at the output and the force can get transformed and transmitted. So, transformed means for example, from rotary to rotary motion.

So, basically torque to linear motion essentially force. So from torque to force or from the torque, of a motor to the output torque of the output link now, the force analysis problem is to determine the input outputs output force transformation relation. So, here I have this example of a transfer device, when this actuator expands this device moves from the sitting position to the standing position.

Now, there will be certain loads on the device and therefore, it is not just the motion that is being transmitted, but also the load is being carried by this actuator. So, I will require some thrust or some force in this actuator, in order to balance or equilibrate the load at the input at the load on the on the device, which is an external load on the mechanism. And, this force of the actuator is the stress produced by other force produced by this prismatic or this hydraulic or pneumatic actuator.

So, I would like to relate W with F at a certain configuration. So, this is about equilibrating forces not about forces that will produce acceleration.

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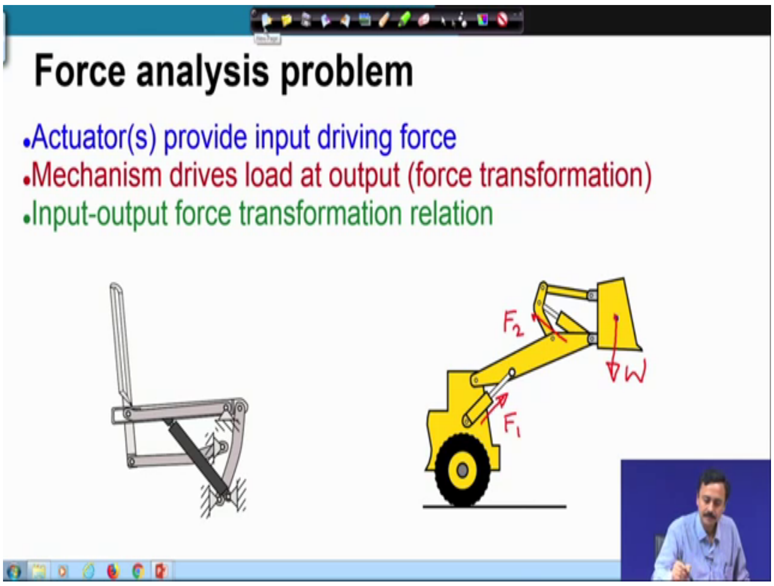

This is another example. So, if I am carrying a certain load, what should be the forces at the actuators to balance this load? So, this is our problem that we are going to discuss here.

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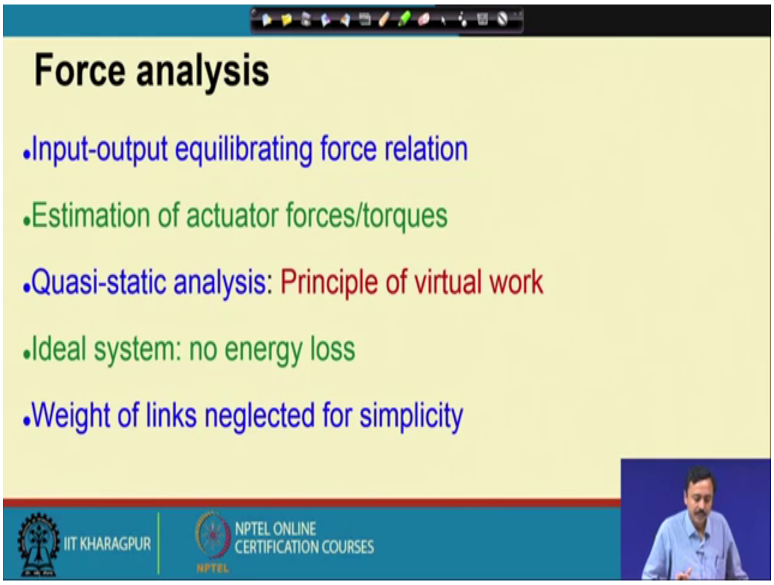

So, under force analysis, we are going to look at the input output equilibrating force relation. This will give us an estimation of the actuator forces or torques that are required that will be required.

This is this will also help us in some way to estimate the size of the actuator. As I have mentioned this is not about dynamic forces that produce acceleration, but forces which maintain the static equilibrium. So, we are looking at quasi static analysis using the principle of virtual work. We will assume that our system is ideal; that means, there is no energy loss. Furthermore, for simplicity we are going to assume that the weights of the links are small compared to the forces that they are carrying. So, under these assumptions and with this goal we are going to discuss the force analysis problem.

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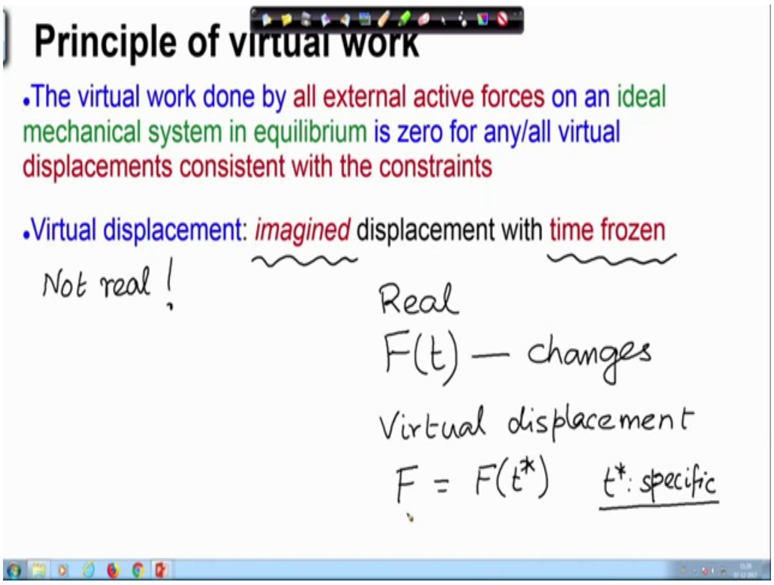

So, we first look at the principle of virtual work. So, the statement of the principle of virtual work says that, the virtual work done by all external active forces on an ideal mechanical system in equilibrium is 0; for any or all virtual displacement consistent with the constraints.

Now, here I will point out certain key words. First of all, the system is in equilibrium. Here, by equilibrium I imply static equilibrium the system is in static equilibrium. Secondly, here I have this concept of virtual work and I also have this concept of virtual displacements. So, I have this concept of virtual displacement through which I am going to calculate the virtual work for a system in equilibrium and I must have this virtual work as 0.

The virtual work done by all these external active forces on the system in equilibrium must be 0. So, let us look at these things one by one. What is virtual displacement? Virtual displacement as the name suggests is not real displacement. So, it is definitely not real displacement it is virtual displacement. So, it is imagined displacement with time frozen.

Now, when you have real displacement, time cannot be frozen. So, if you have forces which are changing with time, in a real displacement the forces are going to change, but in a virtual displacement forces will retain the value at that specific instant of time. So, for real displacement, the force is going to change, but for a virtual displacement, F will be fixed for that specific instant of time. At which we are looking at the equilibrium?

So, we are looking at a specific time instant. So, this is a specific time instant, where we are looking for equilibrium. So, we will fix the force at that value and let the displacement be given and then calculate the work done. Now, I will explain this concept a little further.

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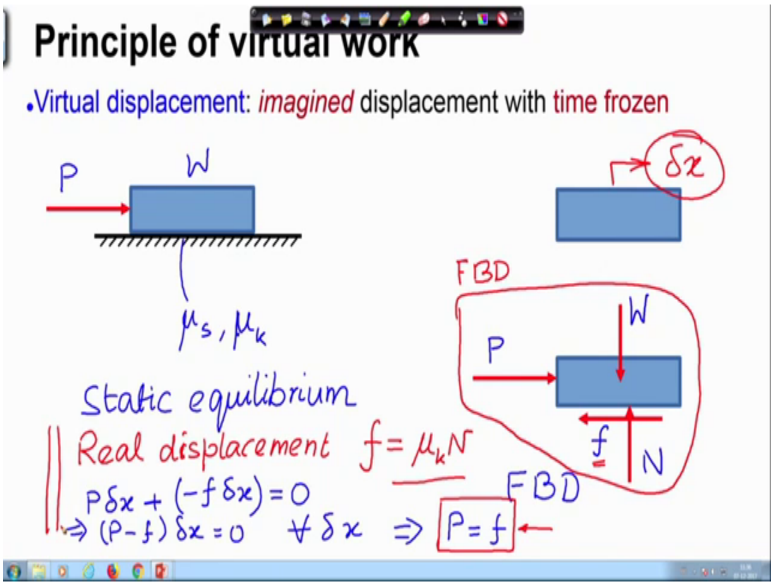

With this example, what I mean by virtual displacement? What I mean by time frozen? What I mean by force being frozen? So, here I take an example of a block of a certain weight on a rough surface with mu S, as the coefficient of static friction and mu K as a coefficient of kinetic friction. So, there is a rough surface on which we have a block of weight W and this is being pushed by a force P. Such that this system is in equilibrium it is in static equilibrium.

So, the block is in static equilibrium under the action of this force P. Now, here I have the free body diagram of the block. So, we have the weight W, we have the normal reaction from the ground N, we have the friction force f and we have the force that is pushing the block P. Now, we all know that under static equilibrium, if you take force balance in the horizontal direction, then P and F must be equal. And by vector directions they are equal and opposite. So, then the block is in equilibrium.

Now, we will come to this same conclusion using the principle of virtual work. Now, what the principle virtual work says that, the work done by the active forces in displacing in virtually displacing the body must be 0. So, what I mean by virtual displacement? Now here, I assume a virtual displacement delta x.

Now, if I say that this displacement is real; that means, the block is really moving then you know immediately. So, for real movement real displacement, the force must be immediately be equal to mu K times N. If this block is moved given a real displacement, then the friction force must be equal to mu times N mu K times N, but under virtual displacement this force f is unknown. It is frozen at the value of force at the time instant of interest at which we are looking at the equilibrium.

So, whatever be the force f, it is frozen it cannot change. When, I give this virtual displacement. So, this is the concept so, if I give real displacement immediately this friction force is going to change it is value to mu K times N. But, if it is a virtual displacement under this condition that is shown by this free body diagram, then this force f is as yet unknown. So, what I have is P times delta x? Now, you see that P the direction of P and direction of delta x is the same.

So, P and delta x are in the same direction. So, therefore, the work done on the block by the external force P is P times delta x, plus the work done by the friction force f. Due to a virtual displacement delta x must be F dot delta x. Now, this dot product if you take the dot product in the vector sense you will get minus of f times delta x and this must be equal to 0. Why this is minus? Because f and delta x are in opposition.

So, therefore, the work done is negative. So, this implies P minus f times delta x must be equal to 0. For an arbitrary displacement delta x, for all displacements virtual displacements delta x, P minus f times delta x must be 0; which immediately implies that, P minus f must be equal to 0; and hence P must be equal to f.

Now, from normal equilibrium equations, this is just a 1 step conclusion, but through the virtual work principle we have come to that same conclusion, but in a slightly different manner. And this is advantageous in a number of situations. In the case of writing the equilibrium equations, we must draw the free body diagram. We must always draw the free body diagram and analyze forces.

But, in the case of virtual work, when we apply the principle of virtual work as I have done here, we need not draw the free body diagram as I will show you. We just need to have the input force output force the displacements at the input and output and the internal forces need not be considered.

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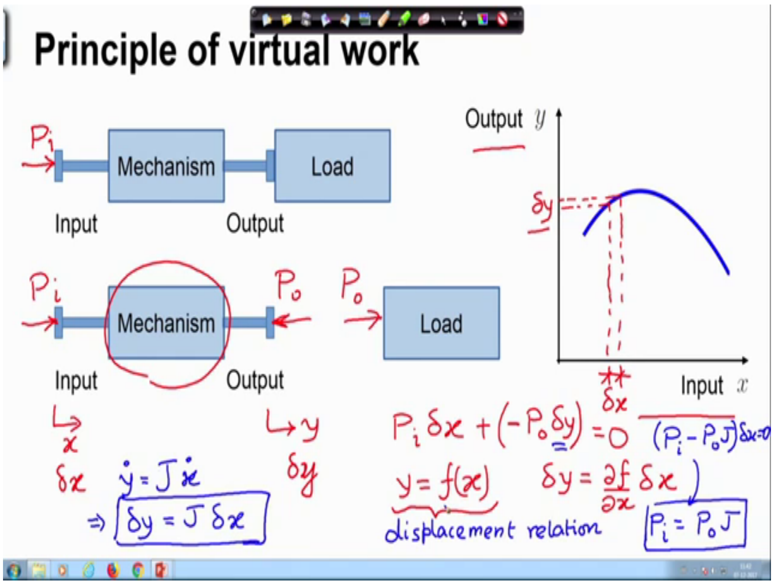

So, here again I explained the principle of virtual work for a mechanism that we are going to use. So, I have a mechanism at the input of which I have some input force. That is a P i and at the output it is driving certain load. So, this mechanism is given some input force and the load is driven by a certain force P o which is provided actually by the mechanism and on the mechanism therefore, by Newton's third law I must have P o in this direction.

Now, if I know the input output relation of this mechanism, which I have shown schematically in this figure. Let us say input is x and output is y with the forces fixed because time is fixed. If the forces are fixed, then I have virtual displacements delta x and delta y. So, I have virtual displacements delta x here and from the displacement relation I should be able to find out delta y, corresponding to the input displacement input virtual displacement delta x, I can find out the output virtual displacement delta y.

Then I can write the work at the input as P i delta x. You can see that P i and delta x are given in the same direction plus the work done at the output is P o times P o dot delta y. In the vector sense if you do that, then you will find that this becomes minus P o times delta y and that must be equal to 0. I need not look; what is inside this mechanism? I just need to know, now the relation between delta x and delta y, which I will know if I know y as a function of x let us say.

If I know y as a function of x which is given by the displacement relation of this mechanism, I can find out delta y as del f del x delta x. So, this is the displacement relation. Now, this could have been obtained even from the velocity relation. Suppose, I know y dot as some function here actually this is the Jacobean as we have seen. Let me call it J, we have already discussed this that the input velocity and output velocity these are related through the Jacobean.

So, therefore, I know the input output relation through this velocity this implies delta y is equal to J times delta x. Because time is being frozen, I can consider that a small change in y and a small change in x with time frozen is related through the Jacobean of the mechanism. Now, this gives us the input output virtual displacement relations in a straightforward manner.

Now, I will substitute this delta y in this place. So, then I have P i minus P o times Jacobean times delta x equal to 0. So, from here I can very easily conclude that P i must be equal to P o times the Jacobean. So, that gives me the force relation. So, what did I require? I require the input output velocity relation or the displacement relation and from there I can very easily find out the force relation. So, this is how the principle of virtual work is used.

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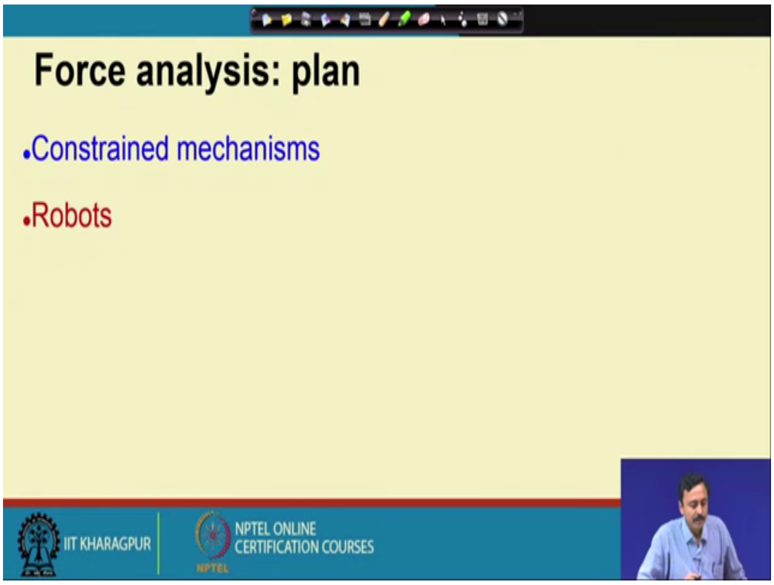

So, our plan is to study constraint mechanisms and subsequently robots.

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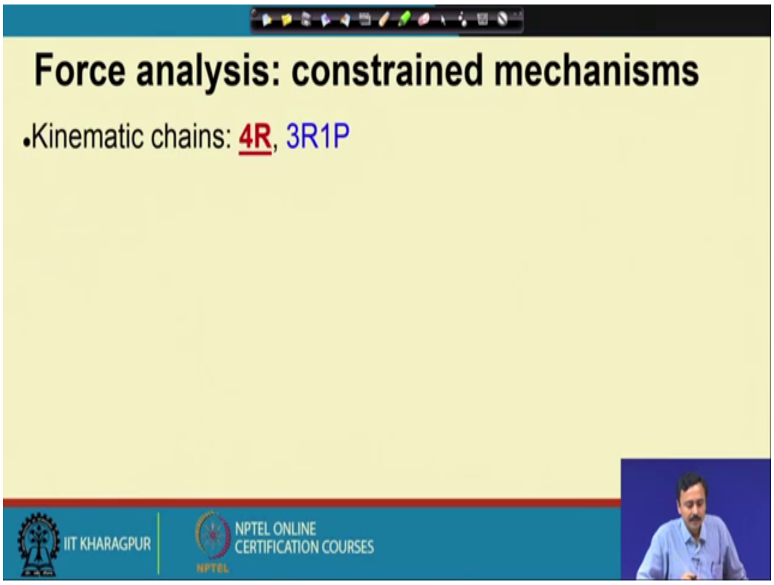

So, force analysis of constraint mechanisms we are going to look aT2 chains. So, here I am going to first discuss the 4R chain.

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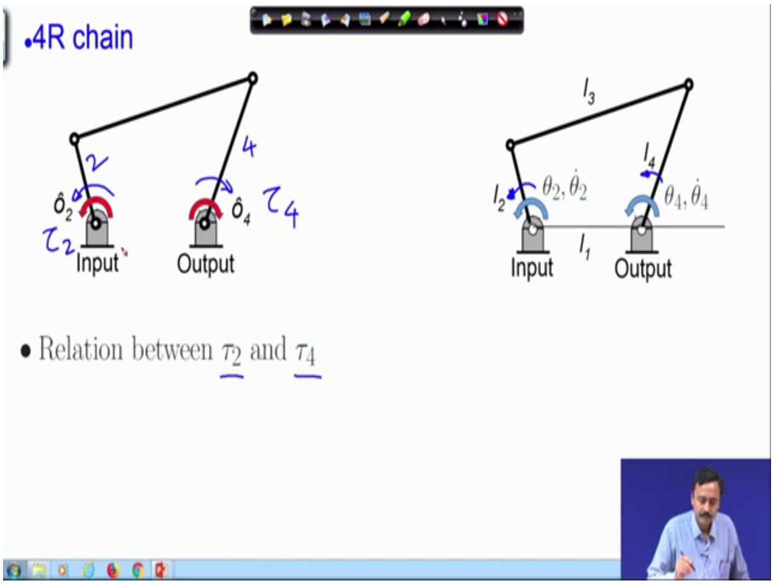

So, this shows the 4R chain I have shown the input torque. So, I have a torque T2 at the input and torque T4 at the output.

Now, I can very well imagine that, if I produce a torque, in the counter clockwise direction. Why I need to produce torque in the counterclockwise direction? Is because the load torque is in the clockwise direction? And that is how this comes about the mechanism is essentially transmitting this transforming and transmitting this input torque to this output link which is link 4, this link 2, this is link 4.

On the other hand, the motion could be this is moving in the counterclockwise, and then this also moves in the counterclockwise direction and driving the load. So, therefore, load must be opposite to the direction of motion at the output. Whereas, load the motion at the input; is in the same direction as the input torque. So, the motion is motion at the input is in the same direction as the input torque. Whereas, motion at the output is in a position with the output torque and this is what is expected.

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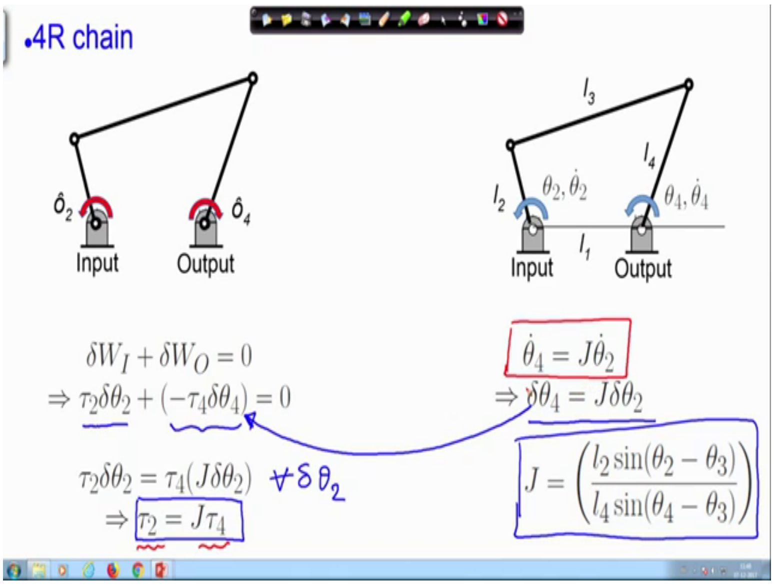

So, therefore, work done the virtual work done at the input plus the virtual work done at the output must sum up to 0 as for the principle of virtual work. Now, what is the virtual work done at the input the virtual work done at the input is torque at the input tau 2 times the virtual displacement at the input delta theta 2.

What is the work done at the output? It is the torque at the output dot the virtual displacement at the output. Now, the torque and at the output and the virtual displacements at the out displacement at the output they are in a position. So, therefore, that product turns out to be negative. Therefore, this sum must vanish now; we already know that the output velocity and the input joint velocity these are related through the Jacobean. And therefore, I can write delta theta 4 as J times delta theta whether Jacobean we have already derived this expression before.

So, therefore, if I substitute this expression of theta 4 in the virtual work expression. What I have is dot times delta theta 2, must be equal to torque at the input times delta theta 2, must be torque at the output times J times delta theta 2. Now delta theta 2 can be arbitrary. So, this should be true for all delta theta 2 any delta theta 2. So, this can happen only when torque at the input is J times the torque at the output.

Now, there is the thing to be noted here, the velocity relation is given as the output joint velocity is J times the input joint velocity. Whereas, the torque relation is the input torque relation is J times the output torque relation output torque. So, input torque is J times the output torque whereas, the input joint velocity is related to the output joint velocity as given here. So, the output joint velocity is J times the input joint velocity. Now, this is very useful in certain mechanisms.

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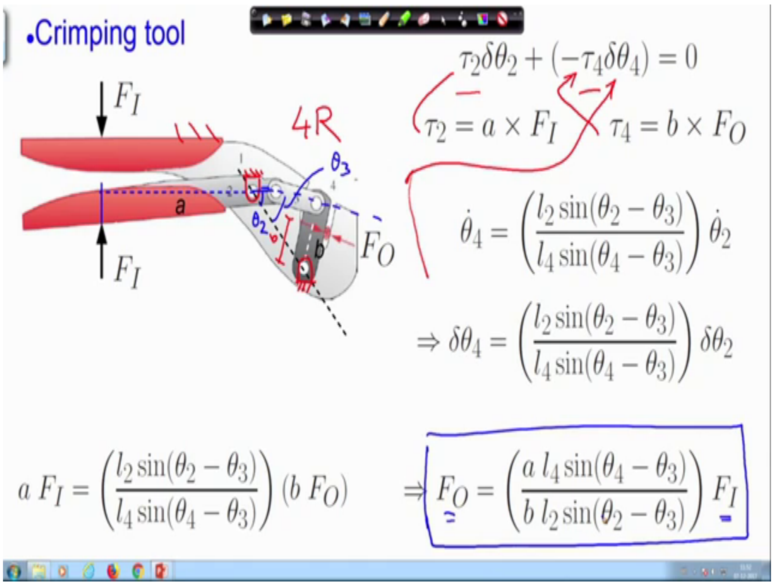

So, here I have this crimping tool and right at the beginning of this course. I had told you that this crimping tool can produce very high forces at the output. Here, I have denoted by FO the force at the output and FI the force at the input. Now, let us see the transformation of forces in this crimping tool.

The virtual work relation, the virtual work statement for equilibrium states that the work done by the input torque; let us say tau 2 plus the work done by the output torque tau 4 must vanish. Now, what is this input torque? Here, you can see that considering that this is a ground hinge and this is a ground hinge, essentially this is a 4R mechanism. This is a 4R mechanism and the links have been numbered.

So, 2 is the handle, 1 is the ground, 2 is this handle and the output link is 4. So, torque at the input is nothing; but a times FI and the torque at the output is nothing but this distance b times FO. So, this is what I have written out torque at the input is a times FI and torque at the output is b times FO. Now, I have this velocity relation input output velocity relation through the Jacobean.

So, therefore, delta theta 4 is Jacobean times delta theta 2. Now, this I will replace. So, this I will replace here and the expression of delta theta 4, I will replace here. And I obtained a time FI is equal to Jacobean times b times FO as I have written out here. So, therefore, FO must be equal to this expression times FI. Now, here we need to look at these angles. So, this is the force relation for us input output force relation.

So, FI is the force at the input FO is the force of crimping. When theta 2 as you can see this angle is theta 2 and the angle made by this link 3 is theta 3. When theta 2 is roughly equal to theta 3 as is shown here:

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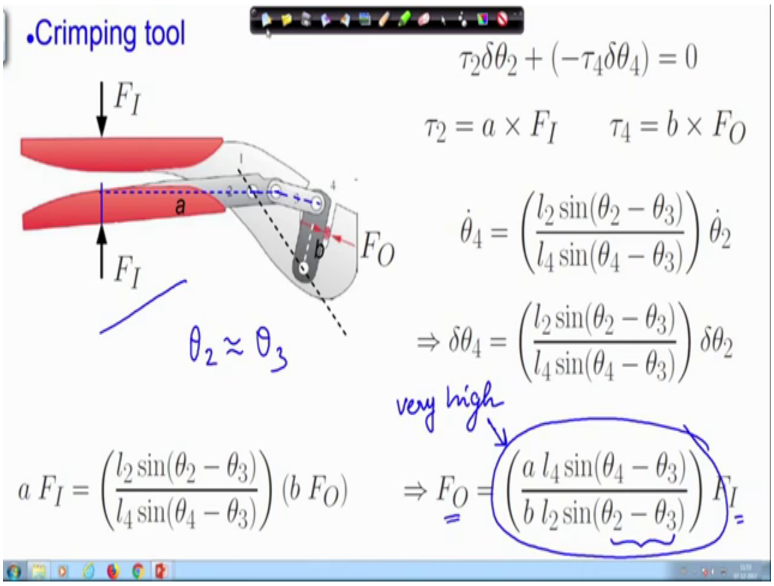

You can see that theta 2 and theta 3. So, link 2 and link 3 are almost collinear which means that theta 2 must be almost equal to theta 3. In that case, this denominator goes to 0 because theta 2 is almost equal to theta 3.

So, sin of theta 2 minus theta 3 must be very close to 0. If that goes to 0 or is very close to 0, this multiplier is very high. It is goes to in the limit it goes to infinity provided that theta 4 is not equal to theta 3 which it is not; as you can see theta 4 is not equal to theta 3. So, therefore, this multiplier is very high. Therefore, the input force gets multiplied by a very high quantity or transform to a very high force at the output. This is not possible using a simple lever.

So, mechanism helps in producing virtually infinite force transmission ratio in a in a mechanism like in a in a tool like this.

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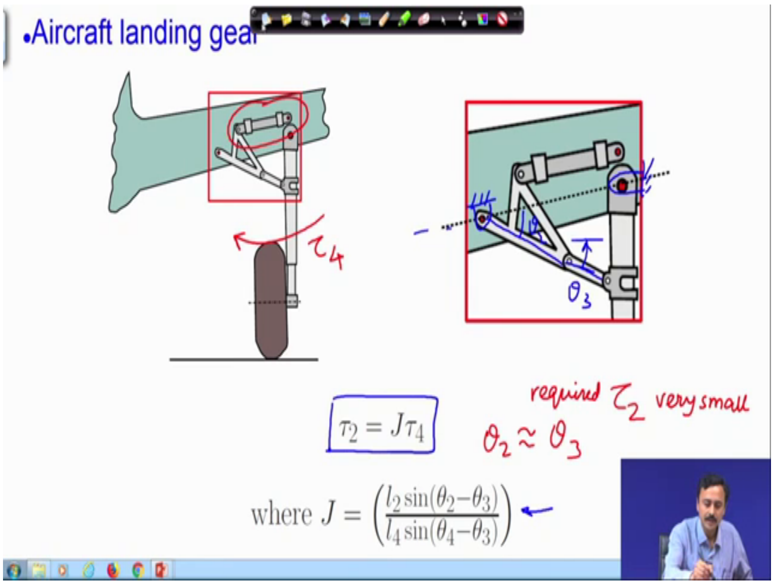

Similarly, in the aircraft landing gear, if you look at the mechanism in the aircraft landing gear, this is again a 4-bar mechanism. And here also, theta 2 this dashed line is the line of frame and this is theta 2 and this is theta 3.

They are very equal nearly equal or equal and hence the torque relation between the input and output tau 2 is j times tau 4 where j is again given by this.

Now, what happens here? Is if I try to drive. So, this is torque at 4, if I try to drive the wheel into the folding position, I will hardly require, because here theta 2 is almost equal to theta 3. So, the numerator vanishes. So, the required torque at the input is very, very small the tau 2 required is very small. So, the tau 2 required to hold the wheel in the open position is very small which can be easily done by locking this actuator.

So, the mechanism takes the major load of the aircraft and the wheel will not get folded. So, this is how you can use the singular or the dead center configuration of the mechanism to resist very large forces.

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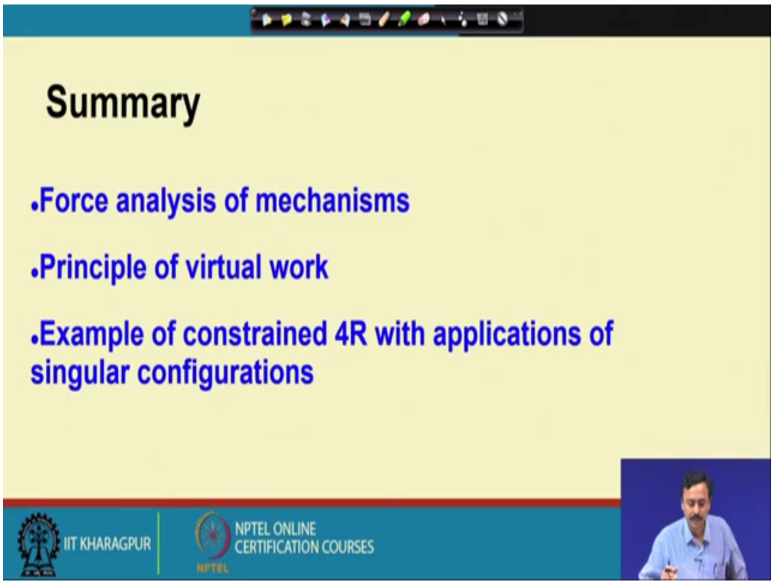

So, to summarize we have looked at the force analysis problem mechanism. Using the principle of virtual work and we have looked at this constraint 4R mechanism and it is application at singular configurations. So, with that I will close this lecture.