Mechanism and Robot Kinematics
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Lecture – 34
Acceleration Analysis – I

In this lecture, we are going to start our discussions on acceleration analysis. Till now we have looked at motion of mechanisms and robots from displacement and velocity perspectives. In this lecture we are going to start our discussion on acceleration analysis.

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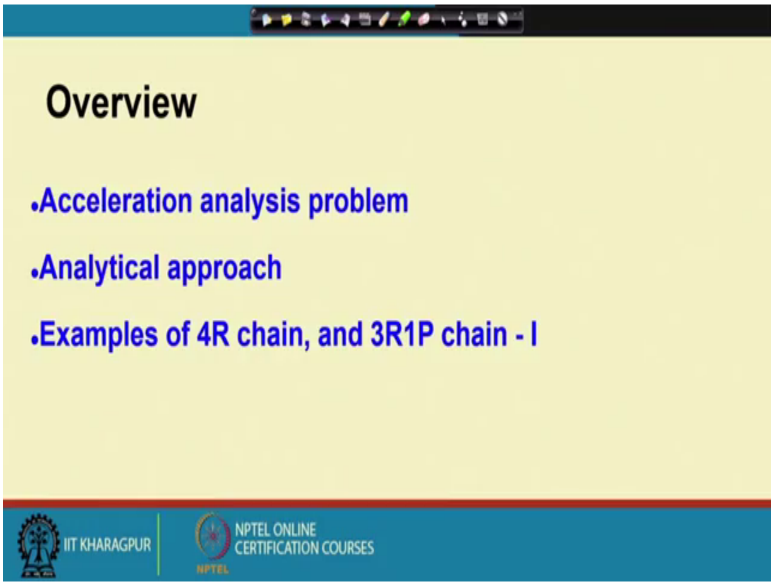

Now, to give you an overview of what we are going to discuss in this lecture, we will look at the acceleration analysis problem and formulate the analytical approach. And we will study this through examples of 4R chain and 3R1P chain of the type 1.

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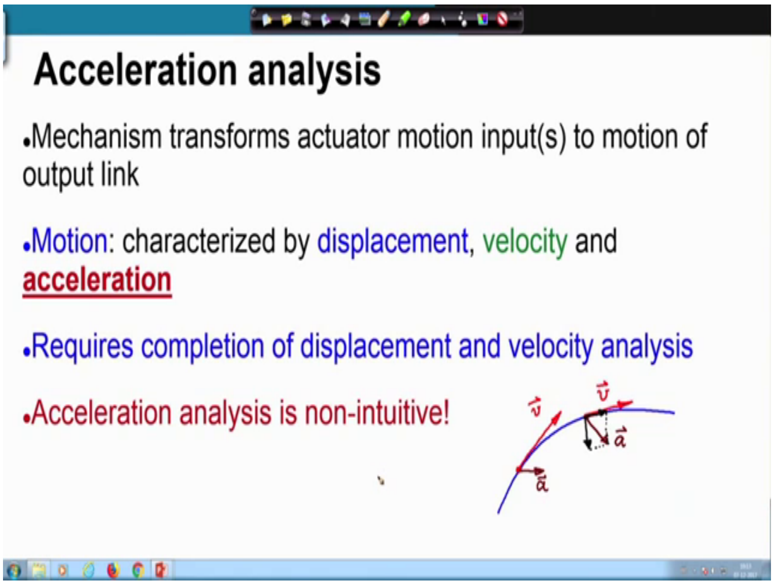

Now, as you know that a mechanism transforms actuator motion which are the inputs to motion of the output link. Now motion is characterized by displacement, velocity and acceleration.

So, we have discussed displacement, we have discussed velocity. So, these 2 characterized motion the next characterization comes from acceleration. So, in acceleration we assume that the displacement and velocity analysis has been completed. So, before we embark upon acceleration analysis we require displacement and velocity analysis to be completed.

Now, acceleration analysis is non-intuitive, what do I mean by that? So, if you consider a point moving on a certain trajectory; then you know that the velocity of this point at any point in the trajectory is tangential to the trajectory, but there is no, such restrictions on acceleration and that makes it non-intuitive. So, what I mean by this; let us consider a trajectory on which the particle is moving.

So, at any point the velocity is definitely restricted along the tangent to the trajectory at that point. So, at any point the velocity vector must be tangent to the path at that point, but when you look at acceleration. Acceleration can have an arbitrary direction. So, acceleration on a path can have an arbitrary direction, but velocity must always be tangent to the path. Now this makes acceleration non-intuitive.

So, as you know that this can have two components, this acceleration can have a component along the path and perpendicular to the path. The tangential acceleration results in increase of the speed whereas; this perpendicular component is because of the path curvature. There is another possibility which occurs in rotating frames or in bodies which are rotating.

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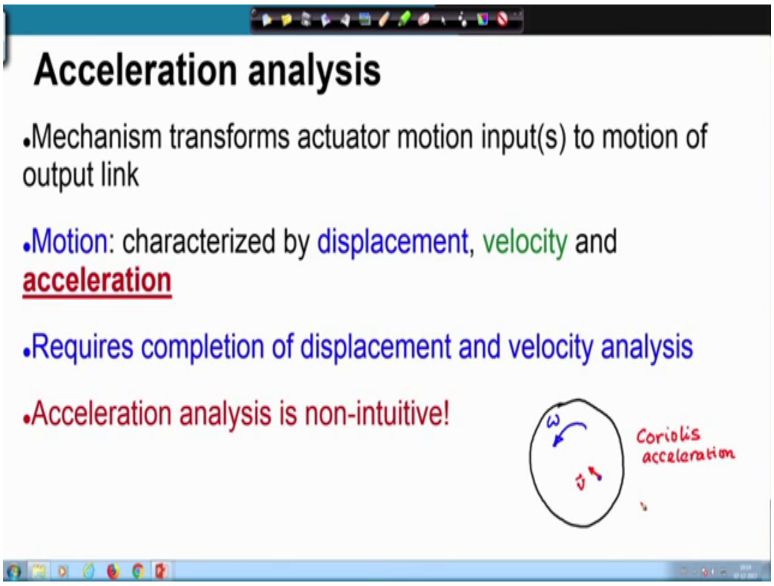

Let us say you have a disc on which you have a particle. The disc is rotating at a constant speed omega.

In this plane let us say and this point has a velocity. Assume that this velocity is also constant I mean with respect to the disc; even then this point has acceleration as we know this point has acceleration which is known as the coriolis acceleration. Because this point this particle has a velocity in a rotating frame. So, this is disc is rotating and the particle is moving with a velocity with respect to the disk; even though omega is constant and v is constant this particle has acceleration. So, this makes acceleration non-intuitive.

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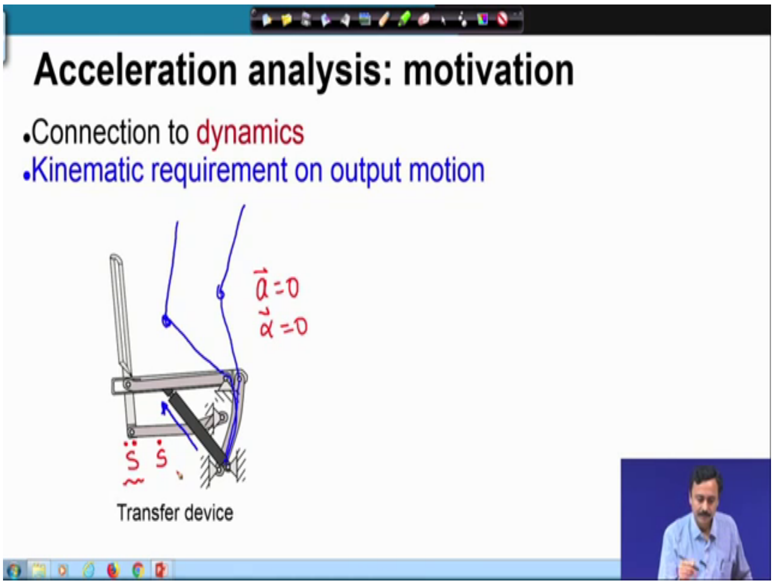

Now, why should we study acceleration? Analysis first point definitely that comes to our mind is it is connected to dynamics we know Newton's second law it requires acceleration it is a connection between acceleration and force. So, in order to relate to dynamics or go over to dynamics we need acceleration of center of mass of the links, but there is another reason why we must study acceleration. We have to go to both to acceleration analysis; to understand this kinematic requirement on output motion of a mechanism or a robot.

Now, what do I mean by this? Let us consider this transfer device as we have discussed when this actuator expands; this device is going to straighten out. So, this device is going to move. Now if I want that there should not be any acceleration at the output of certain point let us say or maybe angular acceleration is 0, in that case I must have restrictions on this expansion rate of this actuator.

The restriction comes from the acceleration of the actuator, acceleration of expansion of the actuator. So, if I want acceleration at the output to be 0 there must be some definite acceleration at the input. If I want some non0 acceleration at the output then also I have to decide on the acceleration I have to know what should be the acceleration at the input. So, that it produces for example, a constant acceleration.

So, therefore, we need to understand the acceleration input output relation for this transfer device.

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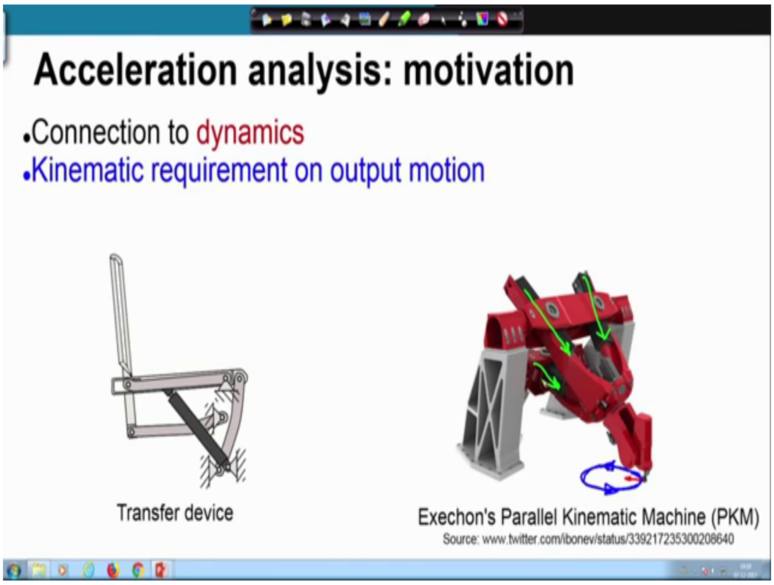

The same holds for this parallel kinematic machine. Suppose I want to move the tool on a certain circular path. Let us say the constant speed; even though the tooltip is moving at a constant speed on a circular path it has acceleration the centripetal acceleration.

So, therefore, I must have certain acceleration at the actuator expansion as well. So, what we need is to understand the acceleration input output relation.

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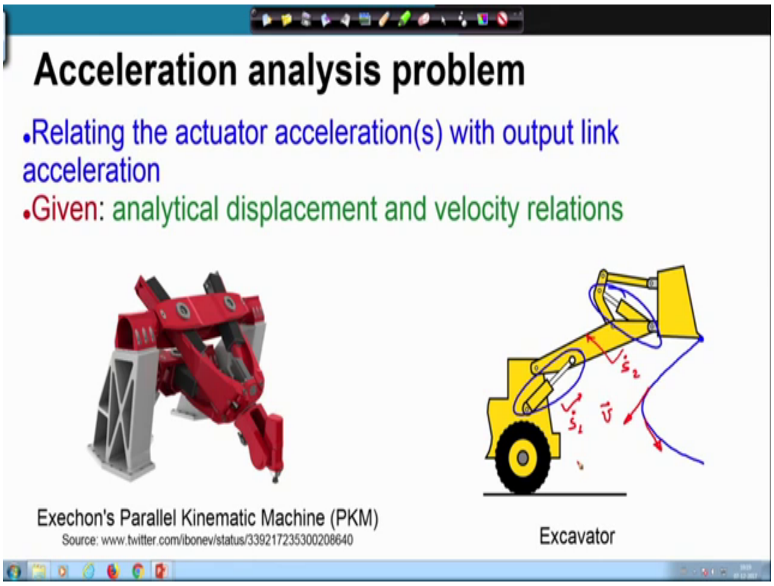

So, the acceleration analysis problem is essentially relating the actuator accelerations with the output link acceleration. So, this is what the problem is about so what are we given we are given the analytical displacement and velocity relations and we have to find out the acceleration relation.

So, here let us say in this excavator, we are given or we know already that the displacement relations of the bin in terms of the actuator expansion. We also know the velocity relations. So, if I want to produce a certain velocity at the output. What should be the expansion rates at the actuators at these 2 actuators. So, then we can embark upon the acceleration analysis.

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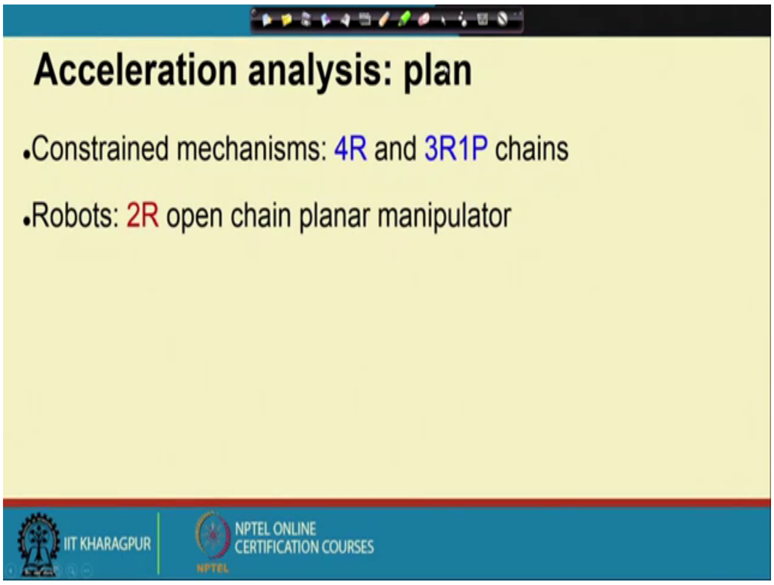

So, our plan for acceleration analysis is given here we will first discuss constraint mechanisms 4R and 3R1P chains after that we are going to move to our open chain cleaner manipulators.

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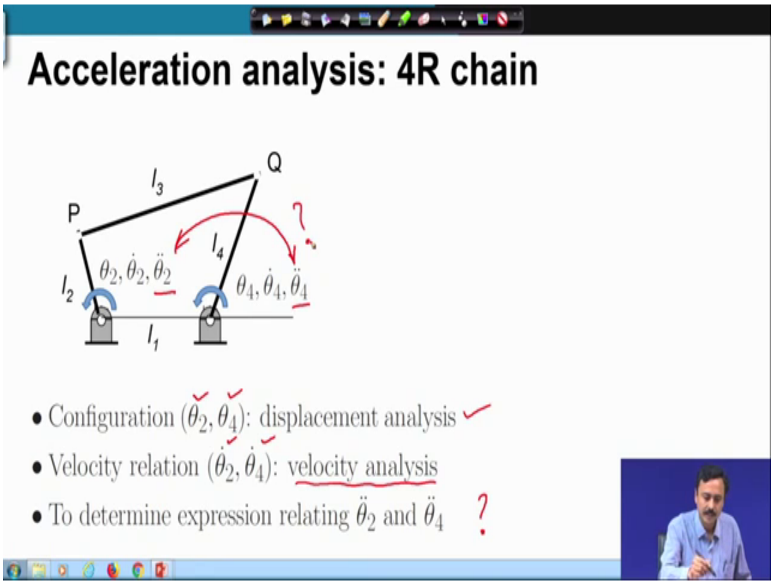

So, here I have and an image of this 4R chain, we already know the configuration so theta 2 and theta 4 are given.

And this relation is through the displacement analysis from the velocity analysis we know the relation between theta 2 dot and theta 4 dot. Then we have to find out the relation between theta 2 double dot and theta 4 double dot. So, we have to relate these 2. So, this is our acceleration analysis problem.

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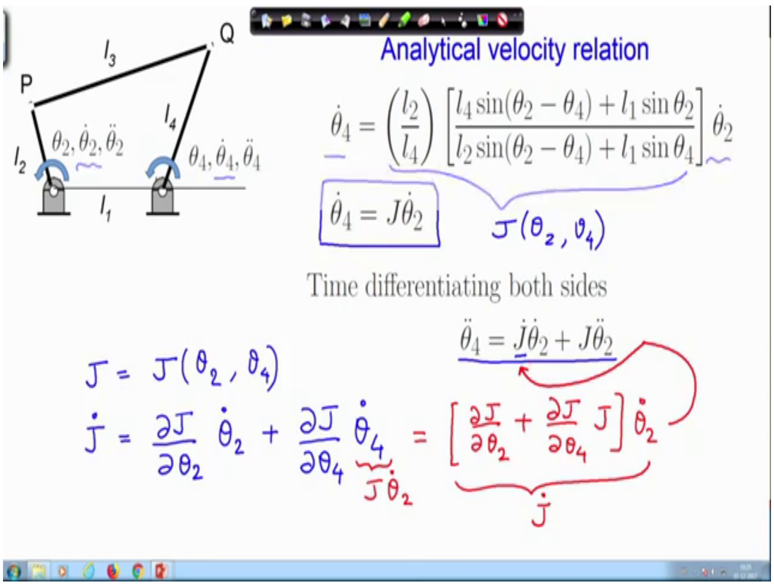

So, let us start with the velocity relation. So, here I have written out for you the analytical velocity relation which we have already discussed. Now here I have an important point to mention we have done this velocity analysis by 2 approaches we had first discussed about the method of ICs or instantaneous centers of rotation. So, using the concept of instantaneous center of rotation we have related the velocity at the input with the velocity at the output.

Subsequently we discussed analytical velocity input output relations. Now when we discuss this method of ICs remember that that velocity analysis depended on the location of the instantaneous center of rotation. Now when we come to acceleration analysis, it is the comparison of velocity of two infinitesimally separated configurations of the mechanism. Now when I move the mechanism from a certain configuration to another configuration you must remember that the IC has also shifted.

So, therefore, when I discuss acceleration analysis the method of ICs is more complicated because I must also take into account the variation of the IC itself the movement of the IC itself. So, therefore, this being complicated we will take recourse to the analytical velocity analysis and we will start from there for our acceleration analysis and that is what we are going to do here. So, here I have written out the analytical velocity relation that we had derived.

So, I have the output velocity. So, it this is the output thither for dot is the output and theta 2 dot is the input. So, the input output velocity relation as you know is related through the Jacobian and the Jacobian is a scalar in this case. Now when you differentiate both sides of this expression with respect to time you have this relation, remember that the Jacobian is a function of theta 2 and theta 4.

So, therefore, when I time differentiate the input output velocity relation I also differentiate the Jacobian. So, Jacobian is a function of theta 2 and theta 4. So, the rate of change of Jacobian I can write as del Jacobian del theta 2 into theta 2 dot using chain rule plus del of the Jacobian del theta 4 theta 4 dot.

So, using chain rule I have this expression of J dot now you will notice that theta 4 dot theta 4 dot is again J times theta 2 dot. So, therefore, this expression I can simplify and write by taking theta 2 out common. So, this is the expression of J dot d J dt. So, time derivative of the Jacobian is therefore, this expression.

Now, this I will substitute here and let us see what we have.

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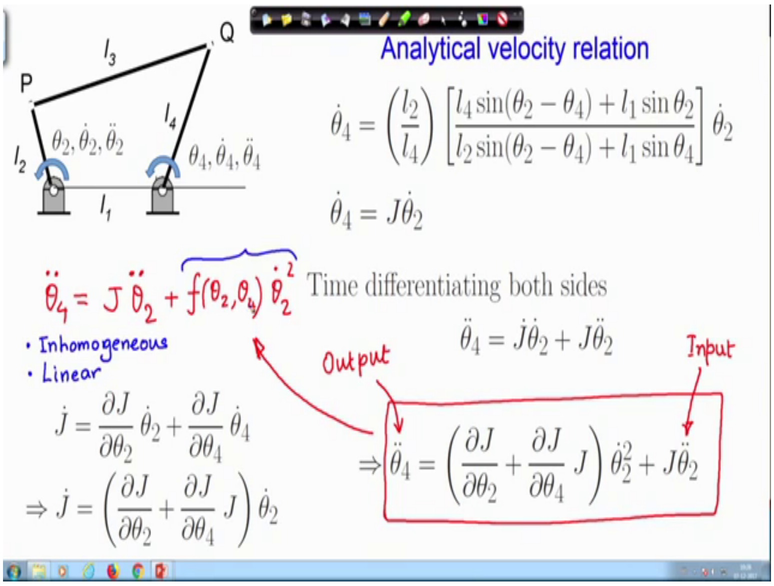

So, this is this step I have just shown you. So, what we arrived at is the acceleration input output relation. So, here I have the input acceleration and on the left I have the output acceleration theta 4 double dot.

Now, there are few things to note here. This input output acceleration relation is inhomogeneous it is of this form let me write this again in a condensed form. So, here I have written out this in a condensed form. Now this relation between theta 4 double dot and theta 2 double dot as you can see is linear.

However, there is this extra term this extra term which is configuration and velocity dependent; such a relation between accelerations will say that it is in homogenous. So, the relation is inhomogeneous though the relation is linear the acceleration relation is linear, the acceleration relation input output acceleration relation is linear, but in homogenous. The inhomogeneous term is because of the velocity.

So, if the velocity is 0 at that instant of time then the acceleration relation becomes homogeneous. So, if the velocity at that instant of time is 0 then the acceleration relation becomes homogeneous otherwise the acceleration relation is inhomogeneous though linear, there is one more point to be noted.

If you require that the output acceleration be 0. If you require the output acceleration to be 0 so let me go to this expression itself.

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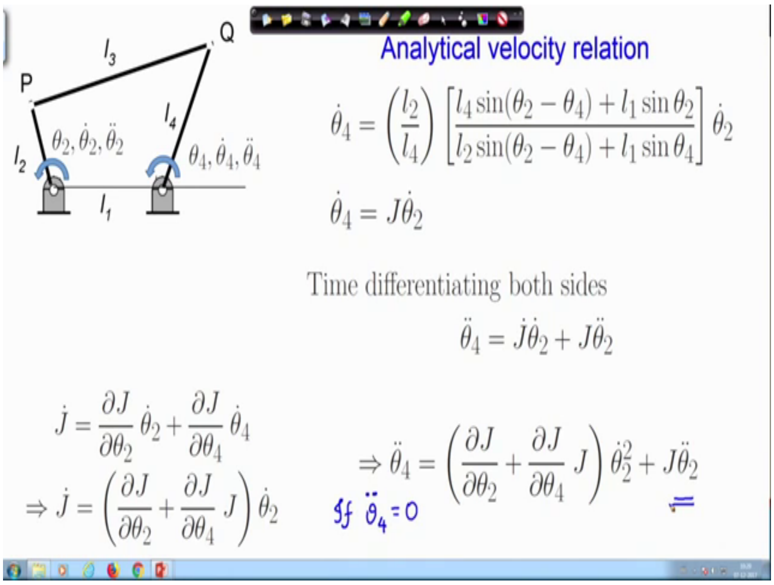

If I need if I need the output acceleration to be 0, then I require an input acceleration. So, what acceleration do I need? So, let us do this.

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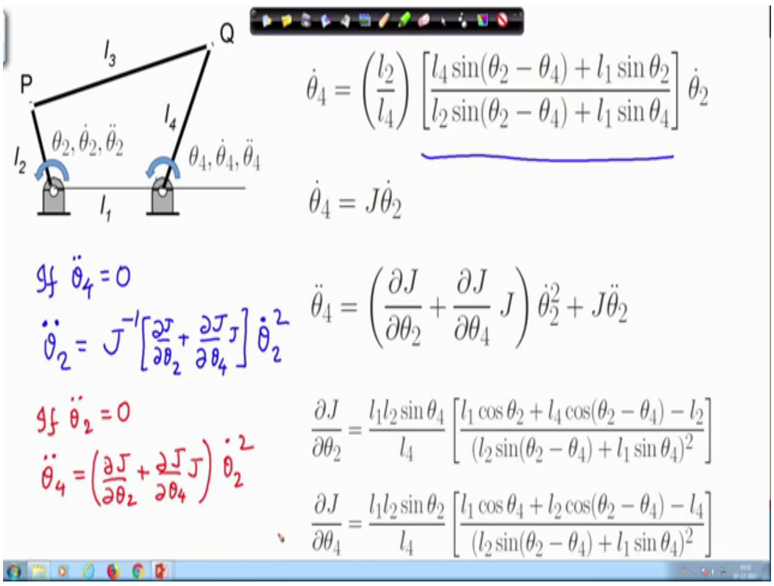

So, here I have written out for you these expressions. Now if you look at the expression for del J del theta 2 and del J del theta 4 which is very easy to compute from the expression of J.

So, these are given here they look complicated, but they can be obtained in a straightforward manner by partial derivative. Now the point that I would like to mention here is that; even if theta 4 double dot is 0 theta 2 double dot in general will not be 0. So, that will become J inverse of del J del theta 2 plus del J del theta 4 into J into theta 2 dot square.

So, even though theta 4 dot is 0 I will have acceleration at the input I must have acceleration of the input. So, that my output does not have any angular acceleration this is not the case with velocity analysis because it that velocity analysis relations velocity relations input output relations are homogeneous, because acceleration input output relations are inhomogeneous that is why even though we desire 0 acceleration at the output we must have acceleration at the input the opposite is also true.

Suppose I do not have acceleration at the input, suppose I am driving at a constant speed at the input. I am driving at a constant speed at the input which means that theta 2 double dot is 0; in that case I will have in general I will have acceleration at the output. So, this is the expression of the output acceleration; even though I am driving at a constant speed which means input acceleration is 0.

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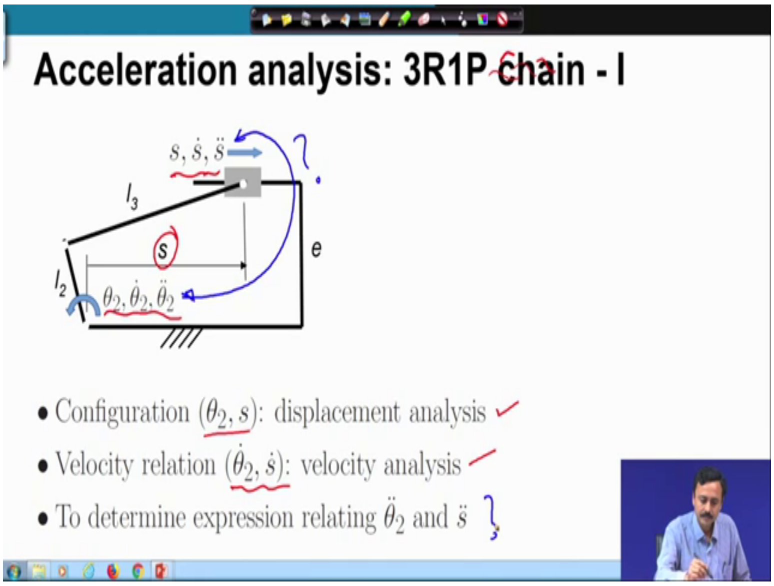

Now, let us go over to the acceleration analysis of 3R1P chain of type 1. So, here we already have completed a displacement analysis. So, which I which means I know the input output displace displacements. I have completed the velocity analysis which means

I can relate theta 2 dot and s dot the output is s and the rates are s s dot and s double dot. The input is theta 2 and the input rates theta 2 dot and the acceleration is theta 2 double dot.

So, what I now need to find out is a relation between this theta 2 double dot and s double dot. So, this is what I need to find out.

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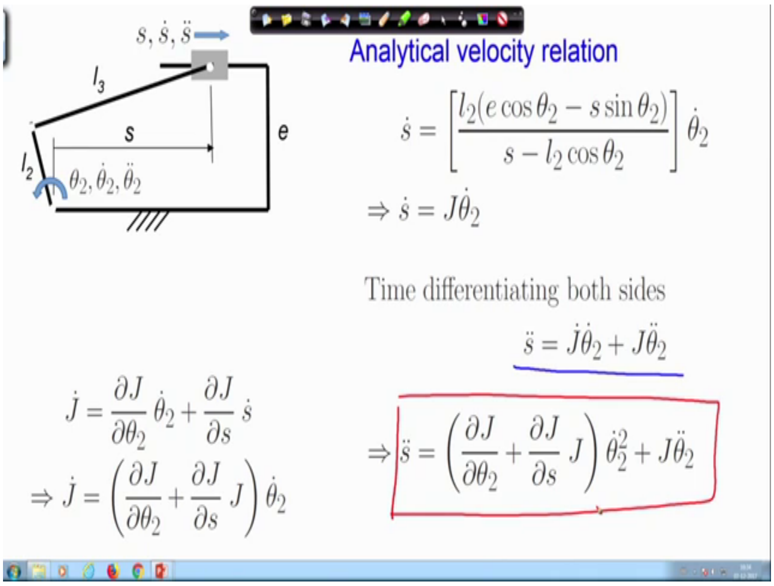

So, here I have written out the analytical velocity relation for the 3R1P chain. Now if you differentiate again with respect to time then you obtain the acceleration relation between s double dot and theta 2 double dot, again we find that it is linear in s double dot and theta double dot though it is inhomogeneous.

Here again you time derivative the Jacobian. So, you have this expression of J dot which you substitute into the expression of s double dot and finally, obtain the input output acceleration relation. So, remember we have started with the analytical velocity relation differentiated that with respect to time and obtain the acceleration relation.

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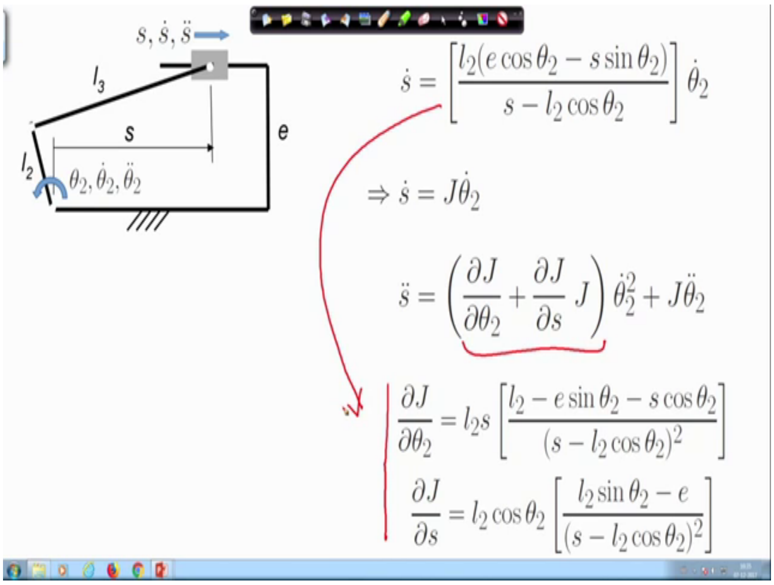

So, here I have written out for you the expressions of del J del theta 2 and del J del s which is required in this term of acceleration relation. So, this can be obtained directly from here by taking these partial derivatives.

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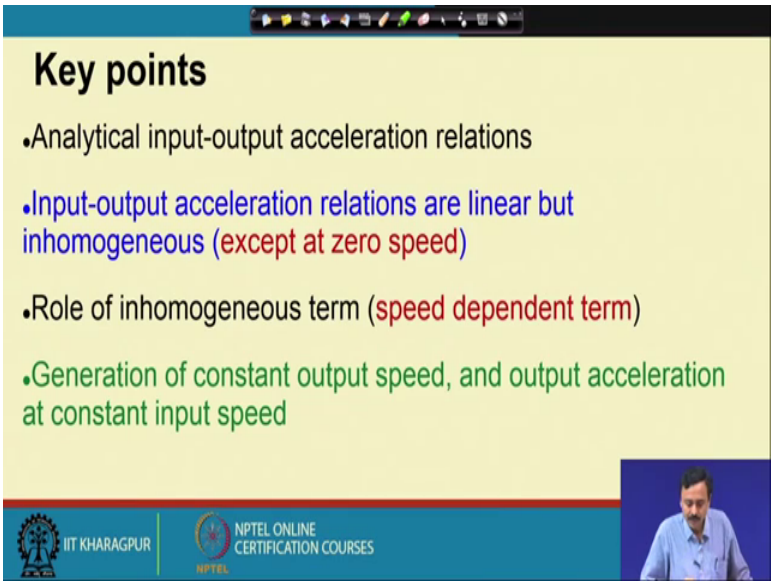

So, what are the key points that we have found, we have derived the analytical input output acceleration relations.

We have found that the input output acceleration relations are linear though they are inhomogeneous except at 0 input speed. Thirdly, we have looked at the role of the inhomogeneous term which is speed dependent we have found that if the speed is 0 the input speed is 0 then the acceleration relations become homogeneous, but. So, in general the acceleration relations are inhomogeneous.

Then we have looked at the constraints that these acceleration relations put on the input output acceleration. So, even though we want to drive the output at constant speed for example, we must have some input acceleration and similarly if you drive the input at a constant speed then you have acceleration at the output.

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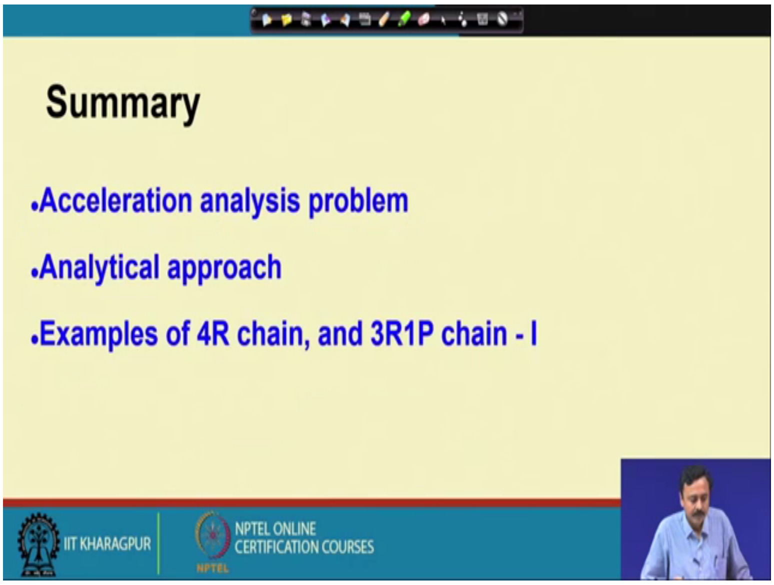

So, to summarize we looked at the acceleration analysis problem we have taken the analytical approach and we have looked at two examples the 4R kinematic chain and the 3R1P kinematic chain of type 1. So, with that I close this lecture.