Mechanism and Robot Kinematics
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Lecture – 33
Path Generation Problem

We have been discussing about the velocity analysis of robot manipulators we have looked at open chain and closed chain robot manipulators. So, the problem of velocity analysis is to relate the actuator velocities to the end effector velocities. Today, we are going to look into the problem of path generation.
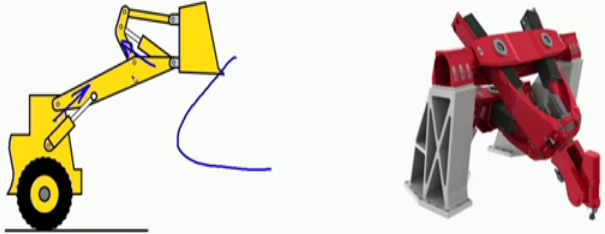
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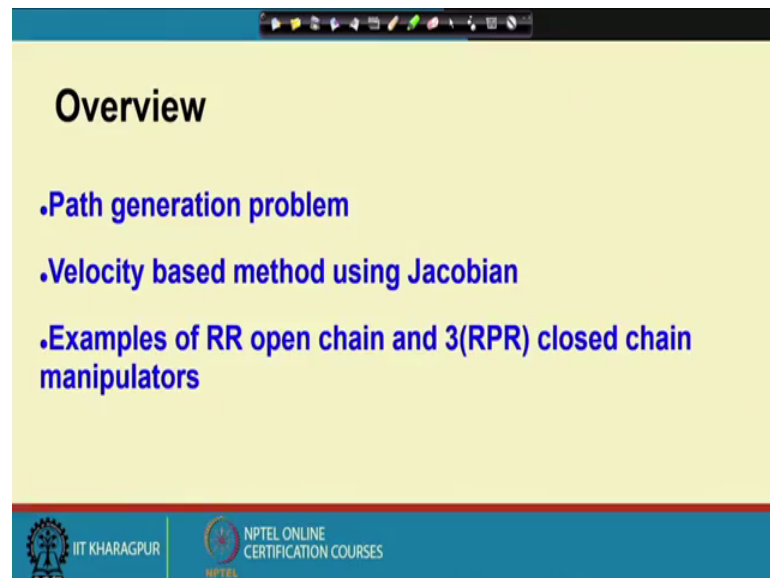
So, to give you an overview of what we are going to discuss in this lecture, we are going to look at the path generation problem using the velocity-based method. We look at examples of the RR open chain planar manipulator and 3 RPR closed chains in planar manipulator.

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So, as I have mentioned that the velocity analysis problem in the case of robots is also closely related to the path generation problem.

So, given the actuator rates are the rates at which the actuators are expanding on or rotating we can find the path that the end effector takes. So, given the actuator rates we can find out the path of the end effector. So, that is the forward problem the inverse problem which is very relevant for various applications is the path generation problem in which for a given path of the end effector; we have to find out the rate of expansion of these actuators. So, that is the inverse problem which is the path generation problem.

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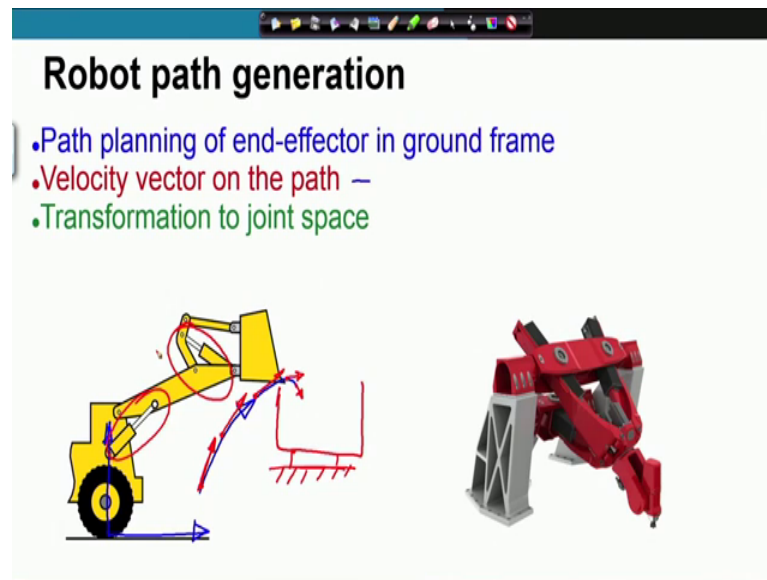
We have discussed briefly what is the path generation problem now the path generation problem has 3 components in it let us go through them one by one. First is the path planning of the end effector in the ground frame which means that given this configuration. For example, this excavator I would like to know in a certain ground reference frame what is the path that I desire.

So, essentially is the representation of the path now this representation of the path will take into account for example, obstacles which might be present or it might be that there is a bin on which this must be dropped in that case it might go something like this. So, based on our requirement we have to first plan the path in the ground reference frame.

The second point is the specification of velocity on the path. So, if I have to generate this path how do I generate I specify the velocities that I desire for the end effector on this path something like this once I have the end effector velocity at every point on the path I transform that velocity back to the joint space. So, essentially, I find out the actuator rates to produce the corresponding end effector velocity. So, that is the plan of this path generation problem. So, that is how this will proceed.

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So, we will look at open chain and closed chain manipulators.

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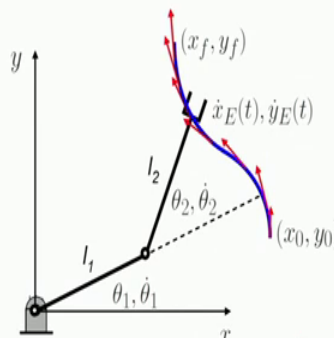
So, this is the problem of path generation. So, essentially this is the second and third part. So, what we have is we have the path already specified or determined.

Now, we have to specify the velocities at each point on this path. So, we are given the end effector path in terms of xE and yE as a function of time. So, these are specified. So, as a function of time I know how xE and yE build we have to determine the corresponding joint trajectory of the joint velocity.
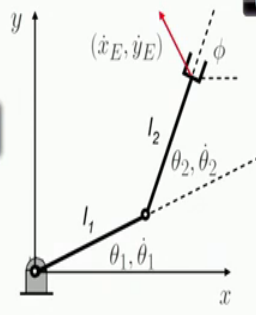
So, the way we will proceed is that given the path we can differentiate at every point and determine the velocity; The velocity of the end effector at a specific point on the path.

So, here we can differentiate and find out the velocity as a function of time and using the velocity analysis we are going to determine the joint velocity which comprises in this case as theta one dot and theta 2 dot.
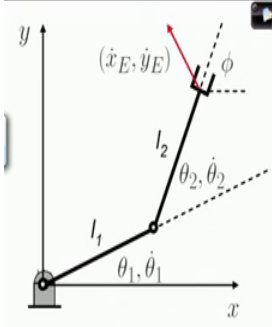
So, let us review what we had discussed in the velocity analysis problem. So, we were given the displacement relations.

$$\dot{x}_E = [-l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2)]\dot{\theta}_1 + [-l_2 \sin(\theta_1 + \theta_2)]\dot{\theta}_2$$

$$\dot{y}_E = [l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)]\dot{\theta}_1 + [l_2 \cos(\theta_1 + \theta_2)]\dot{\theta}_2$$

$$\begin{Bmatrix} \dot{x}_E \\ \dot{y}_E \end{Bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix}$$
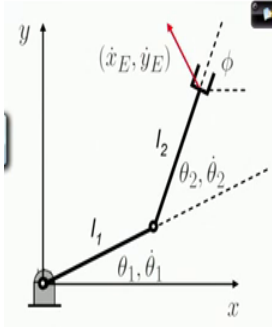
where

$$J_{11} = -l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2)$$
$$J_{12} = -l_2 \sin(\theta_1 + \theta_2)$$
$$J_{21} = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$
$$J_{22} = l_2 \cos(\theta_1 + \theta_2)$$

Then differentiating the displacement relations we arrived at the velocity relation input output velocity relation.

So, on the left I have the end effector velocity vector and on the right, I have a relation through the Jacobean to the joint velocity vector. Where the elements of the Jacobean for the 2 are manipulator has been written out this we have discussed already.

$$\{\dot{X}_E\} = [J]\{\dot{\theta}\}$$

where

$$\{\dot{X}_E\} = \begin{Bmatrix} \dot{x}_E \\ \dot{y}_E \end{Bmatrix}, \qquad \{\dot{\theta}\} = \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix}$$

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

$$\{\dot{\theta}\} = [J]^{-1}\{\dot{X}_E\}$$

where

$$[J]^{-1} = \frac{1}{(J_{11}J_{22} - J_{21}J_{12})} \begin{bmatrix} J_{22} & -J_{12} \\ -J_{21} & J_{11} \end{bmatrix}$$

- **J** is the Jacobian matrix.
- **J** transforms joint velocities to end-effector velocities.

So, we had these relations and finally, we obtained the velocity relations in the forward kinematics forward velocity analysis and this is the velocity relation in the inverse

velocity relation. So, this is the inverse velocity relation and this is the forward velocity relation.

So, we have noted that in the inverse velocity relation we required the inverse of the Jacobean and the inverse of the Jacobean has this denominator term which is the determinant of the Jacobean. So, this determinant must be non-0 in order for the Jacobean to be invertible.

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So, this is our inverse velocity relation now let us look at the example of a straight-line path generation. So, here I have 2 points xo y o and x f yf.

And I would like to connect them by a straight line. Now, every path planning problem can be considered to be a specialization of this problem because any path can be thought of as straight lines between infinitesimal points. So, if I can find this then I can generate any path. So, let us look at this straight line path generation first we are going to represent the path.

So, the first problem will be to represent the path you can really very easily represent this xE and yE at any point on the path as a function of time so, here since we start with x naught. So, this is the x coordinate of the end effector. So, we start at x naught and we move linearly in time because of this additional term. So, you can see at time t equal to 0

I am at x naught and similarly at time t equal to 0 I am at y naught this path has to be traversed in time t f.

So, essentially this time goes from 0 to t f. So, at time t equal to 0 I am at x naught y naught at time t f when t is equal to t f then this factor becomes one. So, this is x naught plus x f minus x naught. So, that becomes x f similarly y E becomes y f. So, at time t equal to t f we are at the final point x f y f.

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So, if you differentiate this function then you have the velocity expression. So, this is the velocity of the end effector point in the Cartesian coordinate in the ground based Cartesian coordinate system. So, this is the velocity that we are going to use. So, once I have the velocity now I can very easily. So, I have the right-hand side here this is the right-hand side.

The velocity of the end effector point from the right-hand side of this inverse velocity relation and through the inverse Jacobean I obtain the joint velocity rates. So, joint it is joint velocity vector.

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Singularity of RR manipulator

Singularity of Jacobian: vanishing of $\text{Det}[\mathbf{J}] = l_1 l_2 \sin \theta_2$
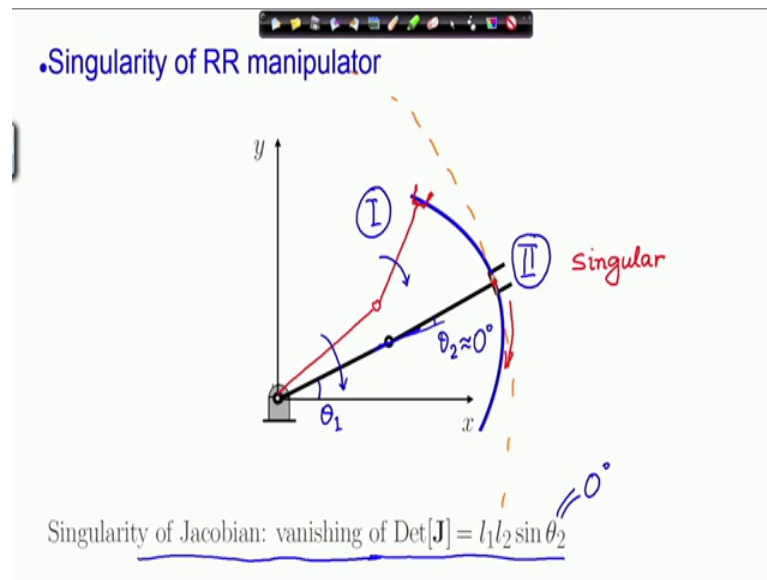
Now, there is this inversion problem of the Jacobean as we have seen here. So, the inverse of the Jacobean has this determinant of the Jacobean sitting in the denominator. So, we must ensure that the determinant of the Jacobean is non0,but there are instances where this might go to 0 or if this might go very close to 0.

So, let us look at this situation what we have here let us say a manipulator which traverse this path up to this point. So, this manipulator moved from configuration one to this configuration 2 the black configuration now here as you can see that this angle has straightened out. So, this angle theta 2 is roughly 0 degree.

So, this is theta one measured from the x axis and theta 2 is the angle made by the second link with respect to the first link now here theta 2 is roughly equal to 0 and we have discussed that the Jacobean is singular when the determinant goes to 0 which in this case it is because theta 2 is almost equal to 0 or 0 at this configuration.

So, in that case the determinant of the Jacobean vanishes and therefore, the Jacobean is no longer invertible. So, this is a singular configuration now when the manipulator passes through a singular configuration there are various possibilities in the sense that I can generate this path, if this path is such that is it passes through the. So, this point let us say on the path the manipulator passes through the singular configuration and the path then again comes inwards as you can see the manipulator is completely extended.

So, this point on the path is actually on the boundary of the workspace. So, this is roughly the boundary of the workspace which we have discussed before. So, this point of the path lies on the boundary of the workspace. So, therefore, the manipulator goes to a singular configuration at this configuration if you want to proceed further on the path which again actually comes into the workspace as you can see it touches the works workspace boundary and then again comes inward into the workspace.

So, therefore, it can be continued. So, at the workspace boundary either you can actuate theta one or you can just actuate theta 2 which means that one actuator can be held fixed the other actuator can be moved.

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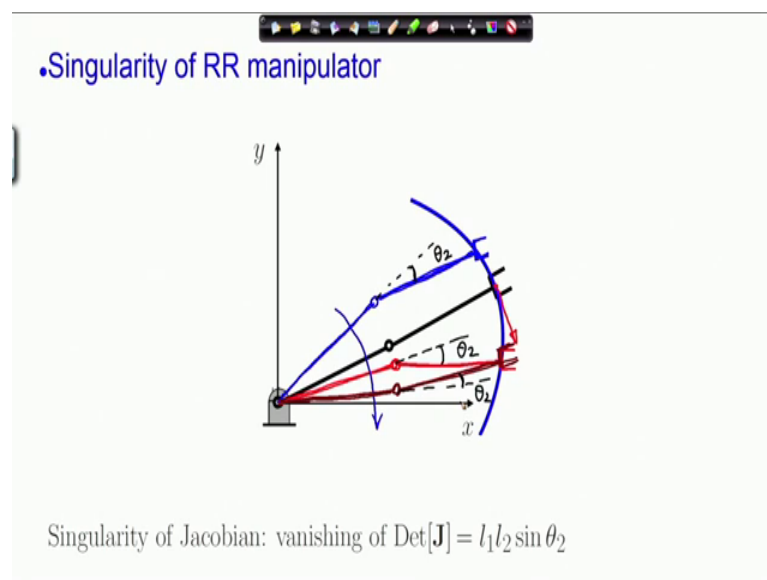
In order to produce a velocity which is tangent because at this boundary point we need to generate a velocity vector of the end effector which is tangent to the path and since this is also the boundary. So, it will be tangent to the boundary.

Now, once you cross then you can come again to a configuration like this. So, you can move from a configuration which I have shown in blue. So, you can move from the blue configuration through the black configuration to the red configuration. There is another possibility that you can move when you go past the singular point you can move to the configuration like this.
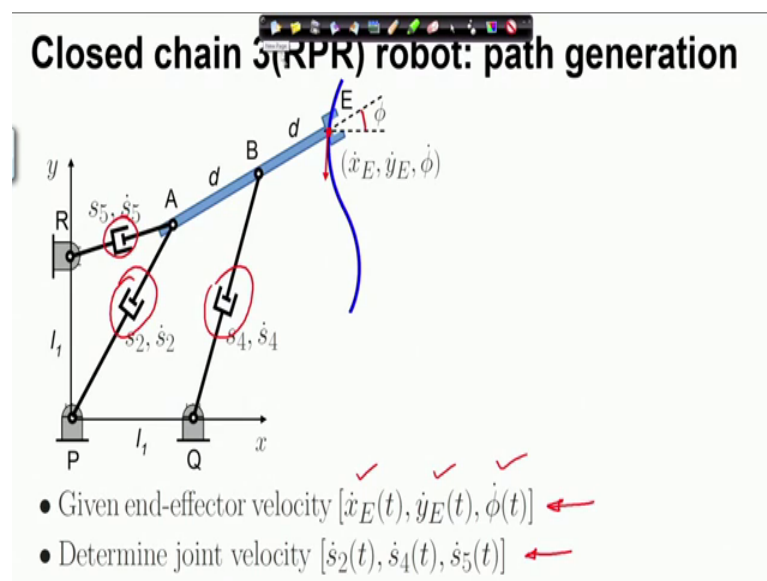
So, this is also a possibility that the angle theta 2 here it was negative goes to 0 and then again goes to negative or theta 2 was negative goes to 0 and flips to the positive side in the brown configuration as you can see here. So, there is a flipping of configuration through the when the manipulator passes through the singular configuration.

Now, usually near the singular configuration because the Jacobean is. So, ill conditioned. So, we have these numerical problems we have numerical problems because the inverse of the Jacobean will involve very large quantities and therefore, once you have very large quantities sitting in the inverse of the Jacobean. Then the joint velocities become very high this can may not be supported by your actuators the actuators may not be able to produce such high joint velocities.

So, you always face a problem. So, the path has to be planned initially such that it does not go very close to a singular configuration of the manipulator if at all it is required that the path has to go very close to the singular configuration then the inverse problem has to be solved carefully. It you have to put additional constraints on the velocity of the actuators or the actuator rates. So, that your motors or actuators do not get saturated.

In that case you lose on the path or you can lose on the velocity that you desired on the path. So, there is a trade off if the path is too close to a singular configuration then you will lose accuracy on that path.

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Closed chain 3(RPR) robot: path generation

- Given end-effector velocity $[\dot{x}_E(t), \dot{y}_E(t), \dot{\phi}(t)]$
- Determine joint velocity $[\dot{s}_2(t), \dot{s}_4(t), \dot{s}_5(t)]$

Now, let us look at the closed chain 3 RPR manipulator here this manipulator has 3 degrees of freedom we are specified the velocity of the end effector point the velocity trajectory of the end effector point through xE dot yE dot and phi dot.

So, phi dot. So, phi is the orientation angle and xE and yE are the positional coordinates in the ground based cartesian coordinate system what we need to find out is the actuator expansion rates. So, this vector s 2 dot s 4 dot s 5 dot. So, here are the actuators you need to find out their expansion rates for a specified end effector trajectory or velocity. So, once given the end effector trajectory we can always determine or calculate the end effector velocity.

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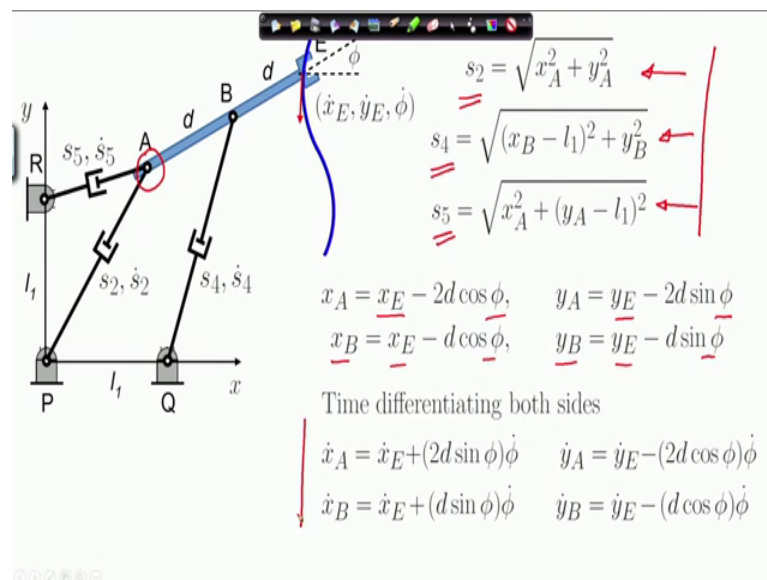
So, let us recapitulate what we had discussed about the velocity analysis problem for this manipulator we have these displacement relations in terms of the coordinates of point A. So, here we have point a. So, we have related the coordinates of point A in terms of the end effector coordinate and the orientation angle phi.

Similarly, the coordinate x B y B they are related again to the end effector coordinates and phi now if you time differentiate these relations you will get the velocity relations which you use in this set of 3 equations which relate to the actuator expansion. So, once I differentiate these relations with respect to time and use the velocity relations that I have here.

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Then I obtain the velocity relations between the end effector velocity vector and the actuator expansion rates.

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So, here we have defined the Jacobean which relates the actuator expansion rate and the end effector velocity vector here this is quite straightforward because we will have the end effector velocity vector already specified through the path.

So, this vector on the right X E dot will be specified this is known to us given the path. We can find this vector once we plan the time of motion therefore, we can directly find

out the expansion rates of the actuators now here again the Jacobean which might look very simple, but these P1 P2 P3 Q1 Q2 Q3 and R1 R2 R3 they have certain denominator terms which might go to 0 and make the Jacobean singular.

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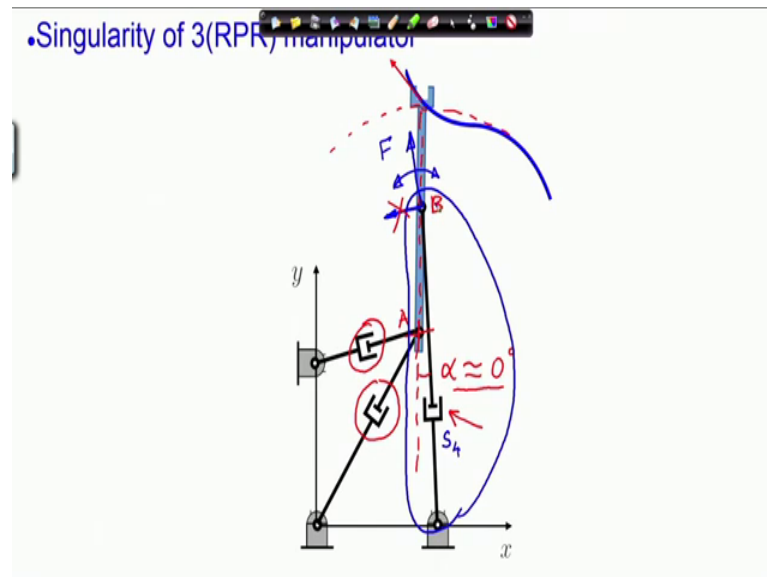
So, let us look at one of these singular configurations here I have drawn a singular configuration or a configuration very close to singularity. This manipulator is very close to singularity because the output link and this actuator angle this angle let me call this alpha is very close to 0.

Now, why this is singular because when alpha is exactly 0 it is a singular configuration and this configuration that I have drawn is very close to singularity now why this configuration is close to singularity or when alpha becomes 0 y is this singular the reason is imagine that I have fixed these 2 actuators. So, this point gets fixed this point gets fixed.

Now, this output link can only rotate about point a point a is fixed now this end effector link can only rotate about point a now who prevents the rotation the this actuator this set of links this should be able to constrain the output link. So, that it cannot rotate; however, when alpha is 0 when alpha becomes 0 this part this leg of the manipulator remember it can only apply force along the actuator this leg of the manipulator can apply force only along the actuator it cannot apply any force perpendicular to the actuator.

In other words, this actuator cannot prevent any rotation of cannot prevent any rotation of the output link about point A. So, therefore, when alpha is equal to 0 the rotation of the output link cannot be prevented by this actuator S4. So, therefore, there will be some rotation. So, the manipulator gains a degree of freedom why because it can rotate though with very small amounts there is a possibility of very small rotations you cannot constrain the output link using S4 when alpha is equal to 0.

So, therefore, this is the typical gaining of degree of freedom in mechanisms at singular configuration which we have seen even for constraint mechanism it gains degrees of degree of freedom at the dead center or singular configuration. So, that is this is precisely the situation here.

So, we have a singular configuration of a parallel manipulator at the dead center or singular configuration of the manipulator.

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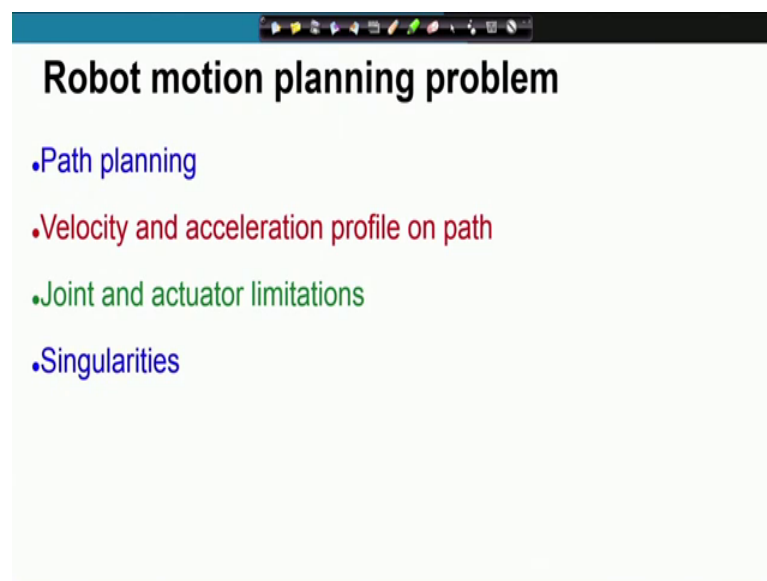
So, we have looked at the path planning problem. So, once you have the path which is planned based on obstacles that might be there you plan a path of the end effector you determine the velocity and the acceleration profile on the path you may have joint and actuator limitations.

So, joint limitations because of physical construction actuator limitations because it may not the actuator may not be able to produce very high velocity or very high acceleration

remember that since we are determining the joint velocity based on the end effector velocity. So, the joint velocity can change very quickly very close to the singular configurations as we have seen. So, we can we can have very high joint velocities and therefore, we will require very high acceleration.

If you require very high acceleration then production of very high acceleration depends on the torque restrictions of the torque or force restrictions on the of the actuators. So, near the singularities the motion planning is tricky. So, it will have very high actuator velocities and accelerations. So, you need more considerations or restrictions on the joint motion near singularities now once you have restrictions you will have errors on the path. So, initial path planning has to be done carefully.

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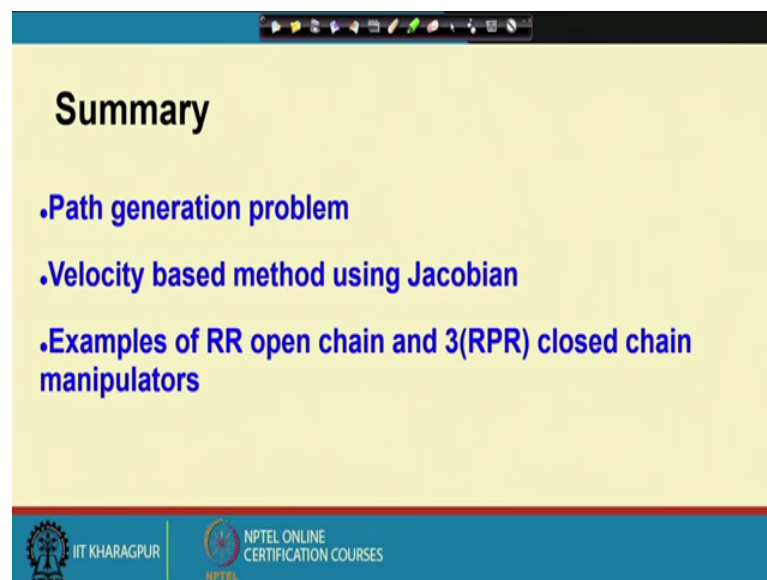
So, to summarize we have looked at the path generation problem using the velocity analysis problem. We have looked at two examples, one is of the open chain 2R manipulator and the other is the closed chain 3 RPR manipulator. So, with that I will conclude this lecture.