Mechanism and Robot Kinematics
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Lecture – 32
Parallel Manipulator Velocity Analysis

We have been discussing about the velocity analysis of robot manipulators. We have already discussed open chain robot manipulators. We have looked at the Jacobean, it is interpretation and singularity of the Jacobean. We have looked at the consequences of singularity of the Jacobean.

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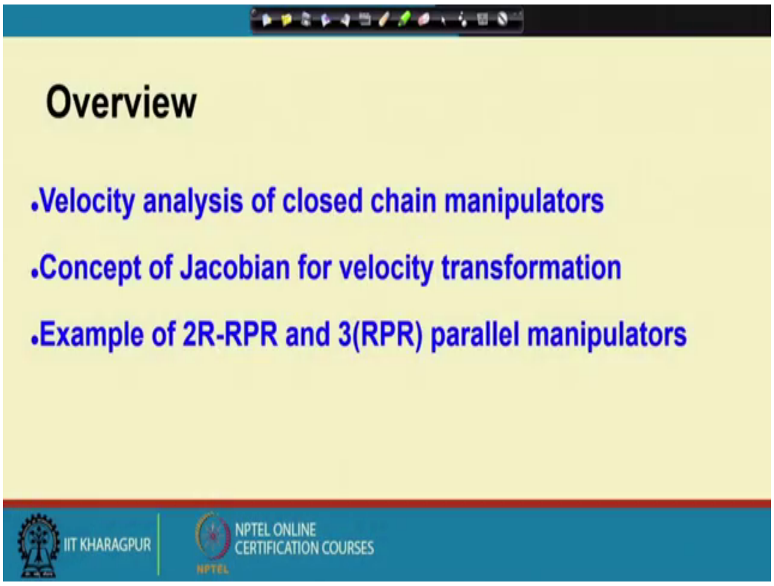

In this lecture, we are going to embark upon discussions on closed chain manipulators. We are going to look at velocity analysis of closed chain manipulators and I will take two examples.

The 2R-RPR plus 3 RPR parallel manipulators these are planer manipulators. We have already looked at the displacement analysis of both these parallel manipulator chains.

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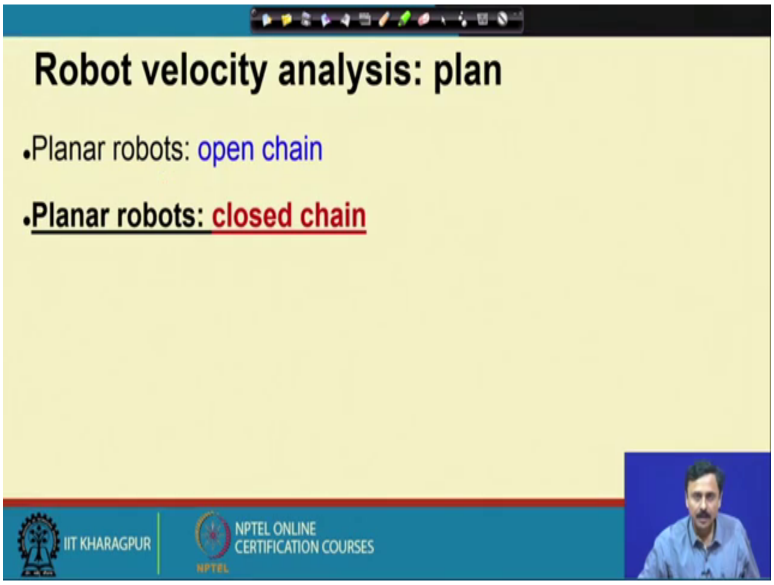

So, in this lecture we are going to look at closed chain planar manipulators.

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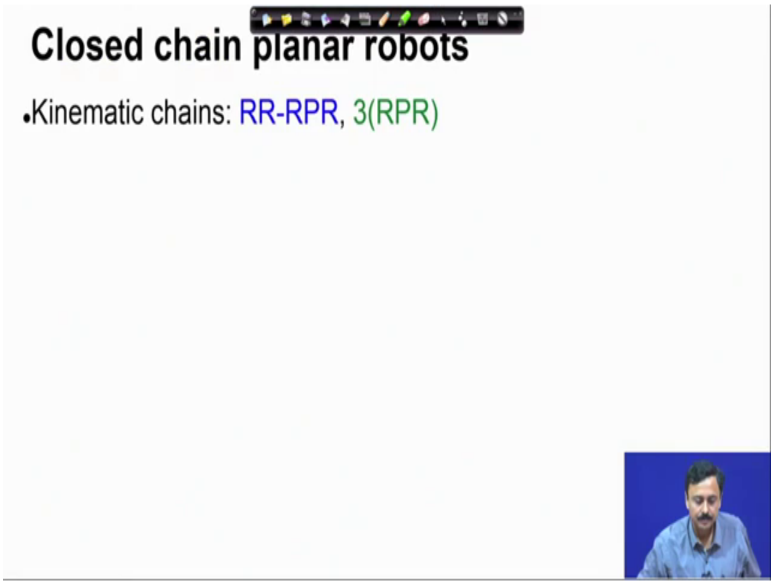

Going to discuss this RR-RPR and 3 RPR chain.

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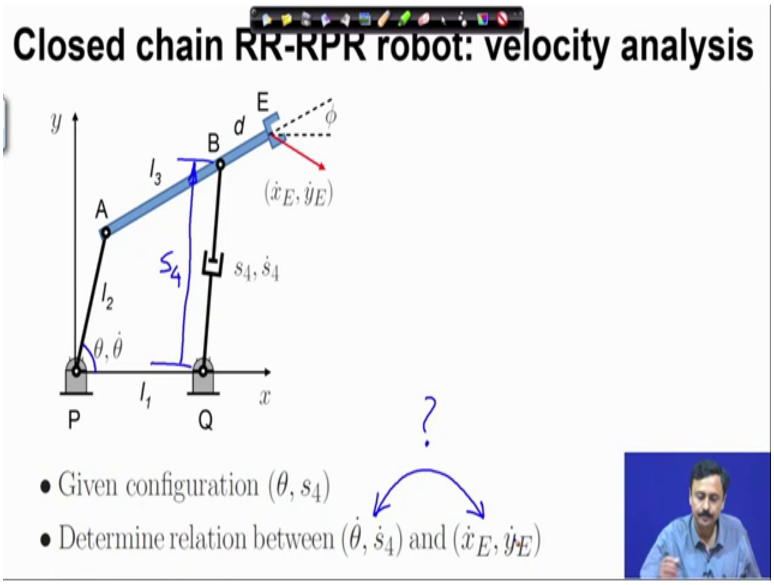

So, let us begin with the RRR PR planar closed chain robot. So, in this manipulator the configuration is specified by this angle theta and the extension of this prismatic pair which I have called S4. So, theta and x S4 they specify the configuration of the manipulator now the problem is to determine the relation between their velocities. So, we have theta dot and S4 dot as the joint velocities are the actuator velocities and xE dot and yE dot as the end effector velocities the velocity vector. So, we would like to find out a relation between them.

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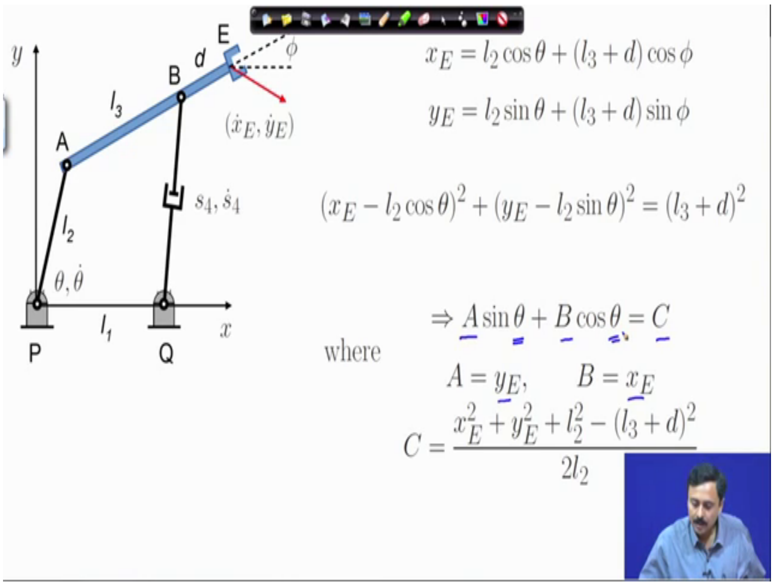

Here I have written out the displacement relations which we have already discussed in a previous lecture the angle phi. The angle phi is this configuration this orientation of the end effector we have also seen in the displacement analysis that given theta and S4 we can find out xE yE and phi. So, we can find out everything.

Now, if you eliminate phi between these two relations we ultimately arrive at a relation involving xE yE and theta here this a b and c they are functions of xE and yE you can interpret this as given xE and yE. It is a relation for theta it is an equation from where we can solve for theta. So, given xE and yE we can solve for theta.

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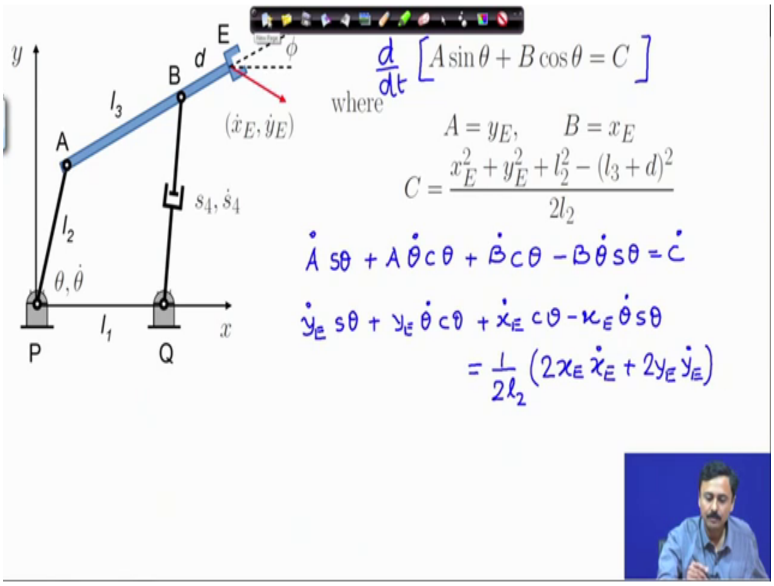

Now, what we are going to do with this relation is that we are going to differentiate this with respect to time. So, this relation we are going to differentiate with respect to time then, what we have is A dot sin theta S theta represents sin theta. This A theta dot cosine theta is B dot cosine theta minus B theta dot sine theta is equal to C dot.

Now, if you use these expressions of A B and C, then you have these relations; this is equal to now I can collect terms of xC dot yE dot and theta dot from here. I can collect terms of xE dot yE dot and theta dot and write in a simplified form which I will show you directly.

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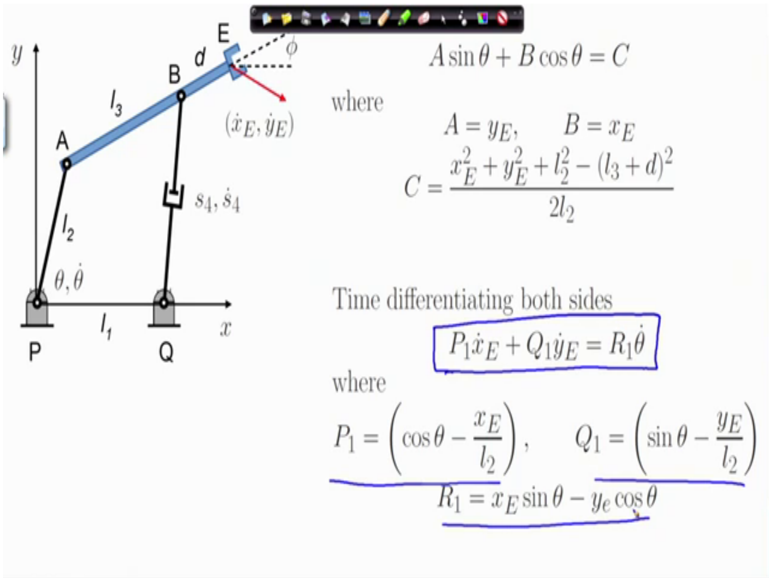

So, I obtain a relation like this by collecting terms of xE dot yE dot and theta dot. So, here P1 is this expression Q1 and R1 which you can very easily derive by simplifying the steps that I have shown you. So, this equation relates xE dot yE dot and theta dot, but this is not enough I need one more relation I must also relate S4 dot to xE dot and yE dot.

So, let us move on so, here I have one relation next, I must bring in S4 dot.

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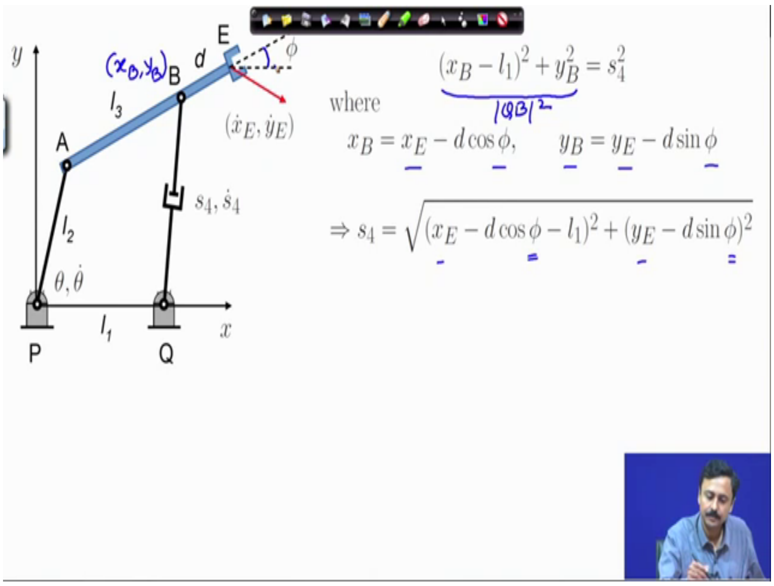

So, that is what we are going to look at now from the previous expressions. So, if you look at the coordinates of point B. So, coordinates of point b xB and yB and the distance between B and Q is nothing but S4. So, therefore, the distance square this is the distance square. So, this is Q B square is equal to S4 square and x b can be related to xE and phi similarly y B can be related to yE and phi.

So, if you use these expressions S4 can be related to xE yE, but you also have phi now phi remember is the orientation angle of the end effector.

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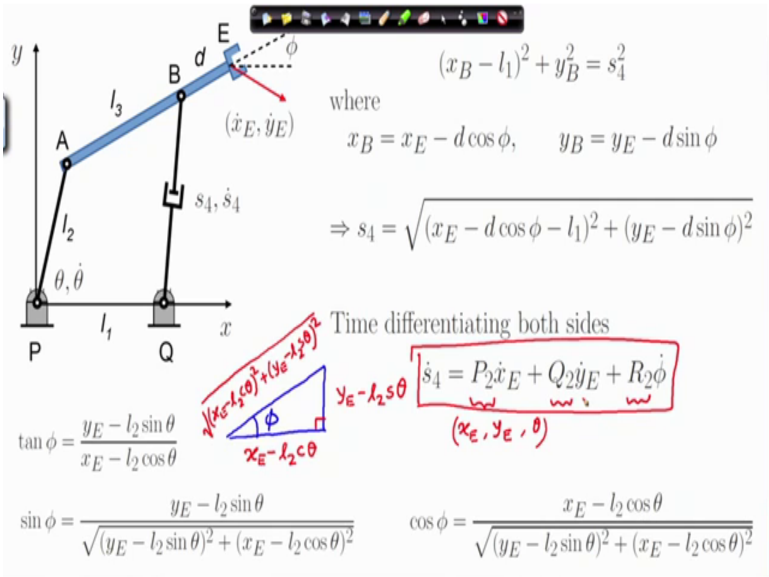

Now, if you differentiate with respect to time both sides of this equation then you can imagine that you will have derivatives of xE you will have derivatives of phi and you will have derivatives of yE. And the coefficients corresponding coefficients are P2 Q2 and R2 which can be very Easily found by differentiating this and this is going to be little complicated.

But it is otherwise straightforward I mean there will be a little bit of algebra involved. So, the steps are straightforward it will just differentiate the expression of S4 with respect to time. Now, we have involved phi. So, let us see how we can find out phi and we have also involved remember this cosine phi and sine phi we also have this cosine phi and sine phi as well as phi dot. So, this P2 Q2 R2 these will be functions of xE yE and phi which are the configuration which specify the configuration of the manipulator ok. Now, this tan phi you can very easily write tan phi in terms of theta.

So, tangent of this angle comes from the displacement relations that we have looked at before.

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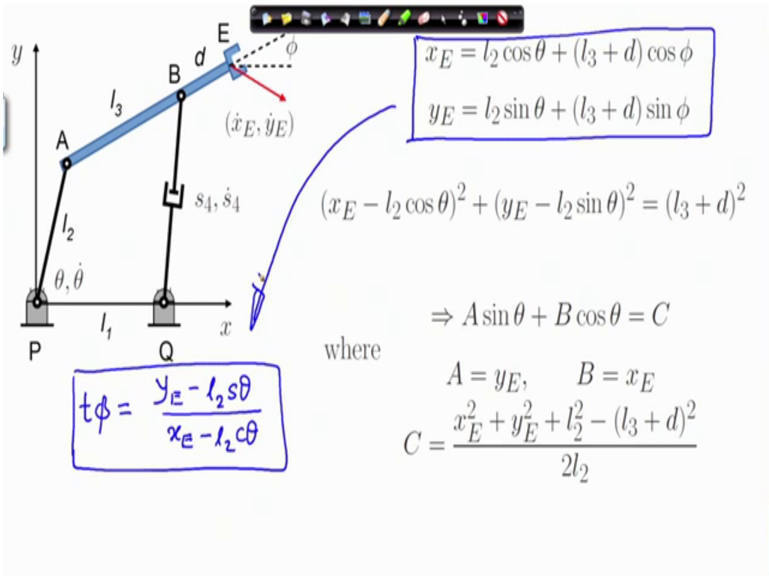

So, these are the relations you can find out tangent phi. So, tan phi is as equal to yE minus l 2 sin theta by xE minus l 2 cosine theta right from here we can obtain tangent phi and this is what we are going to use here. So, this is the expression of tan phi.

Now, once I have tan phi I can also find out cosine phi and sine phi. So, sine phi is this expression. So, how do I find this suppose this angle is phi then tan phi is this ratio? So, if I say that this is yE minus l 2 sin theta and this base of the triangle is xE minus l 2 cosine theta then you can very easily see that tangent phi is this ratio.

So, this hypotenuse of this right-angle triangle is nothing but square root of the base square and the height square. So, therefore, you can relate sine phi and cosine phi which I have written out now in these relations what I have done therefore, is I have replaced sine phi and cosine phi in terms of xE yE and theta.

So, once I have done that these terms P minus Q2 and R2 they are only functions of xE yE and theta P2 Q2 and R2 are functions of xE yE and theta and this relation is between the velocities S4. So, S4 dot xE dot ye dot and phi dot I have eliminated phi the next step is to eliminate phi dot then I would have a relation between xE dot yE dot and S4 dot now how to determine phi dot.

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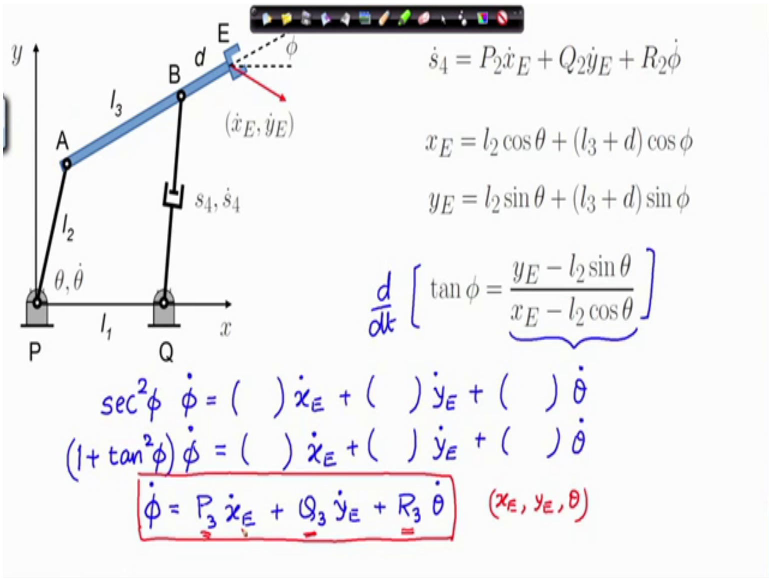

So, here I have rewritten that expression and I have also rewritten the displacement relation and this expression of tan phi. Now, if I differentiate with respect to time I have second square phi into phi dot is equal to the time derivative of the right-hand side of this ratio.

Now, if you think a little bit it will be absolutely clear that the time derivative of this ratio will have a form or can be expressed in the form something times xE dot plus something times yE dot the something times theta dot. Now, this time derivative is straightforward, but there will be a little bit of algebra in this.

Now, second square phi can be again written in terms of tangent phi. So, I have this relation now 10 tan phi is already known in terms of xE yE and theta. So, therefore, finally, I must have let me call it some P3 xE dot plus Q3 yE dot R3 theta dot where remember that P3 Q3 R3 are solely functions of xE yE theta.

So, these are solely functions of xE yE and theta. So, I have found phi dot in terms of xE dot yE dot theta dot xE yE theta.

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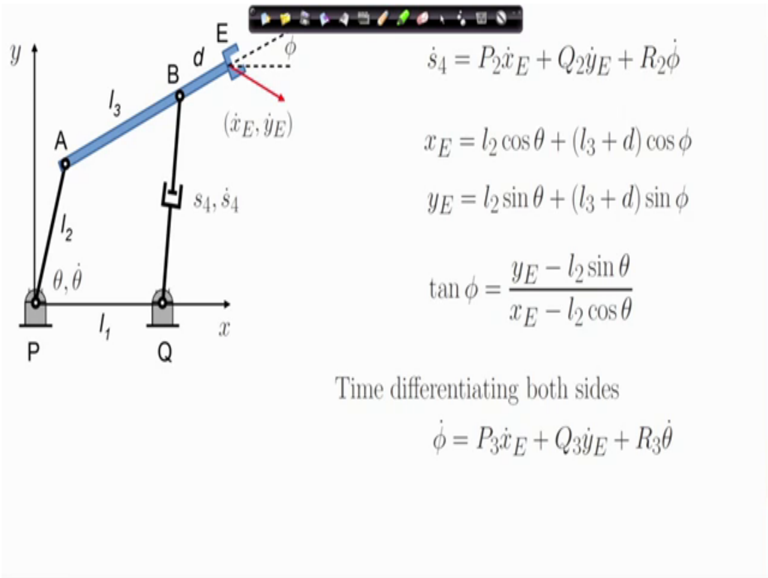

Where this; P3 Q3 R3 are complicated expressions which you can easily find out by a little bit of algebra.

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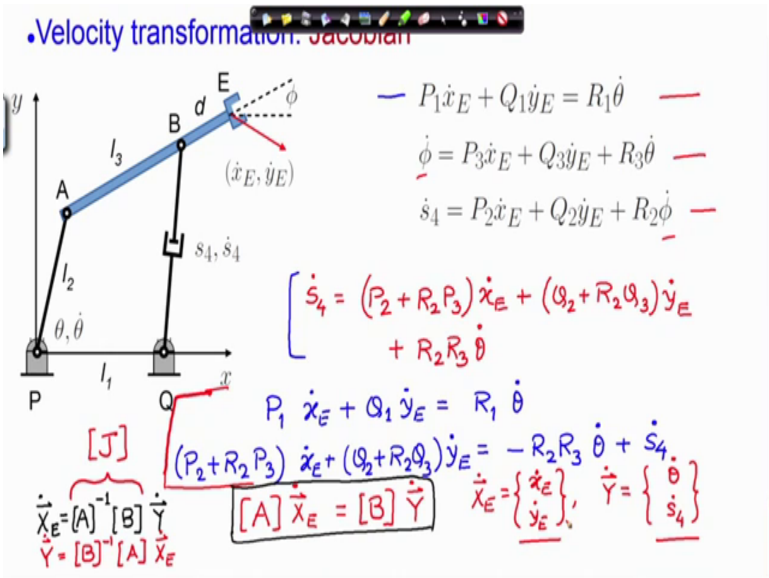

So, here I have collated all the expressions that we had derived we are derived this first and then these 2 expressions were derived.

Now, here I have phi dot which appears in the expression of S4 dot. So, therefore, I can eliminate phi dot in the expression of S4 dot and write this as. So, when I substitute phi dot here I will get this times xE dot plus this terms times yE dot plus R2 RR3 times theta dot.

Now, I can rewrite this as. So, let me write out the first equation here. So, P1 xE dot plus Q1 yE dot is R1 theta dot and now I will write this equation after a little bit of manipulation as that is equal to minus of R2 R3 theta dot plus S4 dot. Now, this I can combine and write in a compact form as let us say some matrix a times xE dot is equal to matrix B times the vector y dot where xE dot of course, this vector xE dot is nothing but xE dot y dot the vector y dot is theta dot S4 dot.

So, the joint velocity vector and the matrices A and B you can very easily read out from these 2 equations. So, this is our velocity relation now it looks a little different than what we had derived earlier, but you can always invert a and rewrite this as such that this is our Jacobean for the manipulator. So, a inverse b is the Jacobean for this manipulator.

If you invert b then you have the inverse Jacobean. So, if you write y dot as B inverse times a xE dot then you have the inverse Jacobean. So, we have finally, determined the velocity relations. So, our joint velocity vector is theta dot S4 dot. So, from here I have related I found a relation to xE dot and yE dot which is the end effector velocity.

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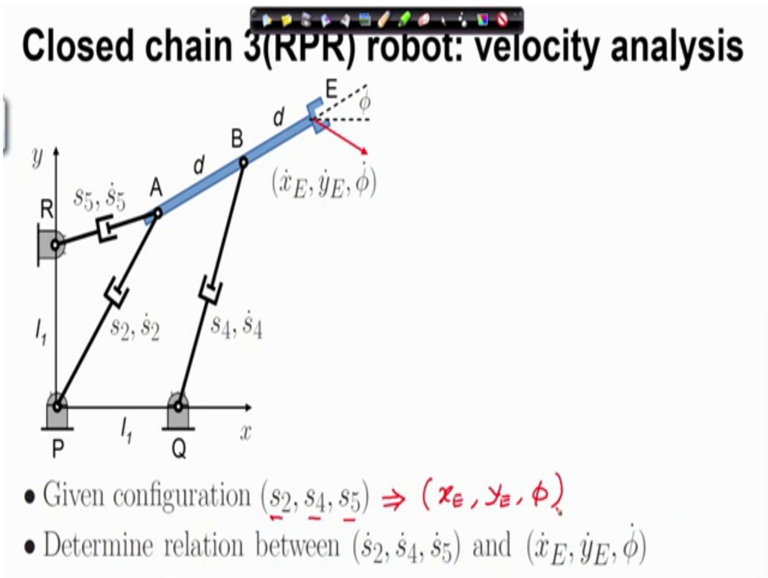

So, let us move forward and discuss the 3 RPR planar closed chain robot manipulator here we need to recall that we had already discussed the displacement analysis the configurations S2 S4 S5. So, given S2 S4 S5 the configuration of the manipulator is specified and we have we had found xE yE and phi.

So, given S2 S4 S5 you can find out xE yE and phi. So, this is a 3 degree of freedom manipulator. So, f 3 inputs S2 S4 and S5 and we can find out sE yE xE ye and phi.

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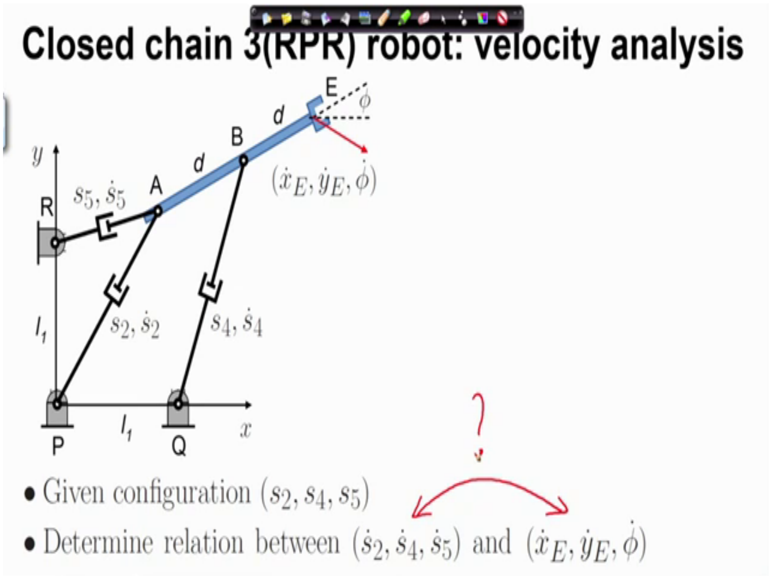

Now, the velocity analysis requires the specification of the relation between S2 dot S4 dot S5 dot and xE dot yE dot and phi dot. So, that is the velocity analysis problem.

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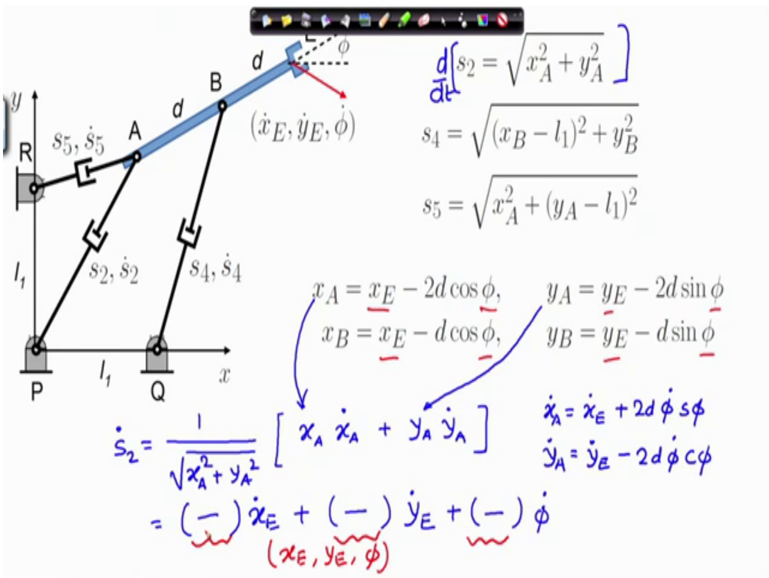

Now, we have derived these relations these are very Easy to see S2 is the throw of this actuator. Similarly, S4 and S5 and they are related in terms of the coordinates of point A and B and the coordinates of P Q and R. So, these relations we have already looked at. So, here you can relate xA yA. So, coordinates of point A and similarly coordinates of point B.

In terms of the coordinates of the end effector and the orientation of the end effector phi so, therefore, you can just substitute them here and start differentiating. So, for example; if you differentiate the first equation which I have written out after some simplification so, this is what you have and now you can replace x A y A in terms of xE yE and phi and you can find out x A dot as xE dot plus 2 d phi dot sine phi.

Similarly, y a dot is yE dot minus 2 d phi dot cosine phi. So, these also you can replace. So, finally, S2 dot will be something times xE dot plus something times yE dot something times phi dot. Now, these bracketed quantities can be easily determined after some algebra you substitute these expressions of xA yA xA dot and yA dot and you can find out this relation.

Now, this bracketed quantities are functions of only xe ye and phi. So, this bracketed quantities are functions of xE yE and phi. So, if you know the configuration of the manipulator these things are already known to you. So, which means this bracketed quantities are known to you in that case I have found a relation between the expansion rate of s 2 which is s 2 dot and the velocity of the end effector which is xE dot yE dot and phi dot.

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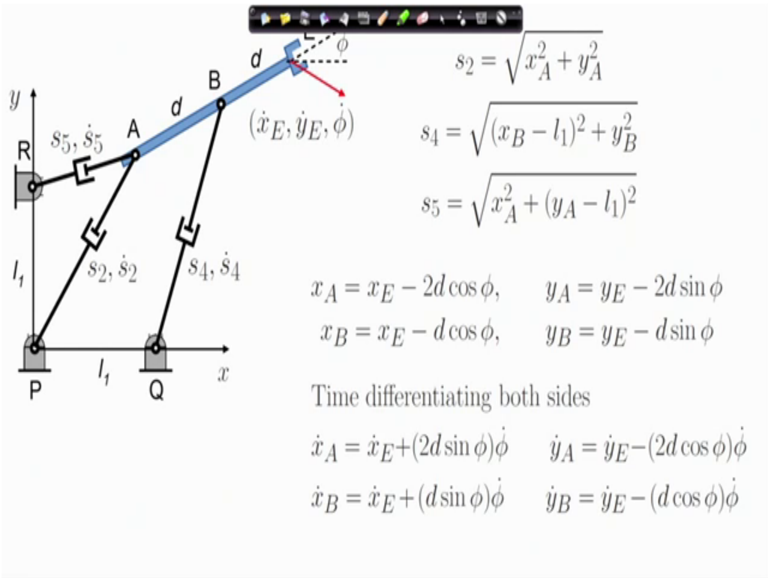

Now, if you continue. So, this is the time derivative.

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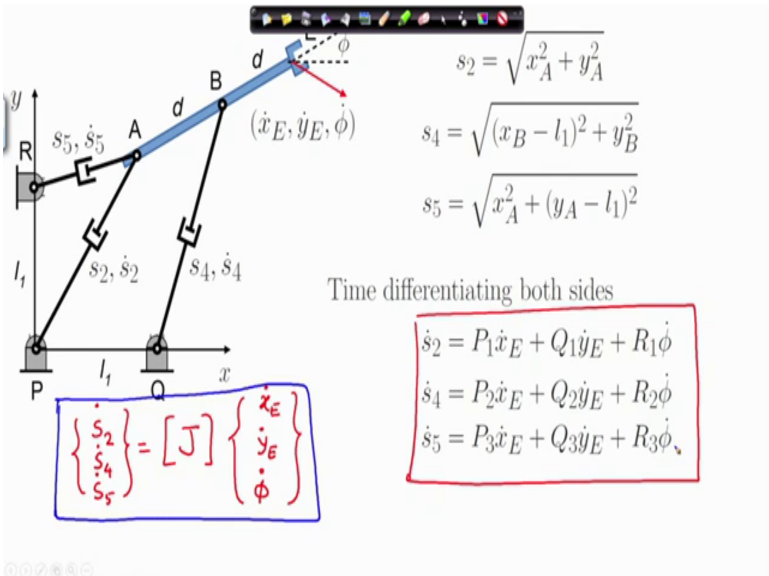

So, if you substitute all these expressions and finally, you obtain these relations between S2 dot S4 dot S5 dot and xE dot yE dot and phi dot. So, which means I have S to dot S4 dot S5 dot is equal to Jacobean matrix times xE dot yE dot and phi dot and that is the velocity relation that we are seeking the Jacobean can be easily read out from these detailed relations.

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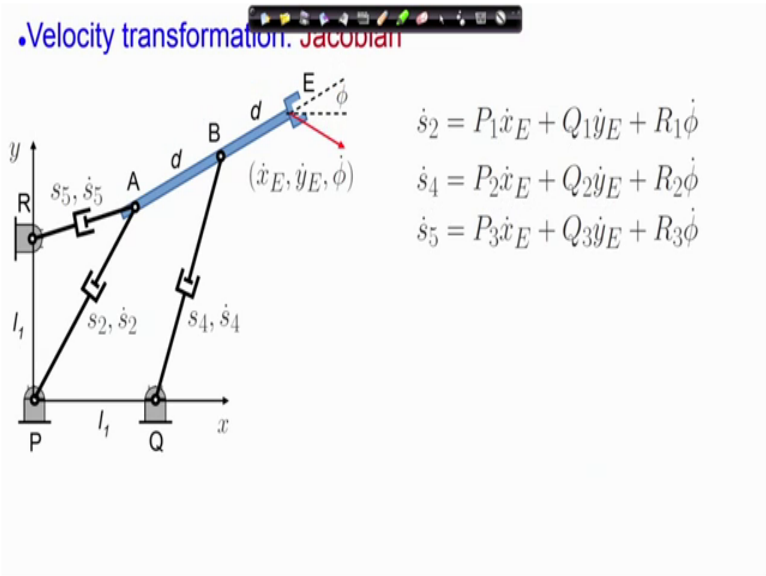

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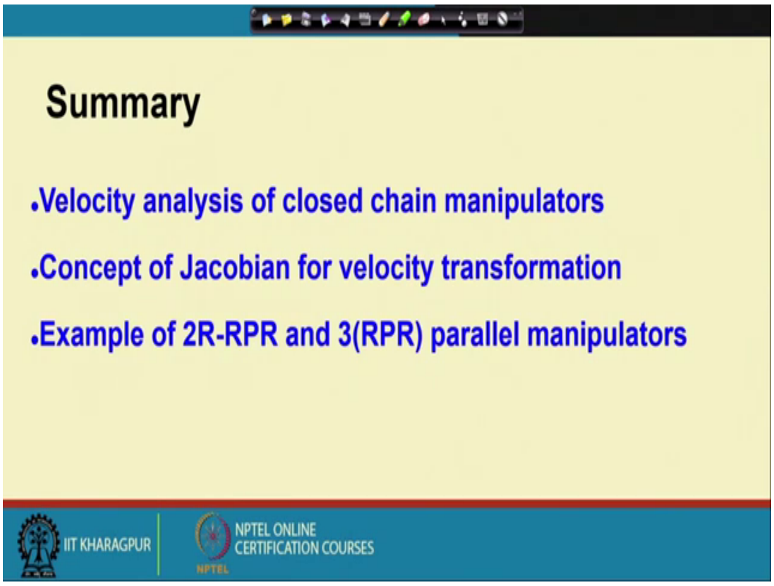

So, to summarize what we have discussed in this lecture we have looked at the closed chain parallel planar manipulators we looked at the velocity analysis problem. We have found the Jacobean of the Jacobean matrix for these manipulators I have outlined methods for determining this Jacobean. We have looked at these 2 examples of 2 R RPR and 3 RP are closed chain planar parallel manipulators.

So, with this we have now discussed the velocity analysis of both open chain and closed chain manipulators now this space wave for discussions on motion planning or path planning a path generation problem for these manipulators. So, this lecture I will close here.