Mechanism and Robot Kinematics
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Lecture – 31
Serial Manipulator Velocity Analysis – III

We have been looking at the velocity relations of manipulators, robot manipulators and we have observed that we can relate the joint velocities to the end effector velocities through the Jacobian matrix. Now this Jacobian matrix, we have found is dependent on the configuration of the manipulator.

Now, why we need to discuss the this velocity relation, is because we have to ultimately generate a path for the end effector. We have to generate a path for the manipulator end effector for performing a certain task. So, the path might be specified and on that path at every point, velocities will have to be specified and this velocity is velocity vector that must be specified should be tangent to the path; this is the condition.

Now, the question is, if I have to produce that velocity vector, I have to produce it through the joint velocities. So, for a given end effector velocity or end effector velocity vector at a certain configuration, I need to determine what should be the joint velocities in order to be able to produce that end effector velocity on that particular point on the on the path.

Now, the point on the path specifies the configuration through inverse kinematics; this we have already seen. So on a path, a certain point where the end effector is presently occupying the that point, we need to solve the configuration which we have already discussed. Then comes the velocity vector along the path which needs to be produced by the corresponding velocities on the joints. And this determination of joint velocities corresponding to a desired end effector velocity is the inverse velocity relation.

We have observed that in the inverse velocity relation, we have the inverse of the Jacobian matrix. We have also commented on this point that the inverse of the JJacobian matrix is not guaranteed at all configurations. So, in this lecture today, we are going to look at first the interpretation of the Jacobian matrix.

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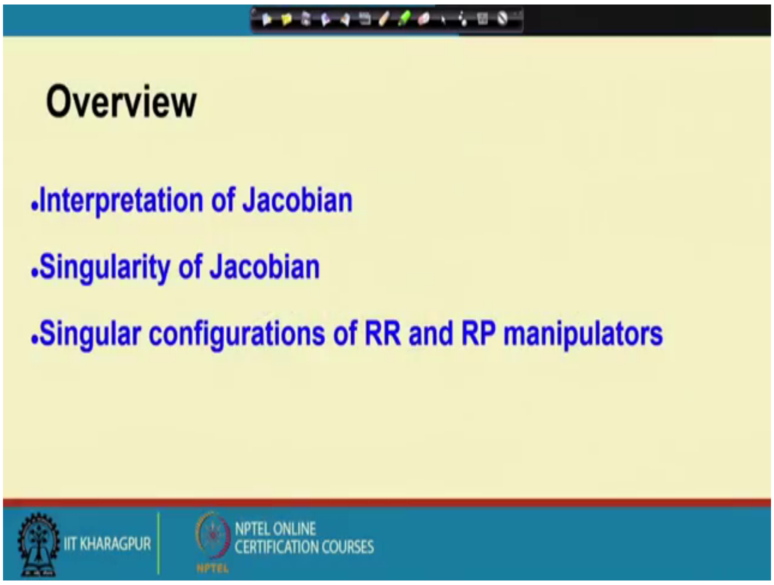

We are going to look at the singularity of the Jacobian matrix with the examples of the RR and RP manipulators that we have discussed.

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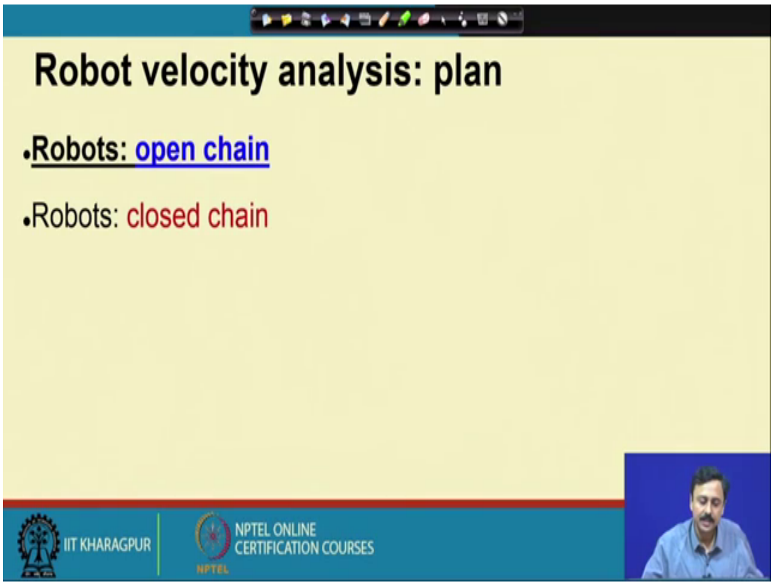

So, we are discussing the open chain planar robots.

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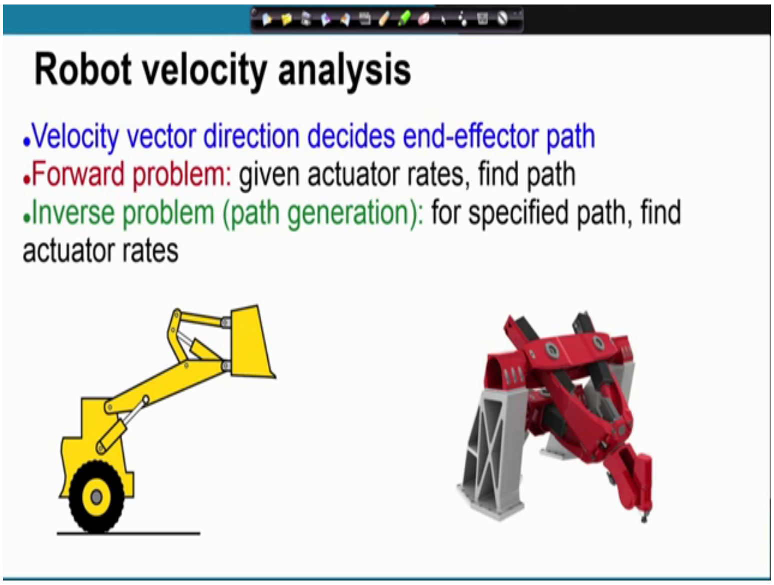

We have already discussed these problems in robot velocity analysis, the forward problem and the inverse problem.

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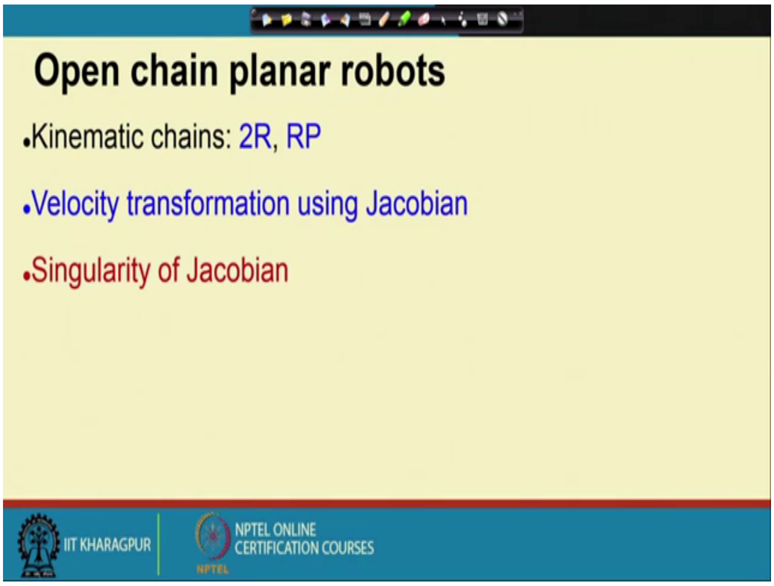

So, in the open chain planar robots, we have discussed 2R and the RP manipulators. We have found, we have determined the velocity transformations using the Jacobian for these manipulators.

In this lecture, we are going to now look at the Singularity of the Jacobian and we are going to look at the consequences of that.

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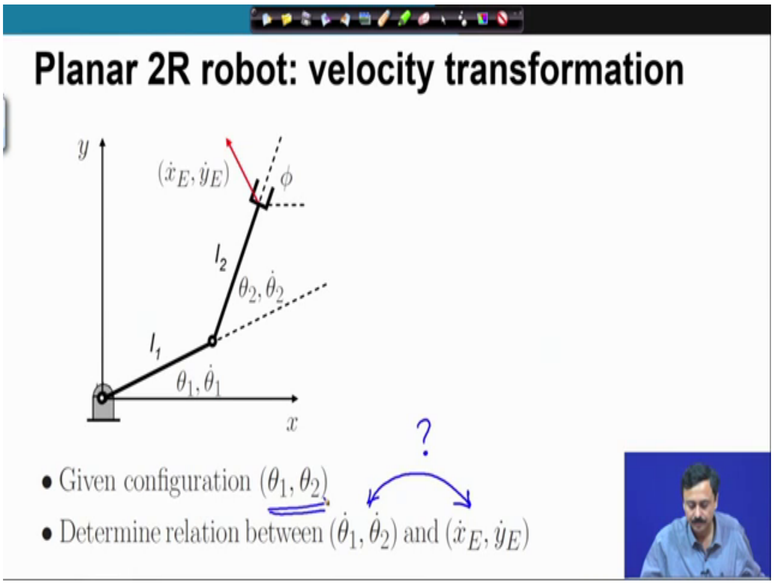

So, just to recapitulate, let us go through the velocity transformation for the 2R manipulator. So, the essential problem was determination of the relations which we have already done, we have already discussed. Theta 1 and theta 2 specify the configuration of the manipulator.

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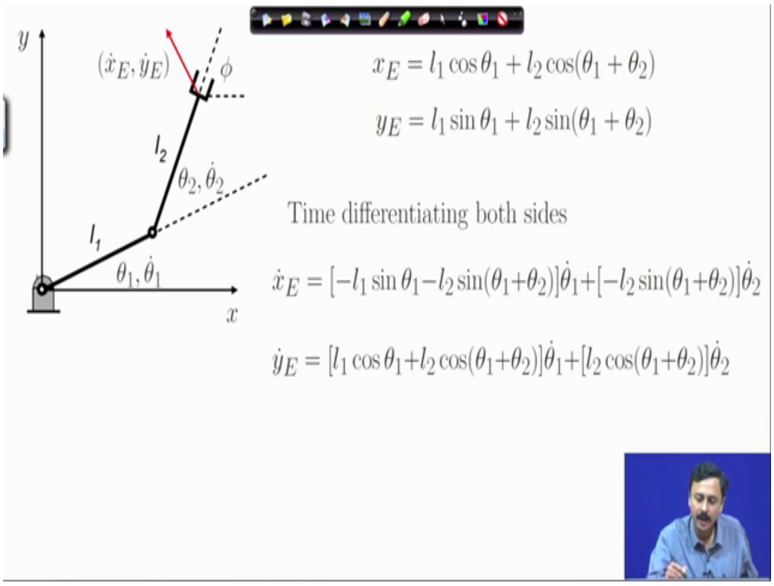

So, we had started off with the forward velo[city] forward displacement relations differentiated with respect to time to obtain the velocity relations in this form.

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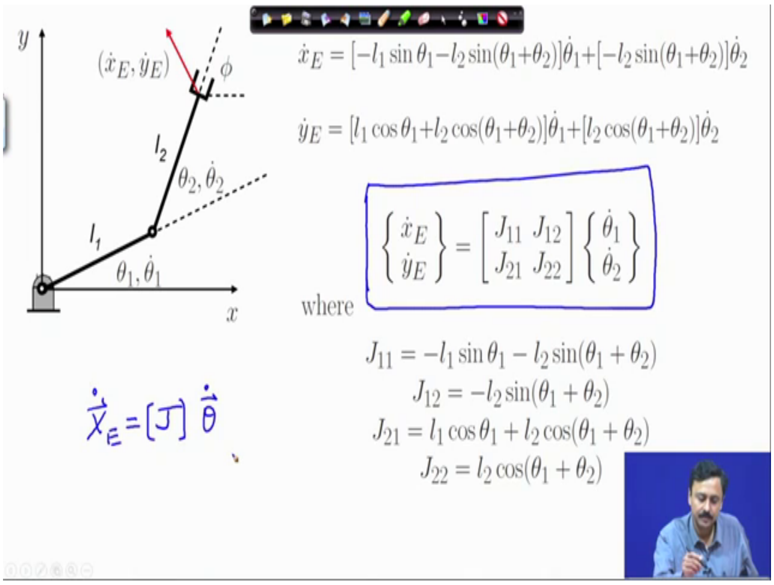

And finally, we had assembled them these velocity relations in the form of this vector relation. So, we have the end effector velocity vector is Jacobian times, Jacobian matrix times the joint velocity vector.

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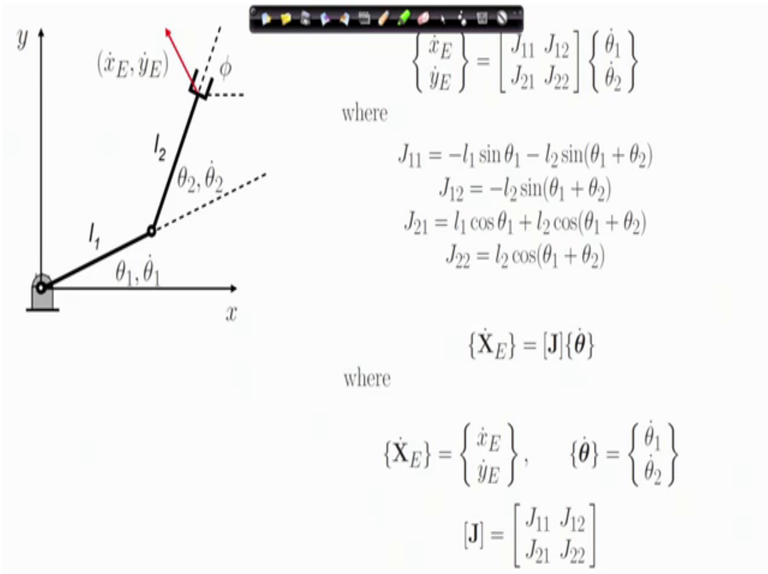

So, this is only a compact way of writing the relation.

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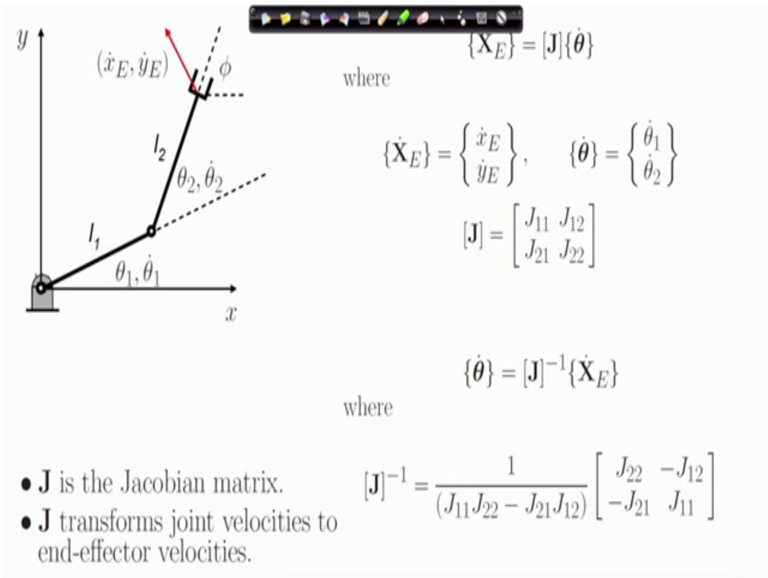

Then we ask the question, how to determine the joint velocities given the end effector velocity vectors, vector. Now this as you know is configuration dependent, the in the Jacobian is configuration dependent. So, inverse is also configuration dependent. In the inverse, we had this determinant sitting in the denominator, determinant of the Jacobian in the denominator and that makes it a little tricky.

So, we must guarantee that this determinant should not vanish. If it vanishes then the Jacobian matrix is singular.

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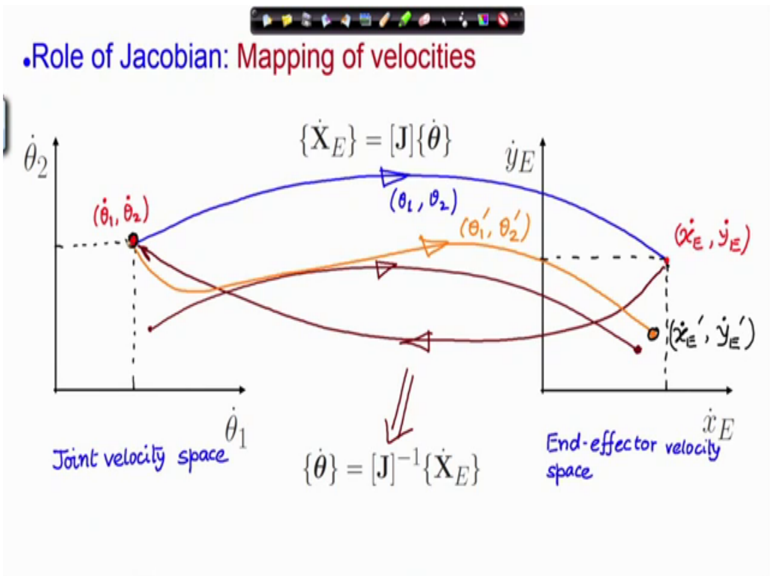

Let us interpret this Jacobian. I have already told you that this Jacobean is something like a proportionality constant, though it is not a constant but it is it is a factor of proportionality. The end effector velocity is proportional to the joint velocity. So, therefore, this end when you come to equality that is through the Jacobian, Jacobian matrix which is configuration dependent.

So, let us interpret the role of Jacobian. So, here I have shown you, on the left we have the Joint velocity space which comprises of theta 1 dot and theta 2 dot. On the right, I have the End-effector velocity space having X E dot and Y E dot on the axis.

Now, this is just for visualization. It is not nothing like that theta 1 dot is orthogonal to theta 2 dot, but this is just for visualization, I have taken the Cartesian frame. So given a point, corresponding to theta 1 dot theta 2 dot. So, given a point in the joint velocity space, we have this mapping which we call the forward velocity relation. We obtain a corresponding point X E dot Y E dot in the End-effector velocity space.

So, this mapping is the forward velocity relation and the reverse is the inverse velocity relation which takes us from the End-effector velocity space to the joint velocity space. So, that is the inverse relations. Now for every point in the joint velocity space, there is a corresponding point in the End-effector velocity space. Now this remember these relations are configuration dependent; for example, this relation might be valid, remember that configuration of a of a 2R manipulator is specified by theta 1, theta 2.

Now, if I change theta 1, theta 2 to something like theta 1 prime and theta 2 prime, but keep these points as it is. I can have another velocity relation. So, I am keeping this point fixed. I am keeping the joint velocity point fixed. I have only changed the configuration of the manipulator. So, this is for theta 1 prime, theta 2 prime and what I have is a new point in the End-effector velocity space.

So, x e prime, x e dot prime and y e dot prime though my starting point; was the same. So, this forward velocity relation and of course the inverse velocity relation as well, they are dependent on the configuration. Now we are going to look at what happens at a Singularity. Now at a Singularity, as I have mentioned this does not exist. So, Jacobian inverse does not exist; that means, the determinant of the Jacobian has actually vanished.

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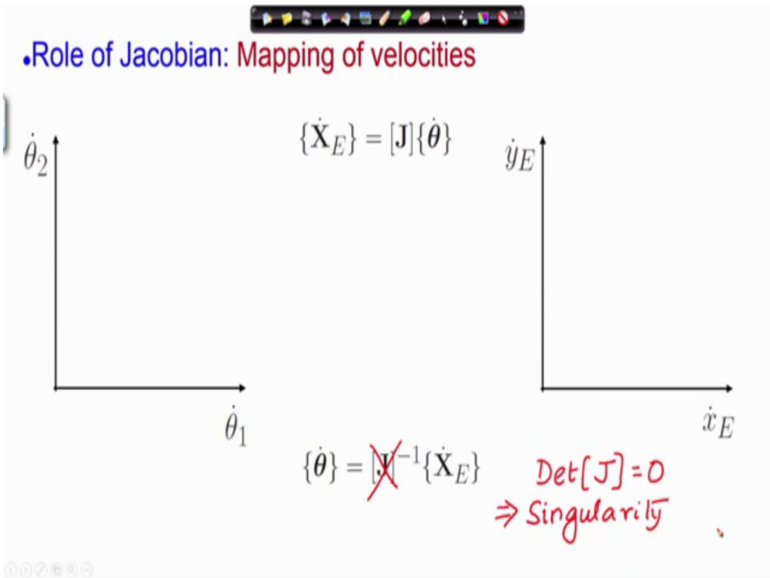

So, this implies Singularity, Singularity of the Jacobian and we also say that is the Singularity of the configuration of the manipulator. So, we do not have the inverse relation. So, what do we observe?

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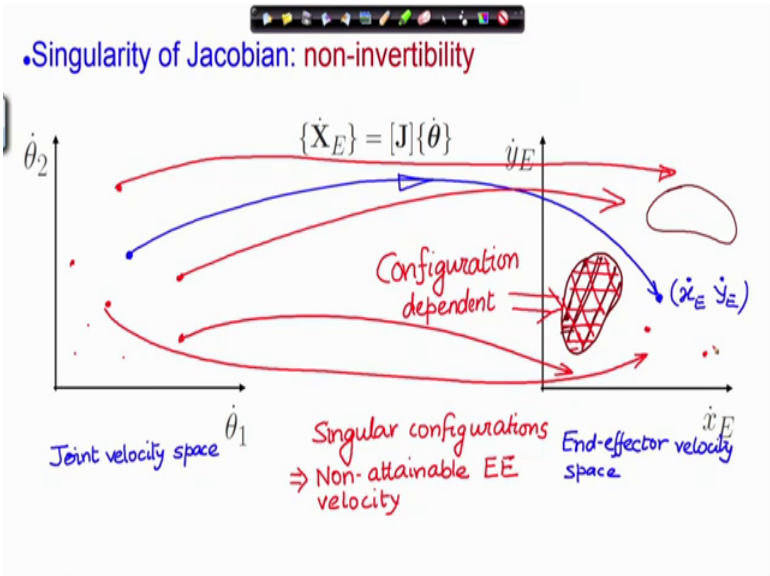

So, given a point here in the joint velocity, so given a point here in the joint velocity space, am I right I have the End-effector velocity space. Given a point in the joint velocity space and a configuration, I arrived at a point in the End-effector velocity space.

But if you now ask, given this point in the end-effector velocity space, what is the corresponding point in the joint velocity space, that question now cannot be answered; which means there is no point in the Joint velocity space for which this velocity is produced at the end-effector. So, this is the effect of non-invertibility. I cannot find a corresponding joint velocity to produce that end-effector velocity, that end effector velocity cannot be produced.

So, which means that whatever you do, there will be certain regions, they may be connected or disconnected that depends. So, in general there will be regions in the end-effector space which cannot be mapped on to the. So, there are regions in the end-effector velocity space which cannot be mapped on to the Joint velocity space. So, there is no point in the Joint velocity that goes into these regions.

So, if you take arbitrary points in the joint velocity, they will go to different points, but they will avoid these regions. These regions cannot be these velocity in these regions cannot be produced. I can choose any point in the Joint velocity space. There is one more very important point associated with these regions, these regions are Configuration dependent, this needs to be remembered. So, these are.

In certain configurations, such regions do not exist; in certain configurations, they do. So, the existence of this region depends on the configuration. So, Singularity is only at those configurations. So, under very special configurations known as Singular configurations, we have these regions. So, we have these. So, only at Singular configurations, we have regions of non-attainability of end-effector velocity.

So, we have non-attainable end-effector velocity. So, these regions, which cannot be with which correspond to velocities which cannot be attained is configuration dependent; they exist only when only for singular configurations. For non singular configurations, everything is achievable.

So, for non-singular configurations, you can have arbitrary velocity, you can produce arbitrary velocity of the end-effector.

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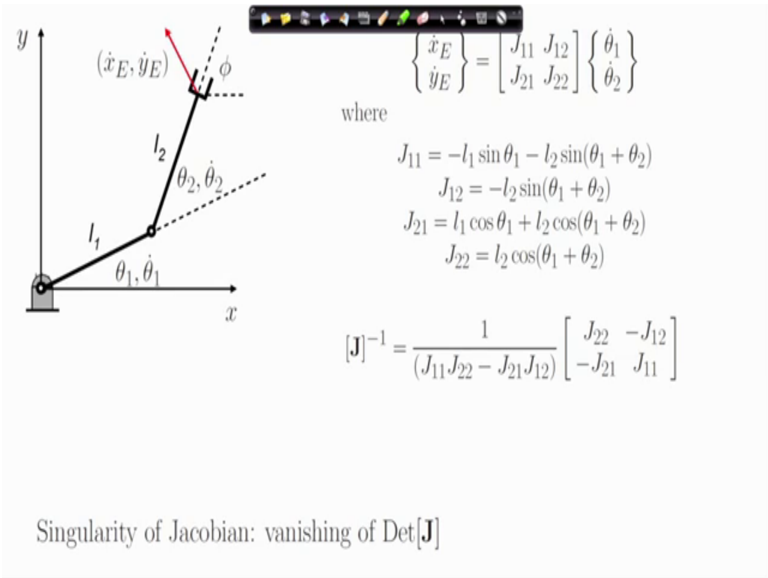

But at singular configurations, we cannot produce certain velocities of the end-effector. So, let us look at these configurations. So, if you take the determinant of the Jacobian. So, this is the inverse as I mentioned that singularity of Jacobian is the vanishing of the determinant.

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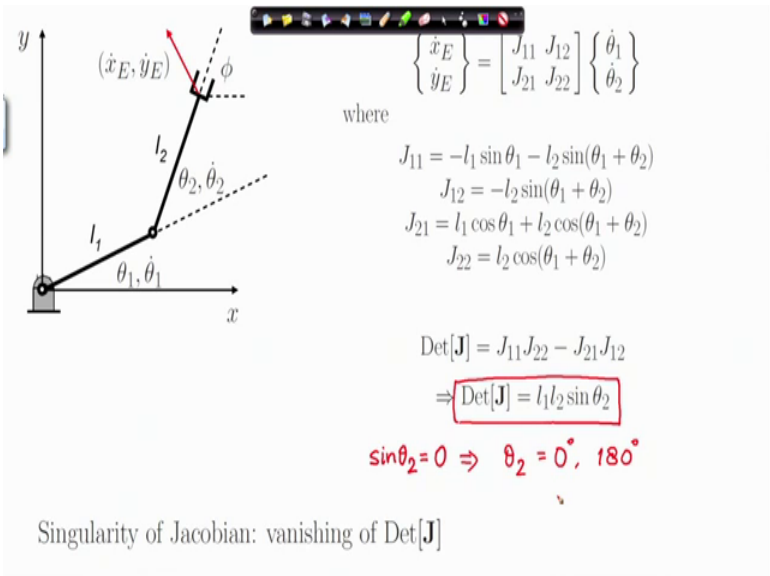

Now, if you take the determinant of this Jacobian for the 2R manipulator, we have this as the determinant. Now at which point will this determinant vanish, when sin theta 2 is 0 because l 1 and l 2 cannot be 0. So, that implies theta 2 is either 0 degree or 180 degree.

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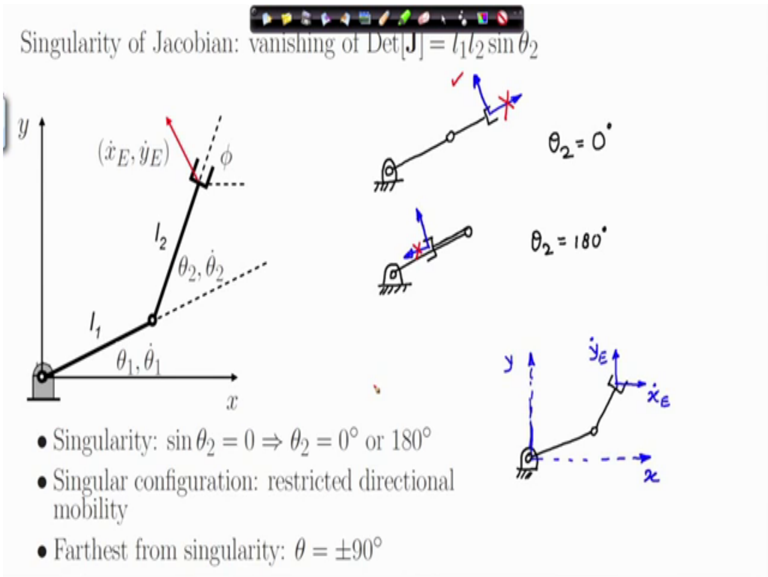

So, Singularity is when you have sine theta 2 equal to 0. So, either theta 2 is 0 degree or 180 degree. So, let us look at these configurations. So, here you have theta 2 equal to 0 degree. This is a singular configuration. Now which end effector velocity cannot be produced? You can produce; you can very easily see this that, you can produce a velocity in this direction.

But you cannot produce velocity in the radial direction. You cannot produce that velocity. So, the velocity that can be produced is the tangential velocity. Now this is only at this particular configuration. If the configuration is somewhat different, now you can produce all velocities, you can produce both these velocities.

So, these were our coordinates. So, you can produce a general velocity vector. But once you are in a singular configuration like this, you cannot produce a velocity in the radial direction. As I have shown you, there is another configuration corresponding to theta equal to pi. So, this is theta 2 equal to 180 degree pi radial.

Once again, you can produce a velocity in this direction, but you cannot produce velocity in the radial direction. So, that is the effect of Singularity.

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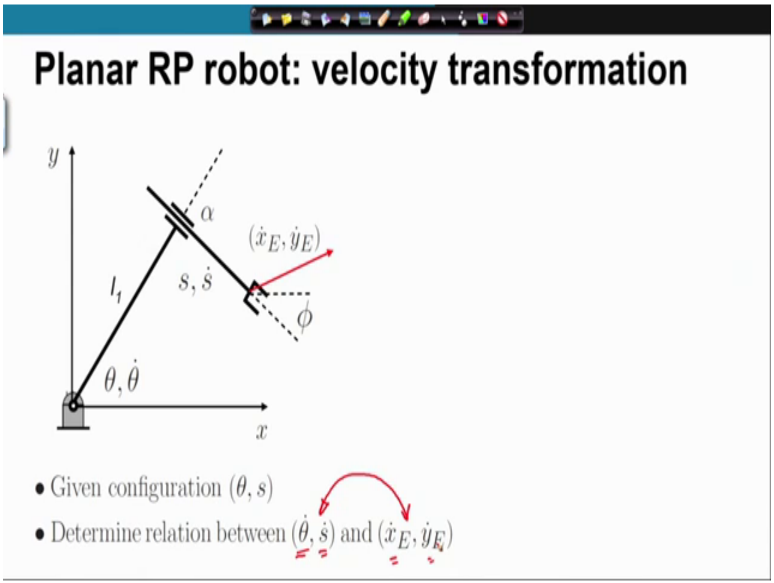

Now, we move on to the RP robot manipulator. Here our velocity relation was between the joint velocities theta dot and s dot and x E dot and y E dot

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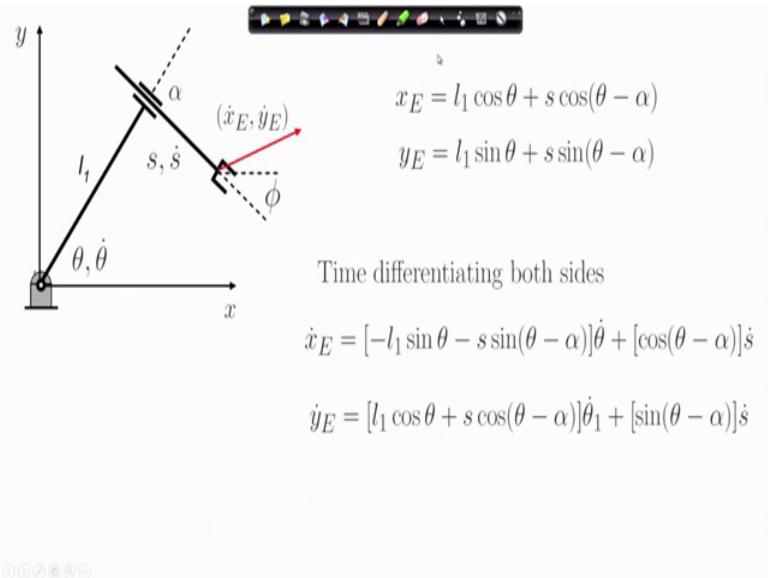

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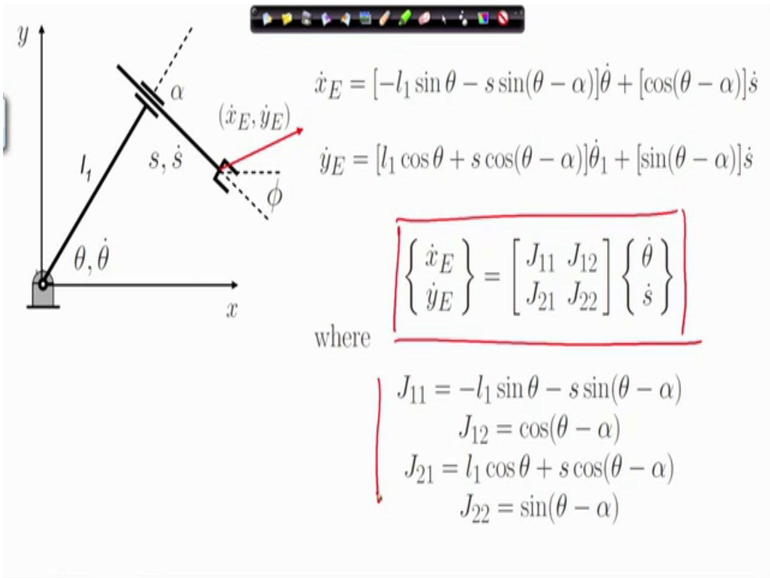

So, just to give you a review of the previous discussions, we had these velocity relations of this form where this J 11, J 12, J 21, J 22 which are the Jacobian elements.

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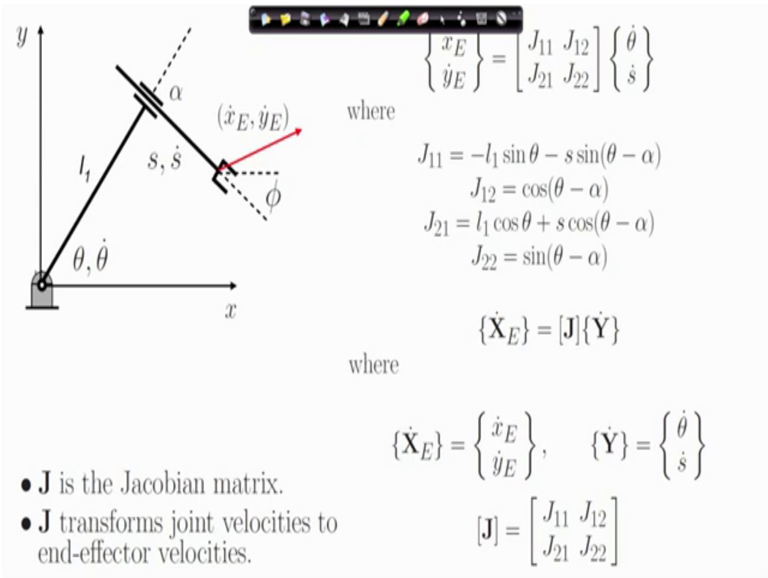

So, they are given here. So, in the compact form, we had written out the velocity relations. Here we have the Jacobian matrix which transforms the joint velocities to end effector velocities.

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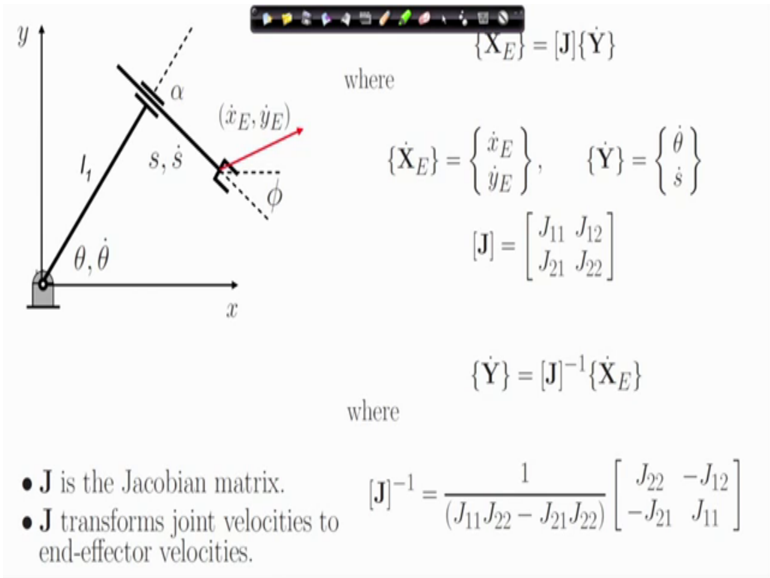

And to find out the joint velocities corresponding to a desired end effector velocity, we have this inverse relation.

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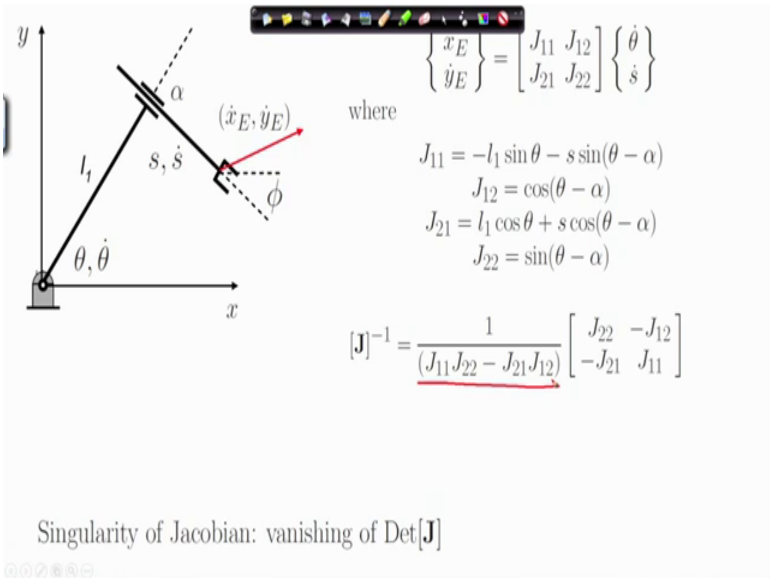

And we are discussing the existence of the inverse of the Jacobian. So, the Jacobian inverse exists if the determinant of the Jacobian is not equal to 0 because the determinant is sitting in the denominator.

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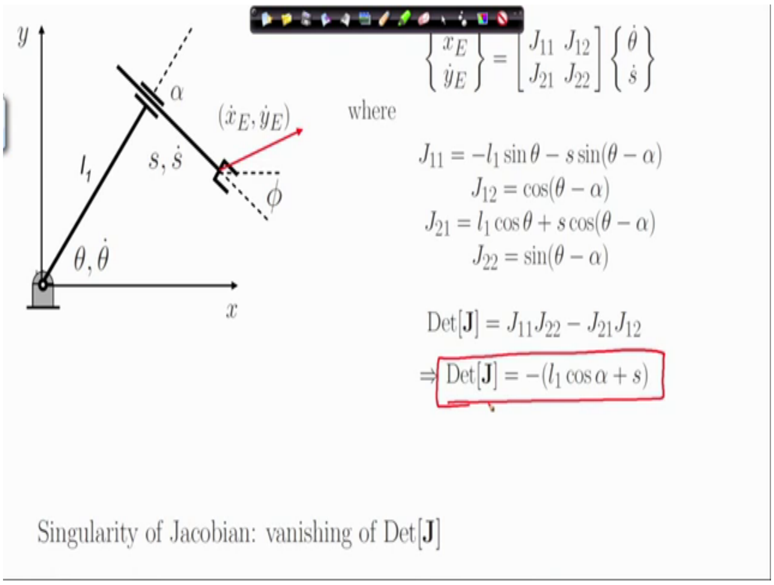

And if you look at the determinant of this manipulator, determine the Jacobian of this manipulator, it is given by this relation l 1 cosine alpha plus s the negative of it.

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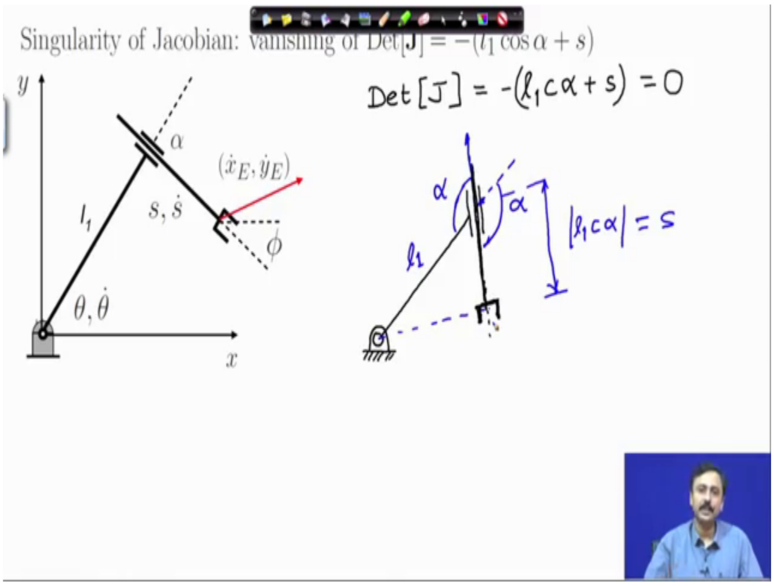

Now what happens if this Jacobian vanishes or where do you have singularity. Now to understand that, let me draw the manipulator. So, what we are trying to determine is the configuration where the determinant vanishes. Now if you look at this configuration, so I have the first link drawn and the dashed path is the path of the end-effector as you change s.

Now, the question is, where is this determinant vanishing; for what value of s. You can see that in the determinant. You have only s l 1 and alpha are constants. So, this angle is alpha, this is l 1. If this is alpha then this is also alpha. So, l 1 cosine alpha is the projection of l 1 in this direction.

Now, because here the way I have shown alpha is greater than pi by 2. So, cosine alpha is negative. So, this has negative projection and that is this, this is the projection. This is the magnitude of it and that must be equal to s which means that if my manipulator is such that the end-effector comes here, then we have Singularity. So, this is a singular configuration where s is equal to l 1, negative of l 1 cosine alpha or the magnitude of l 1 cosine alpha.

Remember that l 1 cosine alpha is negative and s is positive and they are equal in magnitude. So, l 1 cosine alpha plus s must be 0. So, this is the singular configuration. Now let me just summarize a few things what we have discussed in this lecture. We have looked at the Jacobian. We have interpreted the role of Jacobian in velocity transformations. We have studied the singularity when look at the singularity problem of the Jacobian.

We have looked at the singular configurations of the RR and RP manipulator. It is something interesting in RR manipulator, we found that the singularity is where theta 2 equal to 0 and theta 2 equal to 180 degree. Now if I consider my arm as an RR manipulator. So, so this is one R and this is the other R. The revolute pair here and a revolute pair here suppose.

So, I am moving my arm in only a plane. Then as we have shown that singularity means this angle goes to either 0 or 2 pi. So, if it goes to 0 like this; that means, completely extended arm that is singular configuration and when I fold it completely, when this angle is 180 degree, then also this is singular configuration. I also, I had written out that where the Jacobian is farthest from singularity and that was at 90 degree.

So, this is the configuration where it is farthest from singularity because this is singularity, this is singularity, this is farthest from singularity. If you, if you realize, we always when we write we always have this elbow at 90 degree almost. Why, because this manipulability this determinant of the Jacobian defines or quantifies a quantity known as manipulative of a manipulator.

So, how good is the manipulator n moving in different directions. Now at singular configurations; that means, 2 arm fully extended the determinant is 0, completely folded determinant is 0 and it is farthest from 0 at this configuration 90 degree. It is plus minus 90 degree.

So, here I can produce arbitrary velocities very easily whereas to produce velocity in this direction, it was very difficult in the radial direction, similarly in the external position to produce well, this radial velocity is very difficult. So, we usually do not write like this or do not write like this; we write like this most of the times. So, this is because our manipulability is very good at configurations farthest from singularity.

So, these are some of the conclusions that we can draw based on the Jacobian and it is determinants. So, remember that the determinant of the Jacobian quantifies how good the manipulator or how good the configuration of the manipulator is for manipulation purposes. So, that defines something called manipulability of the configuration or of the manipulator at that configuration.

So, with that I will close this lecture.