Mechanism and Robot Kinematics
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Lecture – 30
Serial Manipulator Velocity Analysis – II

We are looking at the velocity analysis problem of robot manipulator. We are going to continue this discussion in this lecture.

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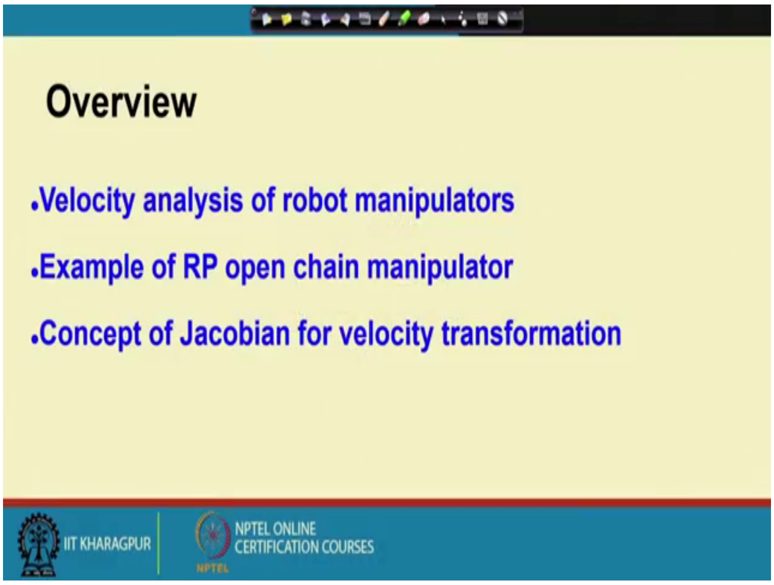

So, let me give you an overview of today's lecture. So, we are going to discuss the RP open chain in our manipulator. So, RP chain has one revolute and one prismatic pair. We are going to derive, the Jacobian relation or velocity transformation relation for the RP open chain planer manipulator.

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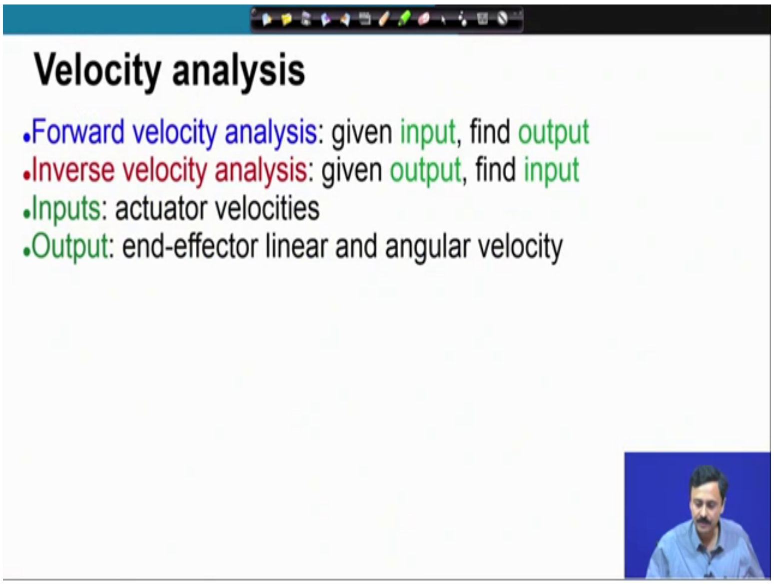

So, as we have looked at the forward and inverse velocity analysis. So, in the forward analysis, we have given the actuator velocity then, we have to find out the end effector velocities and it is just the converse for the inverse velocity analysis.

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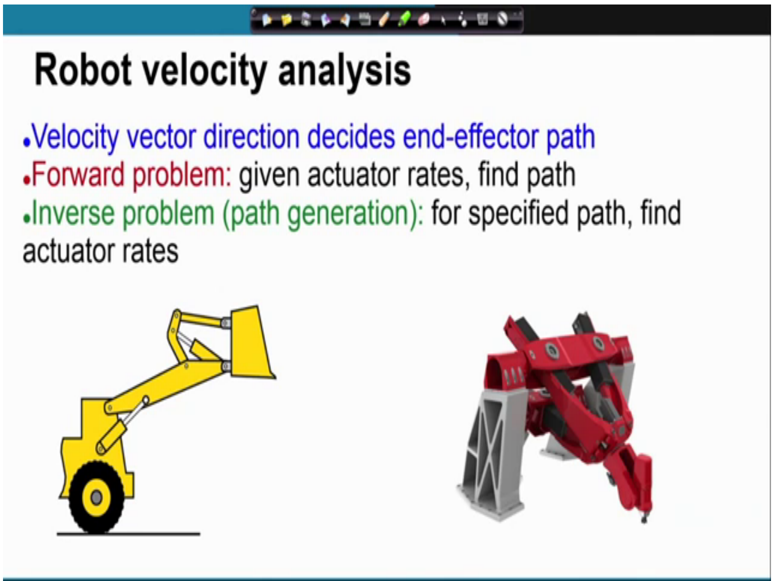

In the case of robots, as we have looked at before, we have these problems a little more involved because here, the velocity vector also decides the path. Or you can also look in the reverse way, given a path, the velocity direction at any point on the path is specified.

So therefore, the velocity analysis becomes a little more extended than in the case of a constraint mechanism. What is very important in the case of robot manipulators is, this inverse problem because, given an end effector path, it tells us how to move the joints. So that, the path is generated. So, in that problem, in the path generation problem, we find out velocities or we specify the velocities at each point on the path and the velocity vector is tangent to the path. And, if I know how to find out the corresponding joint velocities, then I can follow that path or generate that path.

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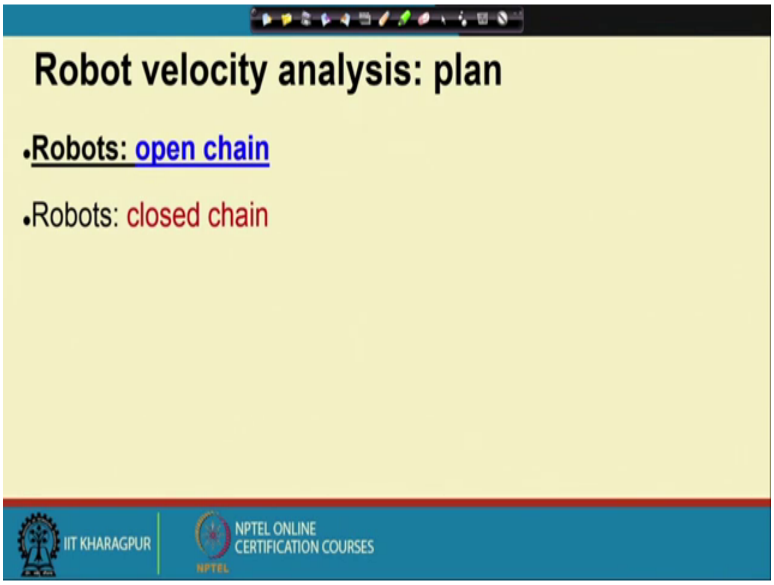

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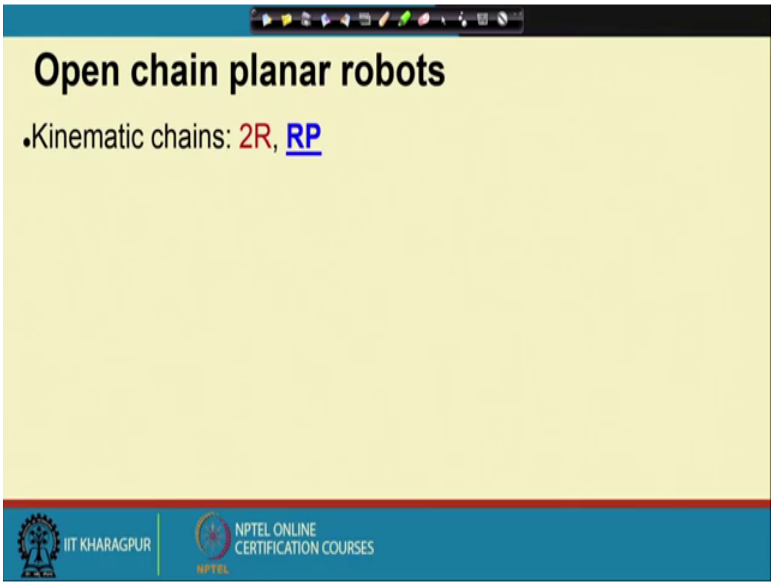

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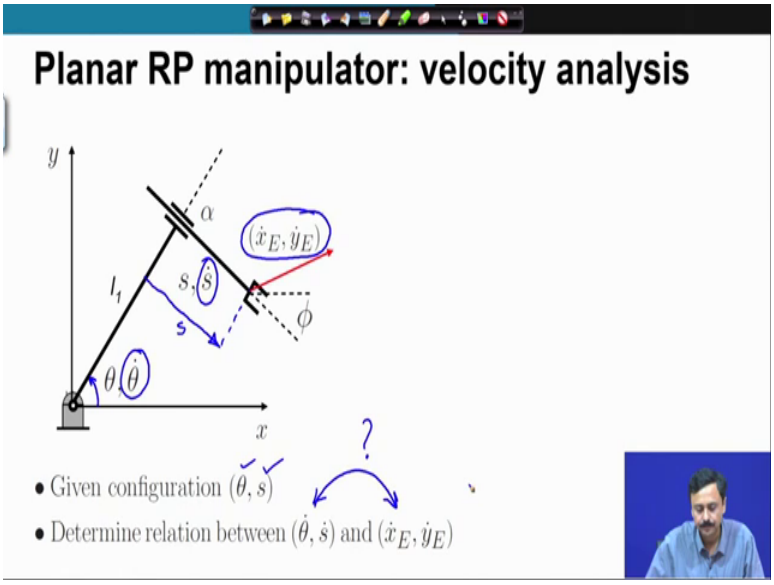

We are discussing, open chain robot manipulators, subsequently, we will look at closed chain as well. So, in this lecture, we are going to look at the RP kinematic chain. So, this is our schematic of the chain, of the RP manipulator. Here, we have these as the joint variables; one is this angle theta and the other, is the extension of the prismatic pair which is, S.

So, this is s and this angle is, theta. So, when a configuration is given, we have specified theta and s. So, that specifies the configuration of the manipulator. Now, what we have to determine is, the relation between theta dot and s dot on one hand and x E dot and y E dot, on the other. So, we have to find this relation, which will give us the velocity relations from the actuators. These are the inputs to the end effector, which is the output.

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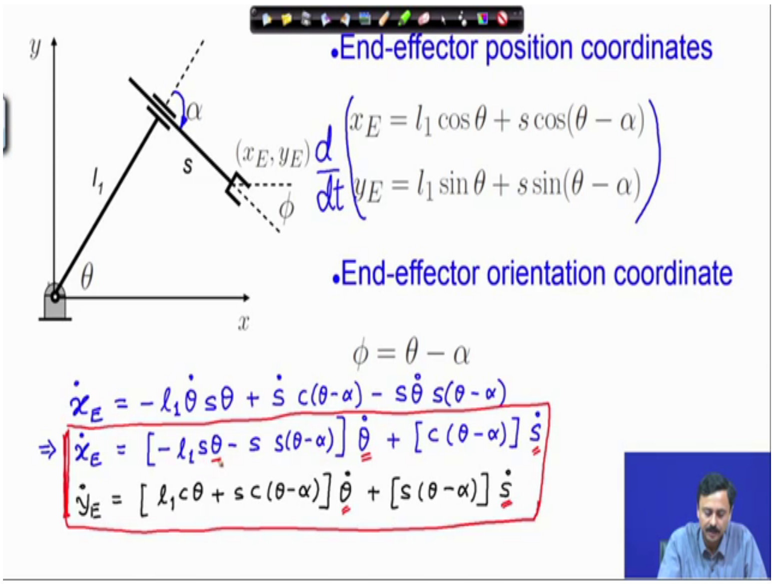

So, you start with the forward velocity displacement relations, forward kinematics relations of the manipulator, which we have derived earlier. And, this is the orientation coordinate of the end effector; where alpha, is this angle. Remember that, this angle alpha, is measured in the clockwise sense, which we have considered to be negative. And, that is how we also had obtained the forward displacement relations. Now, if you differentiate the forward displacement relations with respect to time, so these are the relations.

So, if you take time derivative of these relations, then you obtain x E dot is equal to minus l 1 theta dot sin theta, I am writing s theta in case of sin theta, plus s dot cosine of theta minus alpha and minus s theta dot sin of theta minus alpha. So, this, I can simplify further, by collecting the terms containing theta dot. So that will be, minus l one sin theta minus s sin of theta minus alpha theta dot plus cosine of theta minus alpha s dot. So, that is what I obtained from the first displacement relation. From the second displacement relation, by differentiating the second displacement relation, I have y E dot is equal to l 1 cosine theta plus s times cosine of theta minus alpha times theta dot plus sine of theta minus alpha times s dot. So, these are my velocity relations. As you can see again, that these velocity relations are linear, in terms of the joint velocities. So joint velocities involved, theta dot and s dot and the end effector velocities, x E dot and I E dot. And these relations, involve the configuration, as you can see, it involves theta and s. So, you have, theta and s sitting in these relations. Thus, the velocity relations depend on, the configuration of the manipulator; but in the velocities, they are linear.

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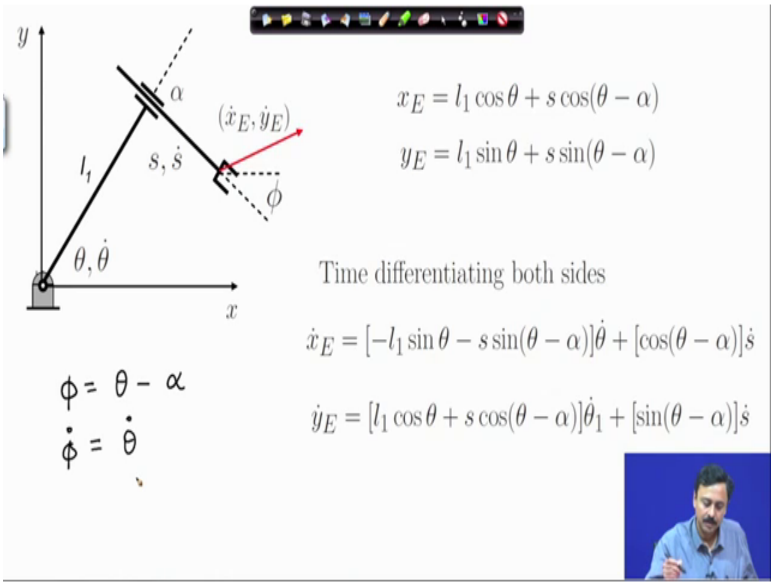

So, let us proceed. So here, I have done this formally. So, I have related the joint velocities and the end effector velocity, as I have derived. There is one more relation that is, the orientation rate. If you remember that, this phi is nothing but theta minus alpha. Therefore, if you take a time derivative of this relation, since alpha is a constant, so phi dot is nothing but theta dot. So, rate of change of orientation is same as the rate of change of the angle theta.

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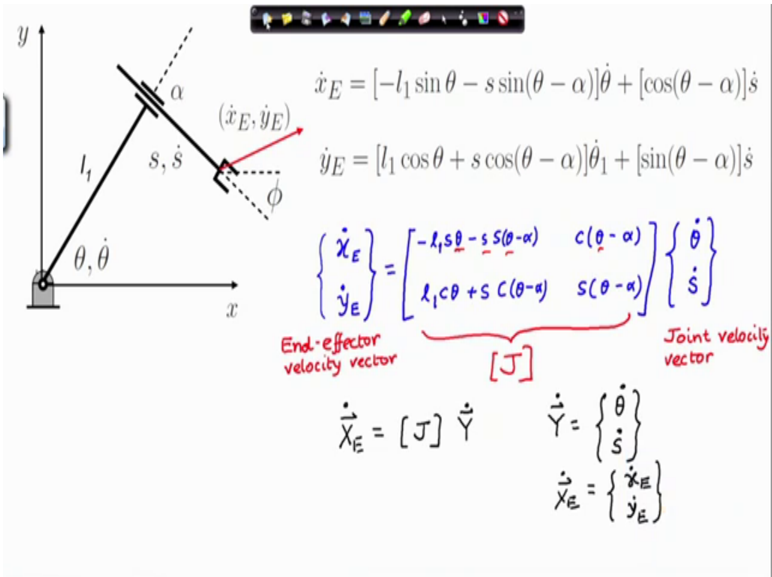

Now, I can assemble these velocity relations in the form of this matrix. So, it is the x velocity of the end effector; it is the y velocity. So, this is the velocity vector of the end effector and that is equal to, I can write this out in the form of a matrix.

So, the right-hand side I have written as a product of a matrix times the vector; now, this vector is nothing but the joint velocity vector. And on the left-hand side, I have the end effector velocity vector and here, I have the Jacobian matrix. So, the end effector velocity vector is proportional to the joint velocity vector and the proportionality is through the Jacobian. The Jacobian is a function of the configuration, as you can see, it depends on theta and s. So, you have, theta s sitting in this Jacobian. So, we write this as the end effector velocity vector is, Jacobian times the joint velocity vector. So here, I define Y dot as, theta dot s dot and of course, x E dot is the end effector velocity.

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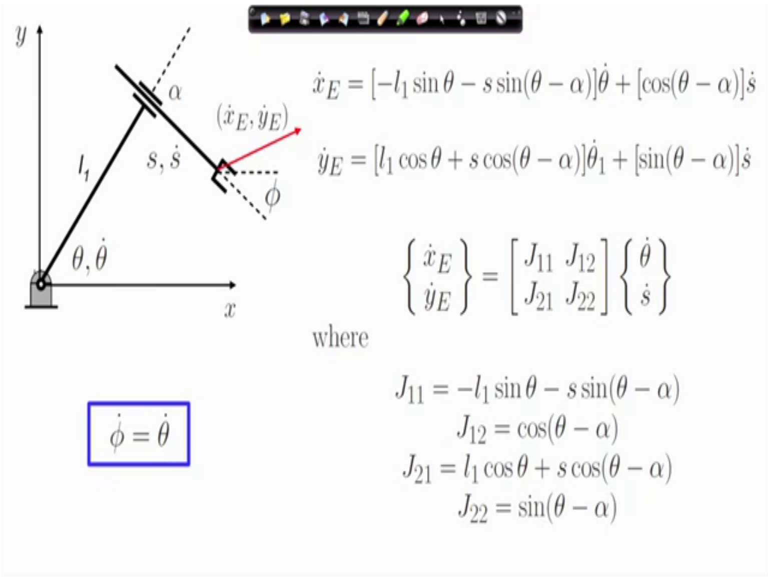

So here, I have written out for you, these relations that I have just now shown. So, given the joint velocities, theta dot and s dot, I can determine the end effector velocities of course, the configuration is assumed to be known; so, the Jacobian is known. Now, the inverse problem. So here, I have the orientation rate, which I have derived already.

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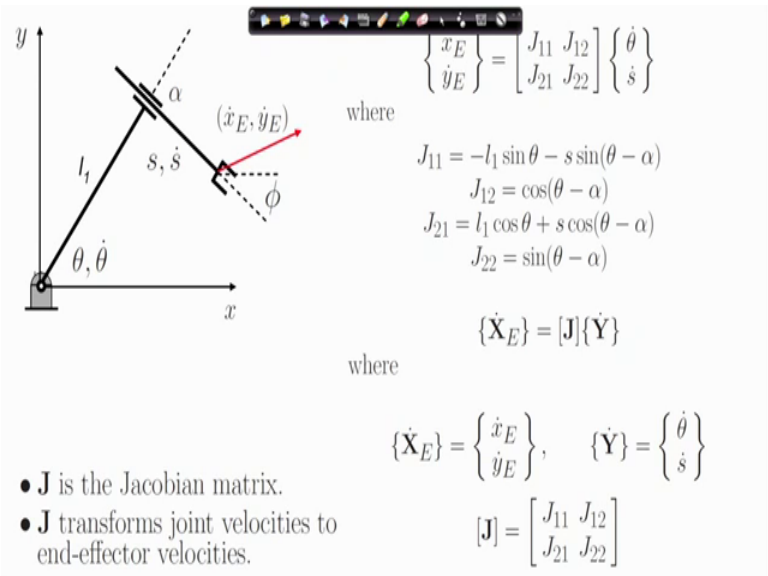

So, this is a compact way of writing the velocity relations that I have just shown you. So here, we have the Jacobian as a matrix, it is a 2 cross 2 matrix. Again, the RP manipulator is a 2 degree of freedom manipulator so this has got a 2 cross 2 Jacobian matrix, which transforms the joint velocities to the end effector velocities. So, we can say that the end effector velocity vector is, proportional to the joint velocity vector and the proportionality is through the Jacobian.

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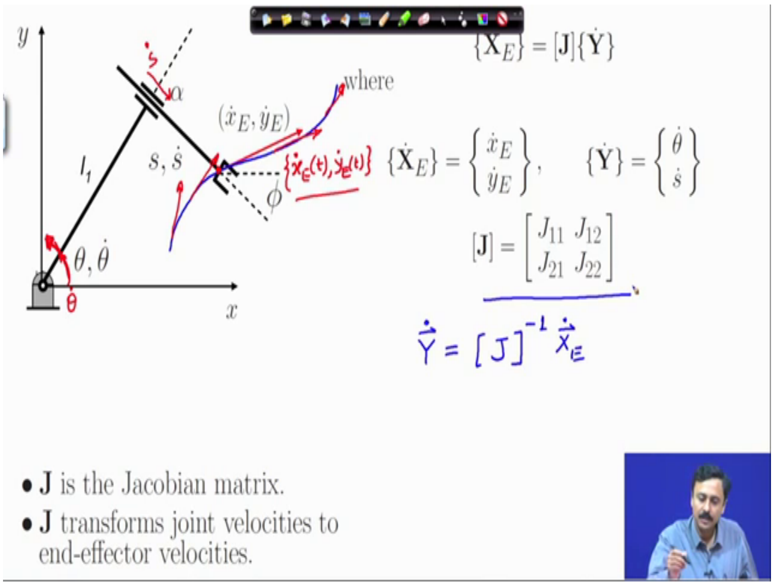

Now, if I ask the inverse problem. So, given the end effector velocity, suppose I want to follow a certain path and I have a velocity specification at every point on the path. So, I know at each instant of time x E dot is a function of time, y E dot as a function of time; so, I know this. So, at this instant, where the end effector is, at this point, x E y E I have this specified velocity vector. Now, how do I find out the joint velocities; so, theta dot and s dot. So, how do I find out theta dot and s dot? So, that will be, obviously, through the inverse relation. So, what I need to do is, I need to find out y E dot in terms of x E dot, which is through the inverse Jacobian relation. So, we need to invert this Jacobian matrix.

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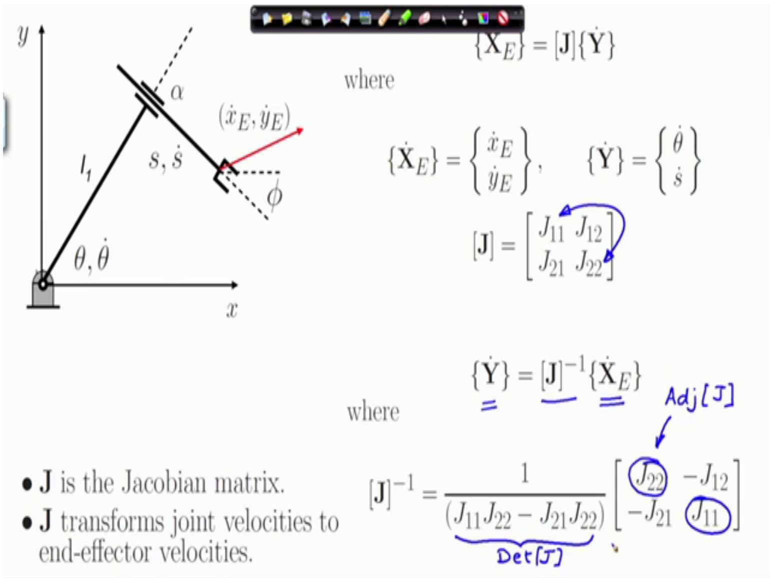

We have already seen how to invert and I will just repeat this here. This is the determinant of the Jacobian matrix and this is the adjoint of the Jacobian matrix. And, the way we find out this adjoint is, we interchange the diagonal terms; that is what I have done here. So here, we have J 2 2 and here we have J 1 1 and we reverse the signs of the diagonal terms.

So, for 2 cross 2 matrices, inversion is very simple. So, this is the adjoint and divided by the determinant. So, once I have the inverse, I can then find out the joint velocities for a specified end effector velocity. There is only one question that remains, whether this inverse of the Jacobian exists. Now, for that to exist this must be non-zero.

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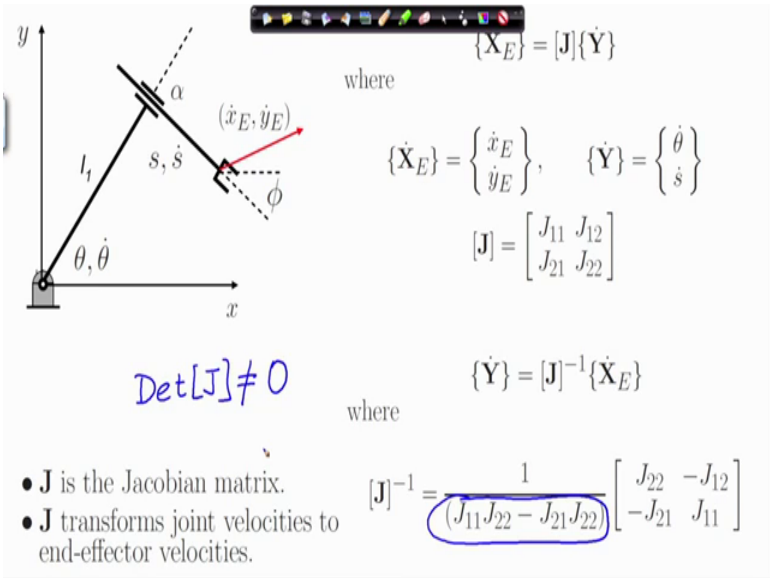

So, the condition for existence of the inverse of the Jacobian is that determinant of the Jacobian should not be 0. If determinant of the Jacobian is 0, then we have a problem. This problem is known as, singularity and we are going to discuss this in a subsequent lecture.

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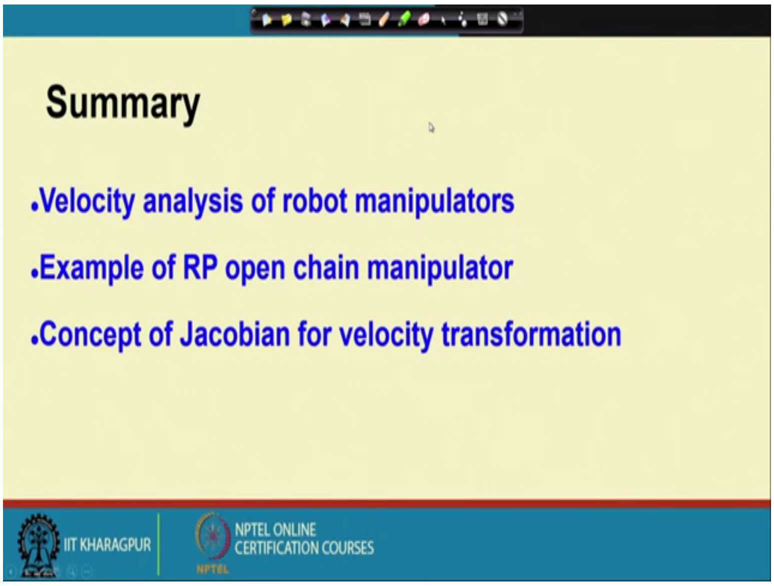

So, let me summarize what we have discussed in this lecture. We have looked at this example of RP open chain manipulator, introduced the concept of Jacobian and we have found that Jacobian of the RP open chain planar manipulator. Now, once we know these velocity relations, we can move forward to analyze or generate parts by the end effector.

So, given a certain end effector path, we just need to find out the velocity or specify the velocity at every point of the path. And when the manipulator is at that configuration, we need to just generate the joint velocities in accordance to, the end effector velocity that is desired. And in this manner, if I move from point to point, I will be able to generate the whole path. So, we are going to look at these problems in a subsequent lecture. So, I am going to close this lecture here.