Mechanism and Robot Kinematics
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Lecture – 29
Serial Manipulator Velocity Analysis – I

We have been discussing about Velocity Analysis of Mechanisms and Robots. In this lecture now we are going to look at the velocity and analysis of open chain manipulators. So, essentially we are going to start with velocity analysis of robots.

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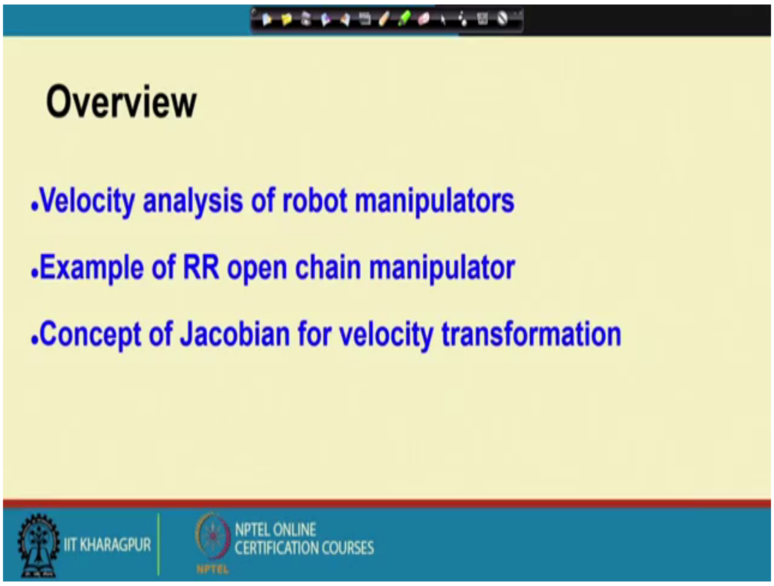

So, let me give you an overview of what we are going to discuss in this lecture we are going to start with this velocity analysis of robot manipulators with an example of an R R open chain manipulator. And we will introduce the concept of Jacobian for velocity transformation as we had done 4 mechanisms.

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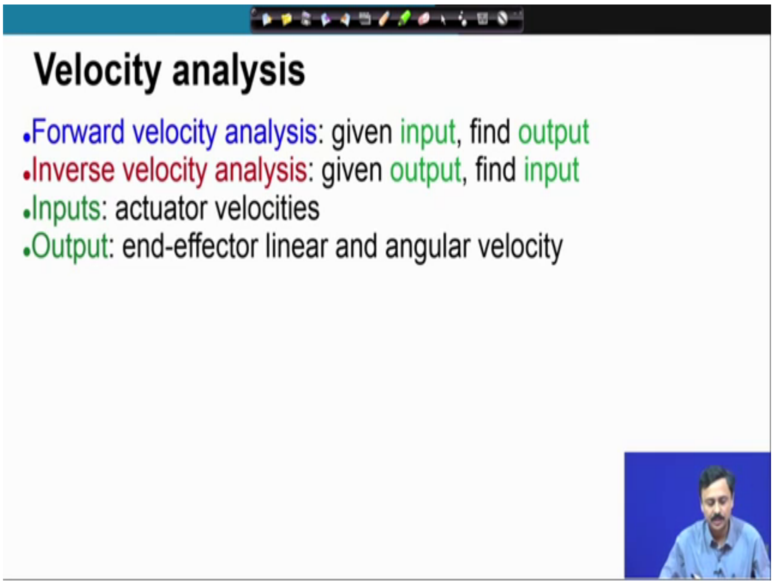

We have discussed this before in velocity analysis problem we have 2 kinds of problems. The first one is known as the forward velocity analysis in which the input velocity is specified, and we have to find out the output velocity were input is the actuator velocity. There can be multiple actuators for example, in robots and we have to find out the end effector velocity.

And it is just the reverse for the inverse velocity analysis where the output is specified we are given a specified output velocity path or we are given time varying velocity field and we have to guide the end effector on that path with certain velocity.

And we have to find out the corresponding input velocity which means the actuator velocities.

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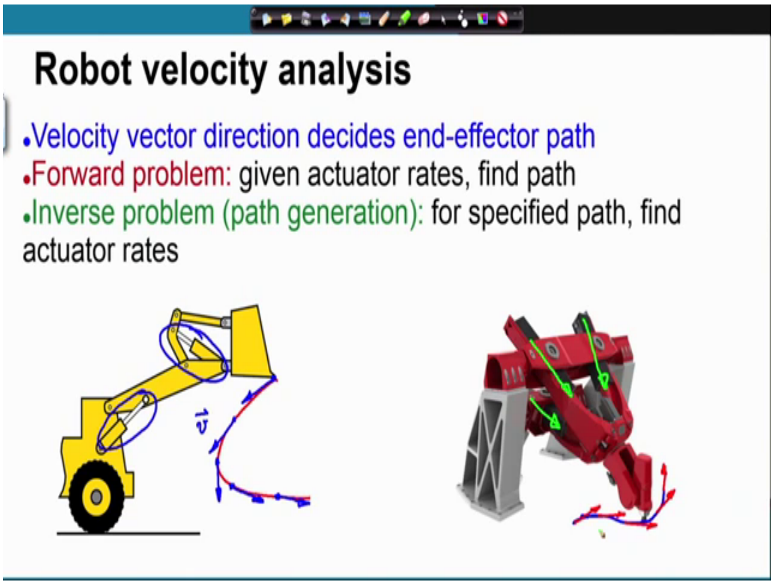

Now in the case of robot velocity analysis the problem is a little more involved as we have discussed. If you consider that this point of the bin has to traverse this path then you need to specify velocities, which are tangent to this path as you know velocity vector is always tangent to the path that a point takes.

So, if you are given the velocity vectors at these points on the path as I have shown these velocity vectors are tangent to the path at those specific points, then you can traverse this path with that velocity profile along the path. So, corresponding to the velocity a certain velocity at a particular point this is a velocity vector you have to find out, the rate of expansion of the actuators to produce that velocity field that velocity vector at the point at this point of the bin.

Similarly, for the case of this parallel kinematic machine suppose you have to make this this tool traverse a certain path then you can do so, by specifying the velocity along the path. And finding out the expansion rates of these actuators the 3 actuators in this parallel kinematic machine so, you have to find out the expansion rates of these actuators. So, that you can produce the velocity at that end effector point at the tool tip so, that you can traverse a desired path.

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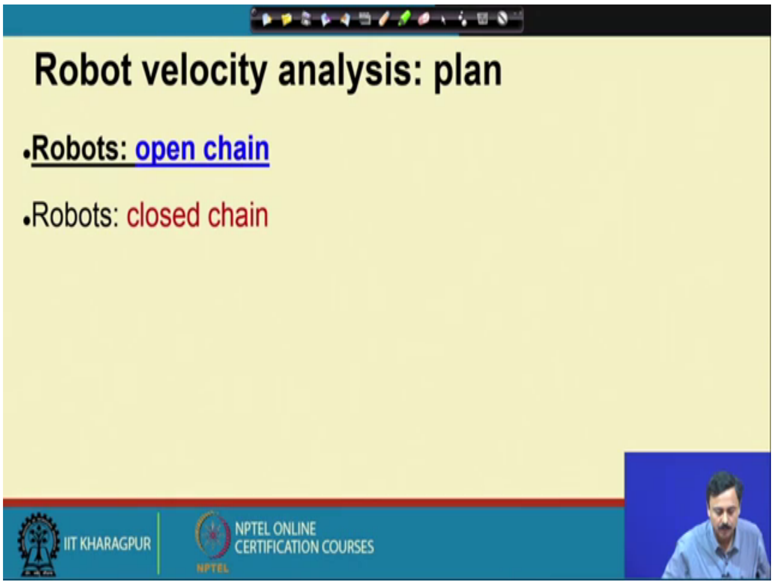

So, we will start our velocity analysis of robots with the open chain manipulator subsequently, we will look at the closed chain manipulator.

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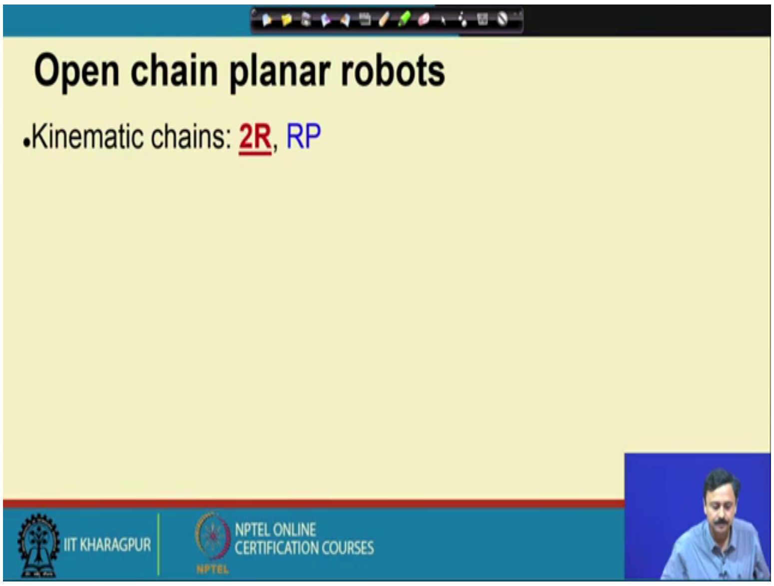

Under open chain manipulators, let us begin with the 2 R manipulator which means there are 2 revolute pairs RR.

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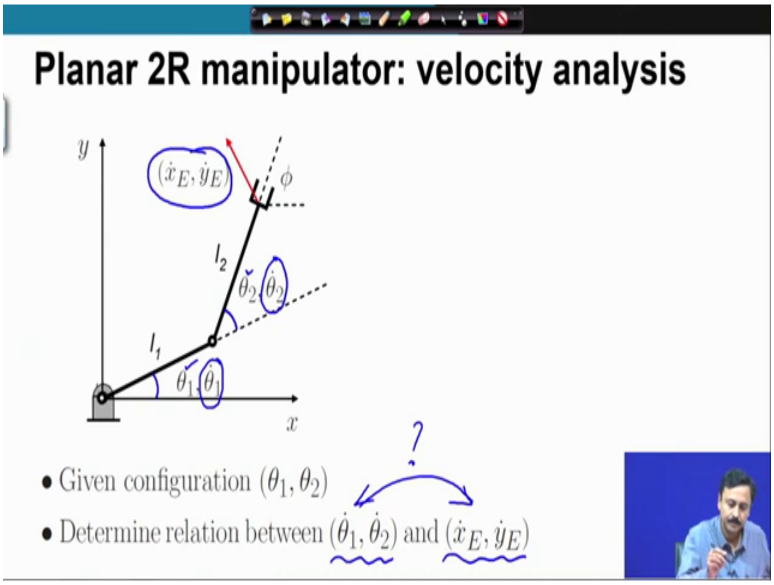

So, this is the schematic of the manipulator with various definitions, let us go through them we are given this configuration theta 1 and theta 2. So, this angle is theta one and remember this angle measured from the extension of link 1 to link 2 that is theta 2. So, we are given the configuration. So, theta 1 and theta 2 are known we have to determine the relation between the velocities.

So, the joint velocities theta 1 dot and theta 2 dot and the end effector velocity, x E dot and y E dot. So, I should be able to find a relation between the joint velocities. So, joint velocities and the end effector velocities. So, this is the relation I want to determine.

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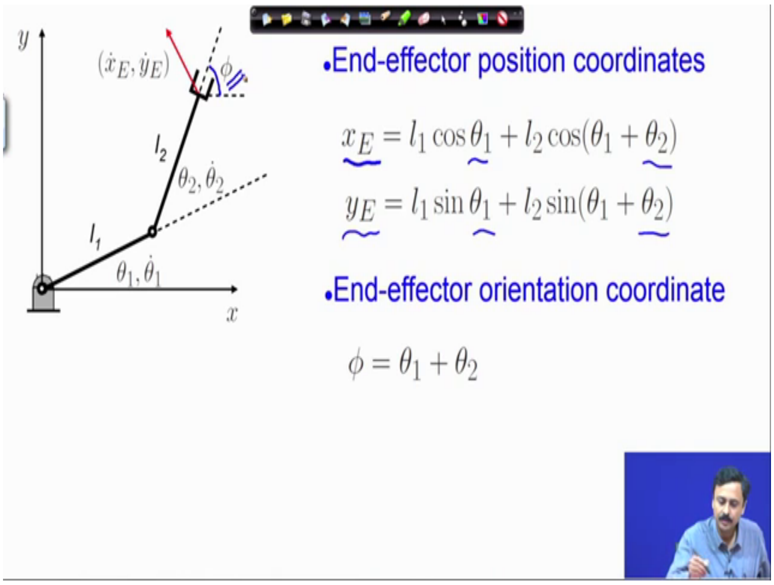

So, let us begin with the forward kinematics relations. So, here I have written out for you the forward kinematics relations, which we had derived earlier. So, I have related here the x position and the y position of the end effector with the joint angles. So, theta 1 and theta 2, we also have a relation between the orientation angle of the end effector, which is phi and the joint angles, which we have also derived earlier. So, once again the forward kinematic relations of this manipulator.

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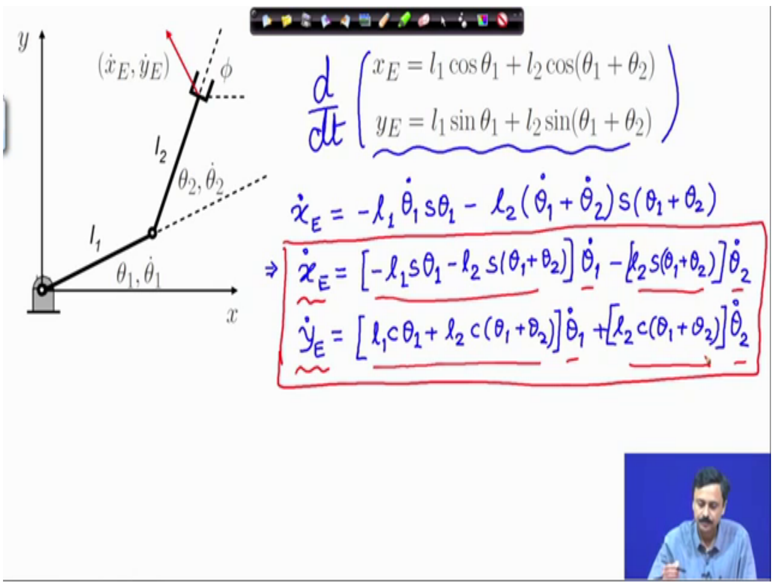

If, I differentiate both sides of these relations with respect to time so both these relations I derivate with respect to time. So, from the first relation what I have is x E dot which is d d t of x E is equal to minus l 1 theta 1 dot sin theta 1. So, I am writing s theta 1 in case of sin theta 1 and minus l 2 theta 1 dot plus theta 2 dot sin of theta 1 plus theta 2.

So, this I can simplify somewhat. So, x E dot I am collecting the terms of theta 1 dot. So, this times theta 1 dot and the other term is. So, that is the second term. Similarly if I derivate the second equation the second relation with respect to time, then I will have y E dot is l 1 cosine theta 1 plus l 2 cosine of theta 1 plus theta 2. So, I have written cosine abbreviated cosine as c. So, this times theta 1 dot plus.

So, these are the 2 relations that I obtained. So, you can see from these 2 relations that I have related the joint velocities with the end effector velocities. And these relations are linear as you can see furthermore these relations are dependent on the configuration. So, as you can see these terms they contain theta 1 and theta 2. So, it is configuration dependent and it also involves the link lengths l 1 and l 2.

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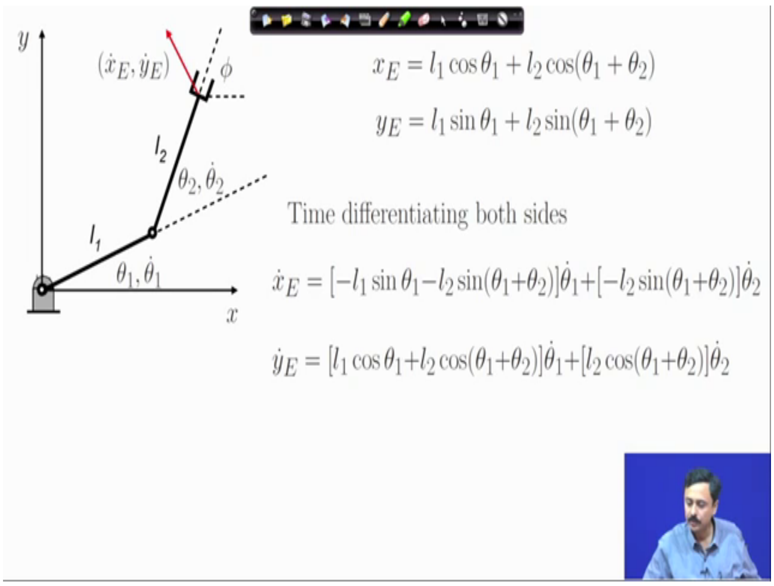

So, let me formally show you these relations. So, when I do this time differentiation finally, I obtain these 2 relations which I have shown you.

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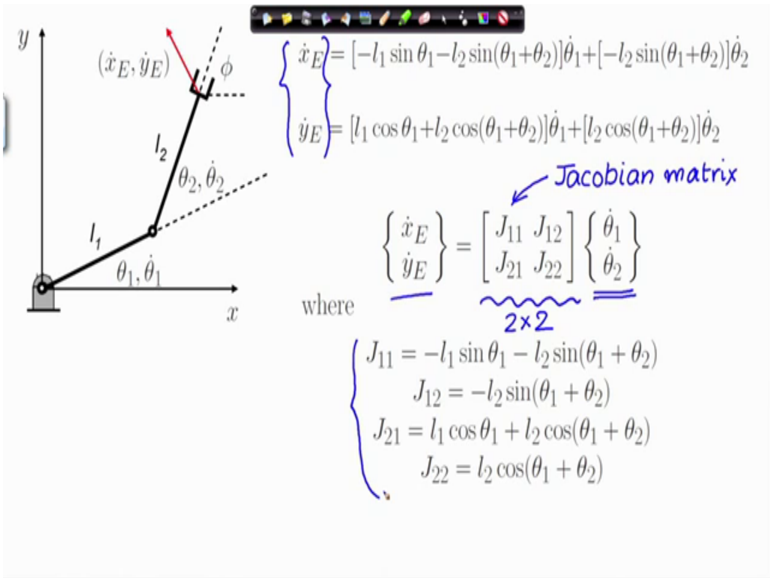

Now, I rewrite these relations and assemble this in this form. So, you can see here that I have collected these 2 velocity terms of the end effector x E dot and y E dot. And represented as a column vector and on the I have also collected the joint velocities theta 1 dot and theta 2 dot as another column vector and related them using what we call the Jacobian. So, this is the jacobian matrix.

So, this is the jacobian matrix is a 2 cross 2 jacobian matrix and the terms I have written out here. Thus in this case of 2 R robot manipulator, we have a jacobian which is now a matrix, which is unlike the case of a mechanism of a constraint mechanism, where it is a scalar quantity?

So, we have the jacobian matrix.

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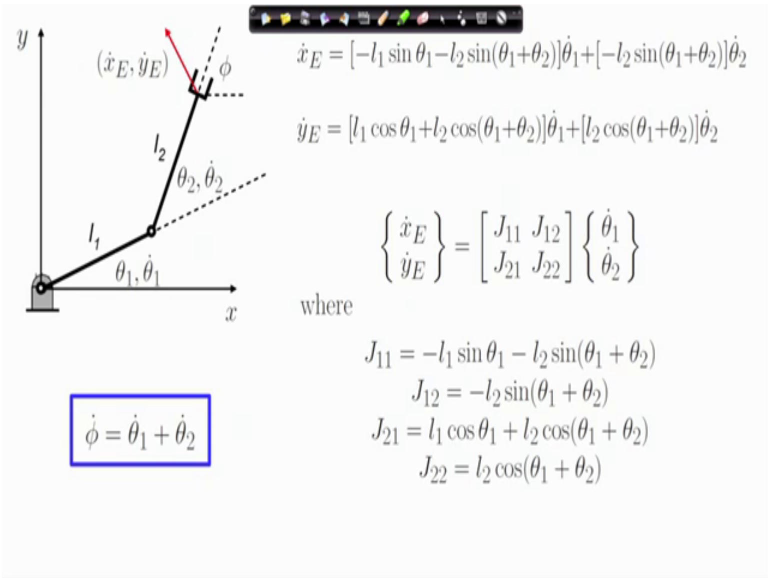

This relation follows by time derivative the orientation relation. So, you have this phi dot as the sum of the joint velocities.

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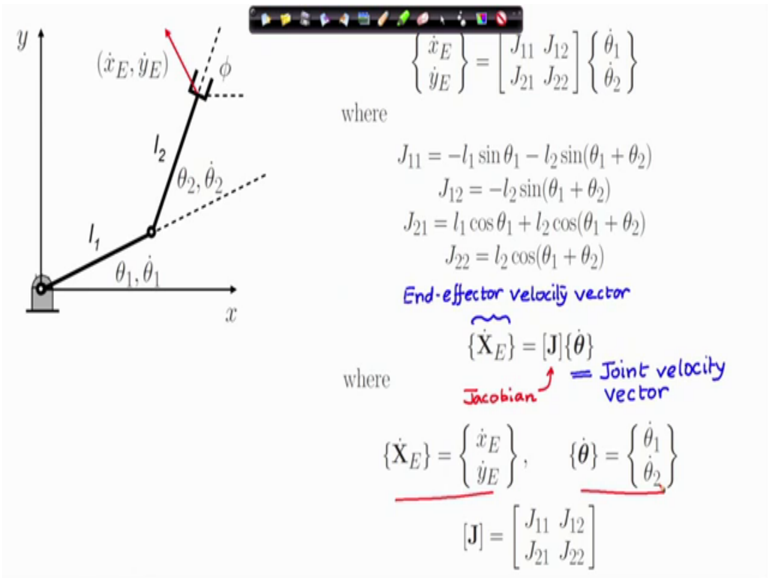

So, we have written out or related these velocity vectors now in terms of the jacobian. So, we have the joint velocity vector. So, here I have further compress these relations. So, this is known as the joint velocity vector and this is the end effector velocity vector.

So, the end effector velocity vector is related to the joint velocity vector through the jacobian. So, this is our jacobian so, where we have these definitions of the end effector velocity vector and the joint velocity vector.

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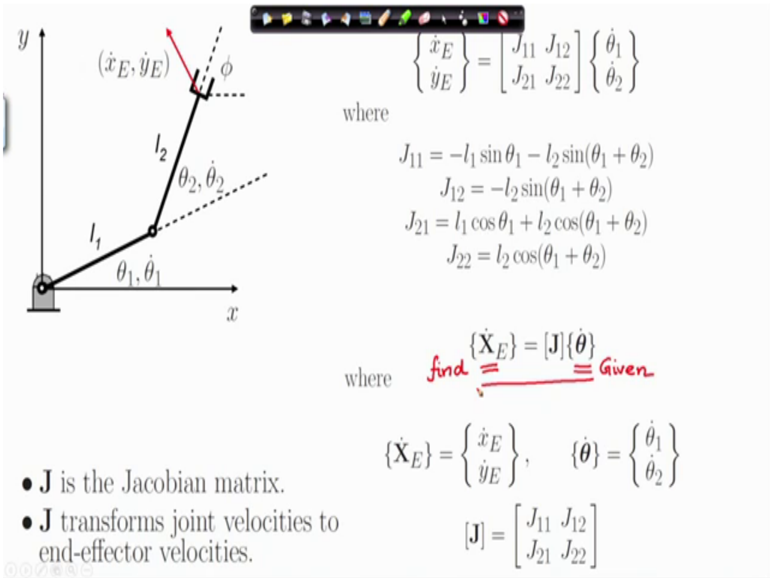

So, this jacobian as I have mentioned is now a matrix in the case of the 2 degree of freedom R R manipulators a 2 cross 2 matrix and it transforms the joint velocities to the end effector velocities, then the question arises how do I relate the end effector velocities to the joint velocities.

In the relation that you see here given the joint velocity vector you can find out the end effector velocity, you can determine the end effector velocity vector. Now I am asking the reverse question given the end effector velocity, how to find out the joint velocity vector? So, that will be the inverse kinematics problem. So, we have these relations.

So, as you know that if I am given this this relation, now how do I find out the joint velocity vector in terms of the end effector velocity vector?

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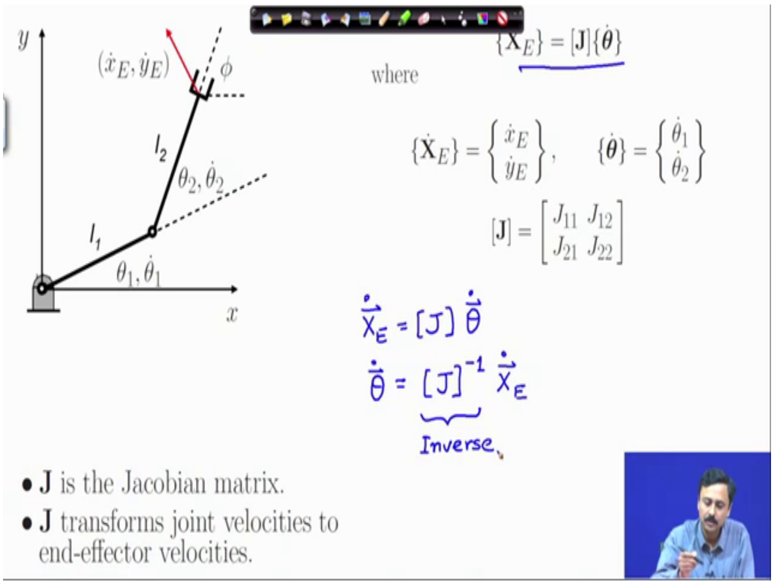

So, I multiply both sides by the inverse of the jacobian if it exists. So, then I can write the end effector velocity vector. So, I relate the joint velocity vector to the end effector velocity vector through the inverse of the jacobian.

Now how do you invert the jacobian. So, how this jacobian? So, that is not very difficult because this is now a 2 cross 2 matrix it is basically inversion of a matrix. So, in the case of 2 cross 2 matrix inversion is not, very difficult.

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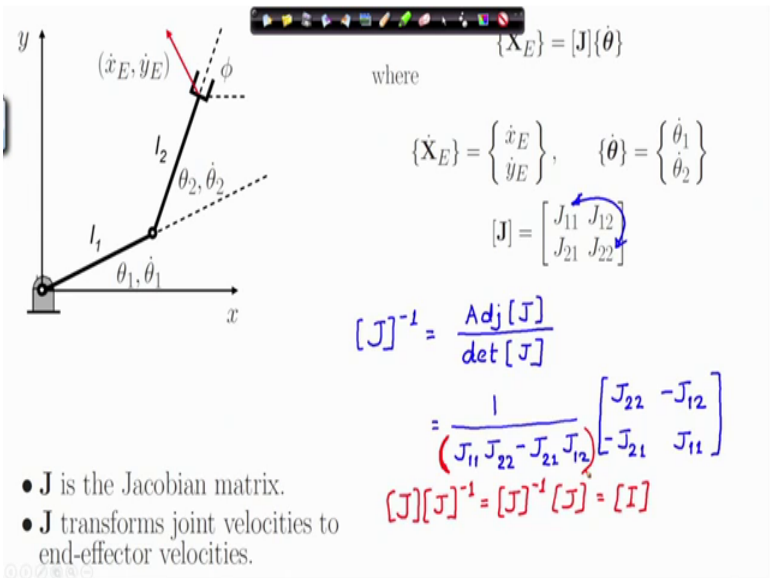

So, let me first define the inverse. So, inverse is defined as the adjoint of the matrix divided by the determinant of the matrix, now the determinant of the matrix simple J 1 1 times J 2 2 minus J 2 1 J 1 2 times now adjoint, adjoint is a transpose of the cofactor matrix.

In this case of a 2 cross 2 matrixes very simple you have to exchange the diagonal terms; that means, here you will have J 2 2 and here you have J 1 1. So, you exchange the diagonal terms and you change the sign reverse the sign of the of diagonal terms that is the adjoint. And you can check very easily that J times, J inverse is also equal to J inverse times J and that is the identity matrix this is the identity matrix this you can check very easily then this J inverse is correct.

So, we have looked at the inversion of the jacobian, now I have also told said that the inversion the existence of inversion is something we must look at. Because here in the denominator we have this jacobian terms the elements of the jacobian. So, the determinant of the jacobian, if it vanishes then we have a problem. And we are going to look at this problem in detail in a subsequent lecture.

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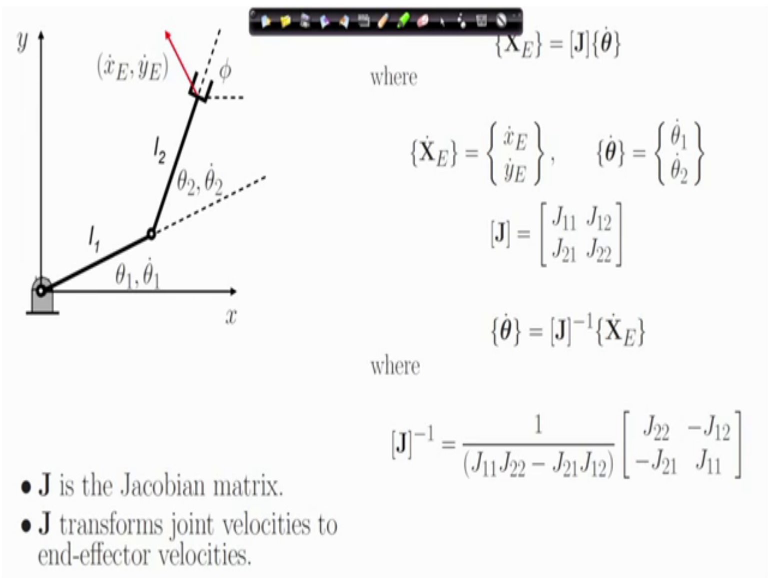

So, right now let me formalizes. So, as I have mentioned we have this inverse obtained by writing the cofactor of the jacobian, divided adjoint of the jacobian divided by the determinant of the jacobian and the adjoint is the transpose of the cofactor matrix.

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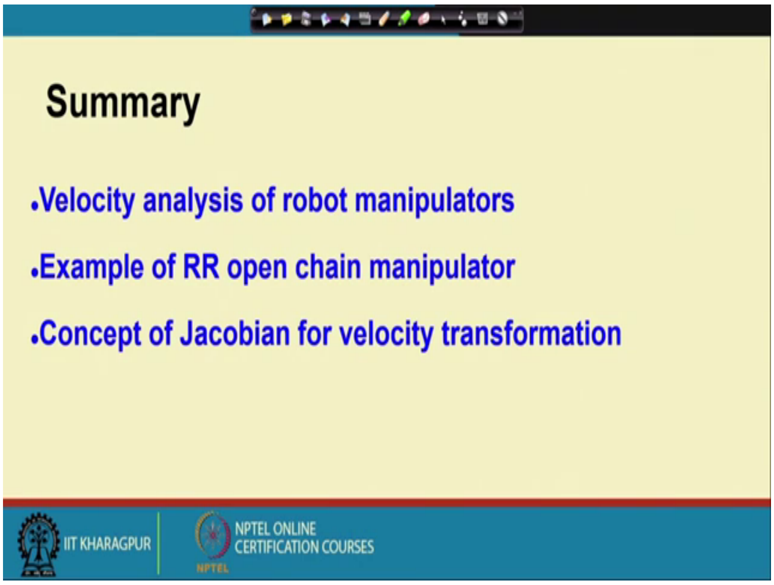

So, let me summarize what we have discussed in this lecture we discussed the problem with velocity analysis problem of A 2 R manipulator, we have looked at the velocity relations the velocity relations between the end effector velocity. And the joint velocities and we found that these relations are linear and the factor the factor which relates these 2 velocities, the end effector and the joint velocities through the linear relation is the jacobian.

So, it is something like velocity of end effector is jacobian times the velocity of the joint the of the joints velocity vector of the joints. And this the relation the proportionality constant. So, to say is the jacobian and the jacobian is dependent on the configuration as we have seen, and we have also inverted the jacobian to find out the joint velocities in terms of the end effector velocities. So, this will be important or used in the inverse velocity relations.

So, when we are given the end effector velocities or desired end effector velocities how do we find out the joint velocities? So, for that we need to invert the jacobian we have also said that the inversion of the jacobian is dependent on the non-vanishing of the determinant of the jacobian.

Now what happens if this determinate vanishes is something that we are going to look at in a subsequent lecture. So, I will close this lecture here.