Mechanism and Robot Kinematics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 27
Velocity Analysis: Analytical Approach – II

We are going to discuss further on the velocity analysis problem in today's lecture. We are following the analytical approach which is based on the displacement relations, displacement input output relations from where we are deriving the velocity relations. So, this analytical approach we are now going to look at we are going to apply to another kinematic chain, which is the 3R1P chain of type 1.

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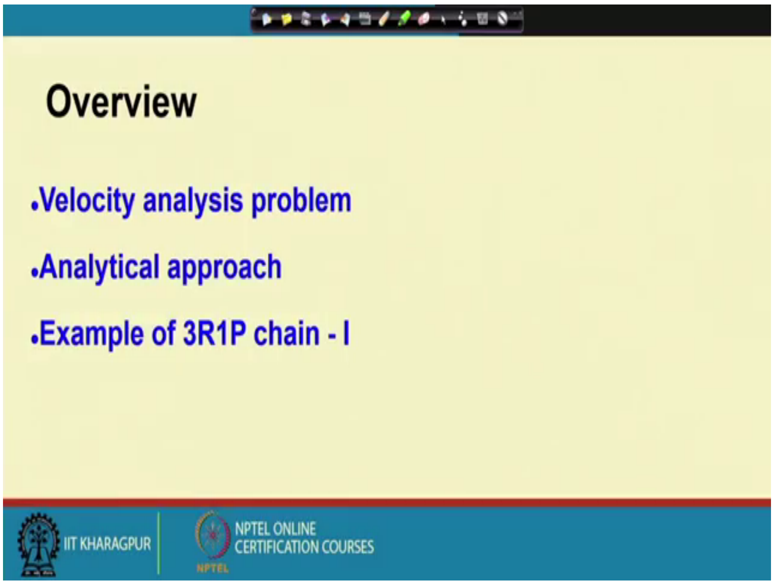

So, to give you the overview of what we are going to discuss in this lecture going to take the analytical approach for velocity analysis with the example of the 3R1P chain of type 1.

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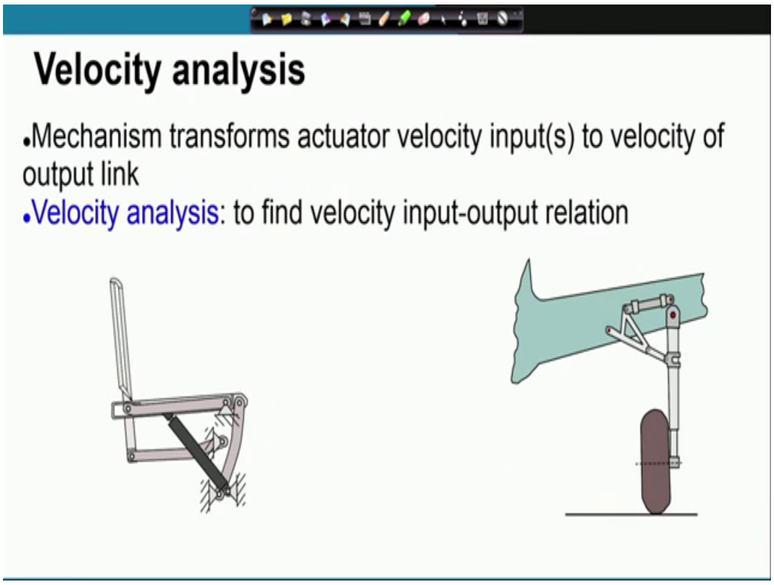

So, we have already defined this velocity analysis problem which is to determine the input output velocity relations.

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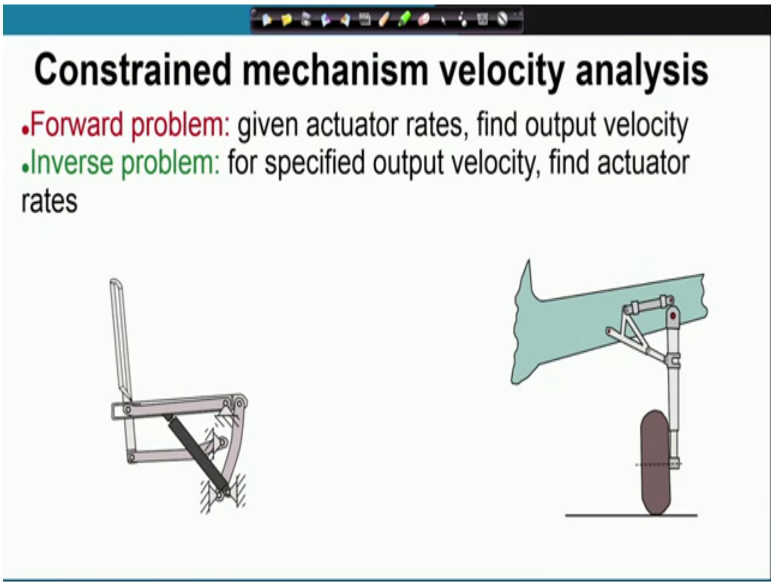

We have 2 problems the forward and the inverse problems.

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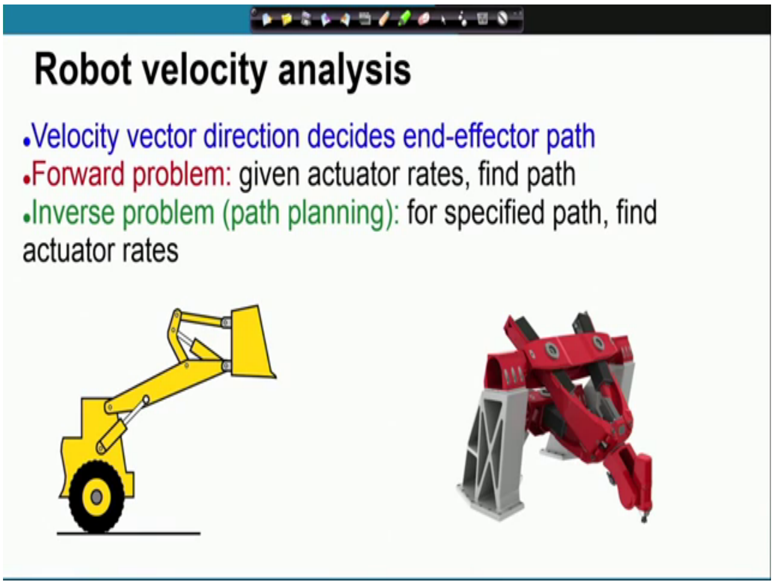

And for robots we have the path planning problem, which is the inverse problem which is very useful for in a generation of output trajectories by a robot manipulator which we were going to discuss very soon.

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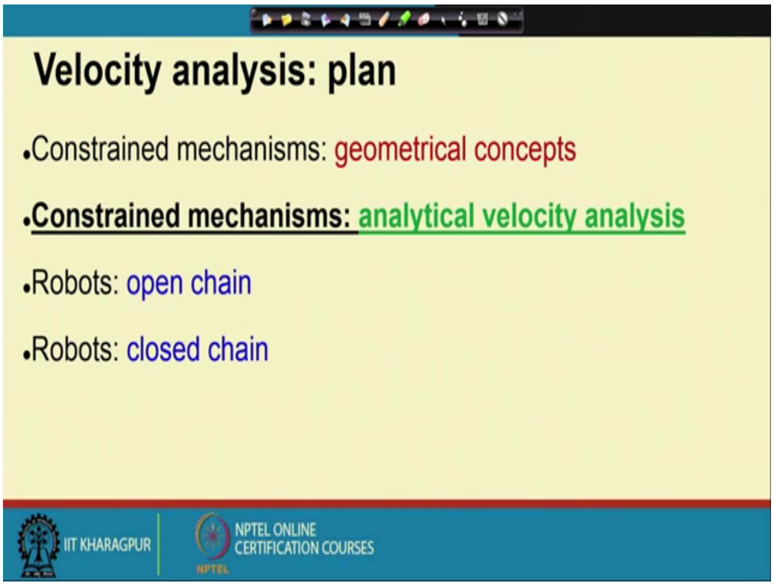

We will continue with the analytical velocity analysis for constraint mechanisms, later on we are going to look at robots and how paths can be generated.

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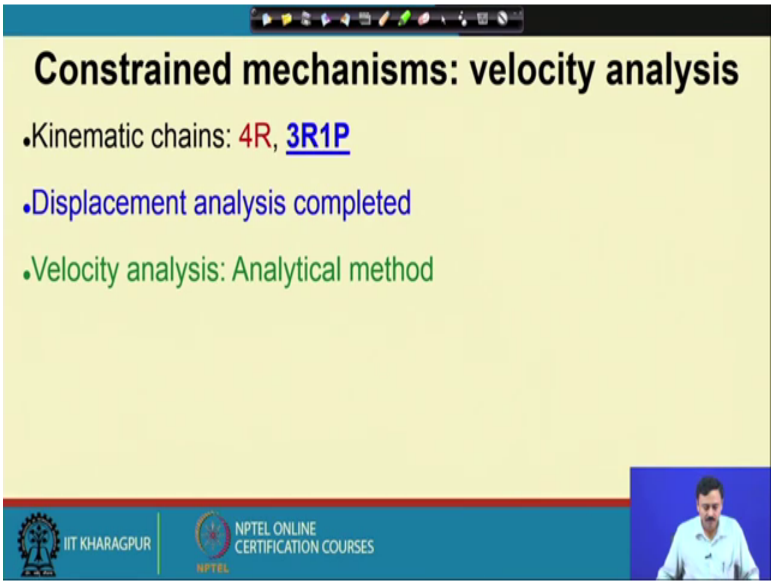

So, in this lecture we are going to look at the 3R1P chain of type 1. We will assume that the displacement analysis is completed, which means that I know the configuration of the mechanism of course, I am given one input because this has one degree of freedom I can calculate the other angles using the displacement relations. We are now going to embark upon the velocity analysis problem.

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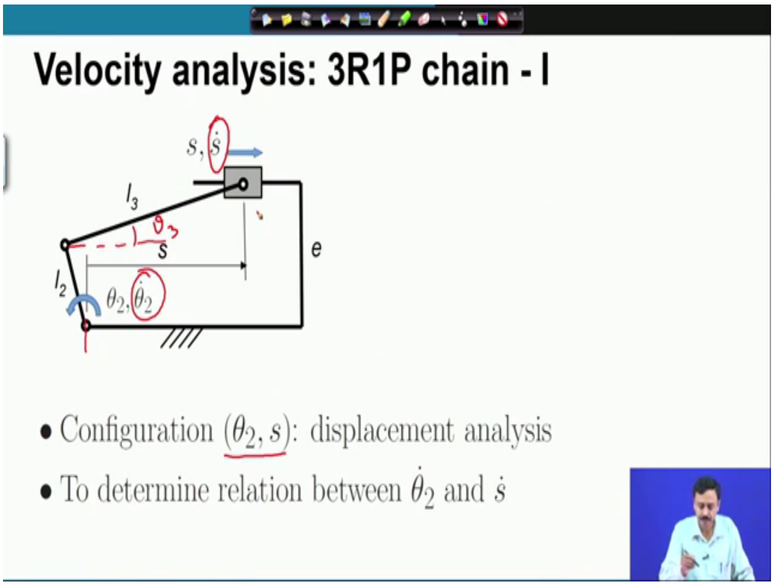

So, this is the 3R1P chain type 1. As I have mentioned that the configuration in terms of theta 2 s and also of course, theta 3. So, if I know theta 2 for example, I can find out s I can find out theta 3 if I am given s I can find out theta 2 I can find out theta 3 all this we have discussed under displacement analysis. The problem is now to determine the relation between theta 2 dot and s dot.

So, this s dot or s the displacement s is measured from this reference s dot is the velocity of the slider.

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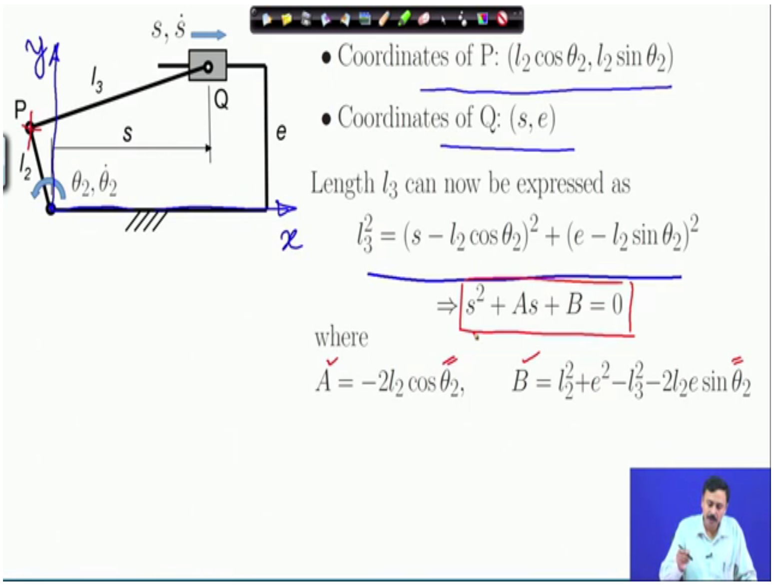

So, as in the displacement analysis problem we begin with the coordinates of point P what we have here, let me draw out the coordinate system. So, this is our coordinate system, just to recapitulate that we have this coordinate system with the origin at this thinge. So, in this coordinate system I am expressing the coordinates of point P and coordinates of point Q, using these coordinates I have expressed the length of link 3 which we have done before we have done all these simplifications and we have seen that this leads us to this kind of an expression relating s and theta 2. So, we have the relation between s and theta 2 theta 2 is in these 2 expressions of A and B we have theta 2. So, we were previously finding out s given theta 2 this was the displacement analysis problem.

But now what we are going to do and of course, we can also find out this angle.

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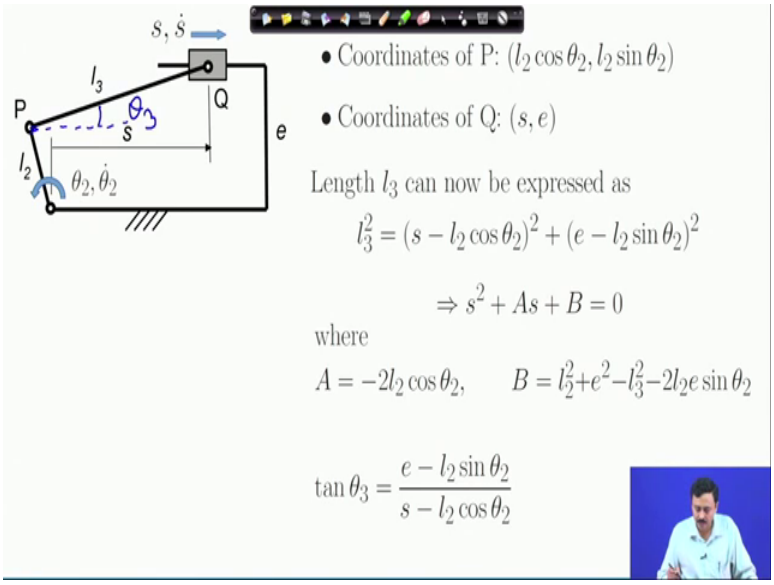

This is our angle theta 3, we can find out this angle theta 3 in terms of tangent theta 3, in terms of s and theta 2. So, once I have solved for s from this quadratic equation I can find out theta 3. So, this we have looked at before. Let us move forward. So, we have this quadratic equation of s.

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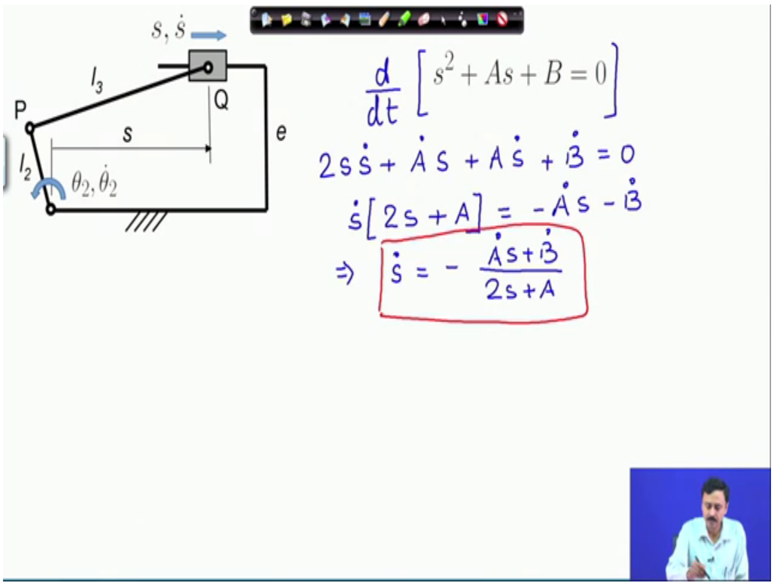

In terms of A and B which are functions of theta 2. So, if I take time derivative of this expression, what do I have? I have 2 s s dot this is d d t of s plus e dot s remember A is the function of theta 2 plus A s dot, then B dot. So, therefore, I can collect terms of s dot and take the other terms on the other side

So, therefore, I have this expression of s dot. So, I have this expression of s dot in terms of the time derivatives of A B and the expression of A. So, on the right-hand side I have s theta 2 and derivatives.

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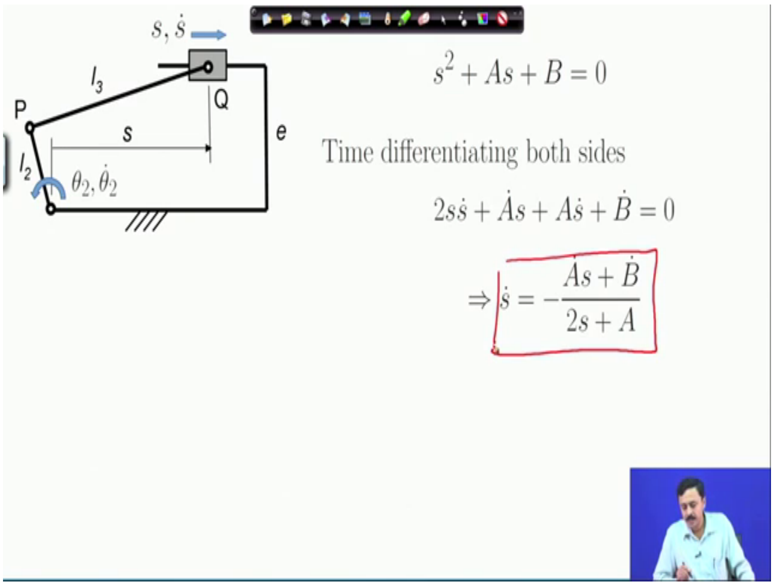

So, let me write this out formally so this is what we have just now derived.

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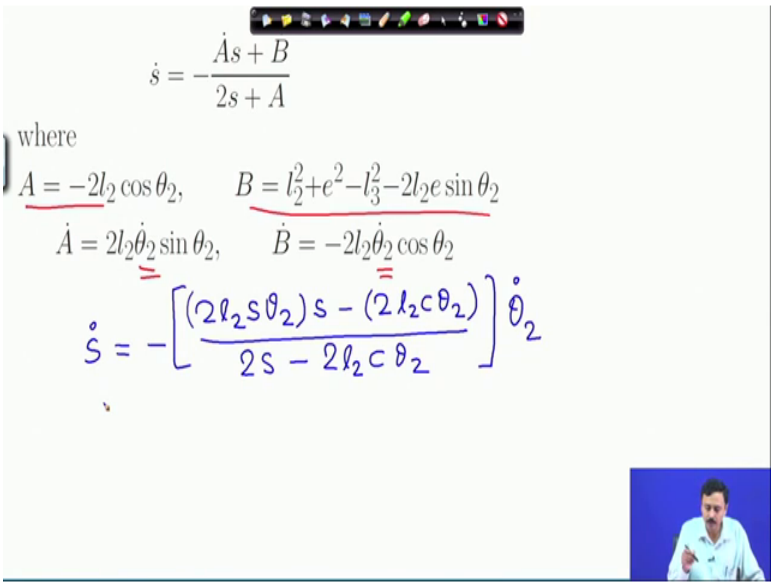

So, therefore, s dot is A dot s plus B dot divided by 2 A, 2 s plus a in the denominator and there is a negative sign as you can see. Here A and B have these expressions which we have seen. So, if you differentiate these with respect to time so you will get A dot and B dot and you will notice that these expressions of A dot and B dot involve theta 2 dot. So, A dot and B dot are proportional to theta 2 dot. So, if you put back these expressions of e dot and B dot in the expression of s dot. So, you have 2 l 2 sine theta 2 s. We have theta 2 dot I want to pull out theta 2 dot. So, I will write it later minus 2 l 2 cosine theta 2. Whole divided by and this whole thing is multiplied by theta 2 dot. So, you can simplify this expression further so we have related s dot and theta 2 dot in terms of s and theta 2.

This is our final relation you can of course, simplify this which we are going to do now. So, let me write this out formally.

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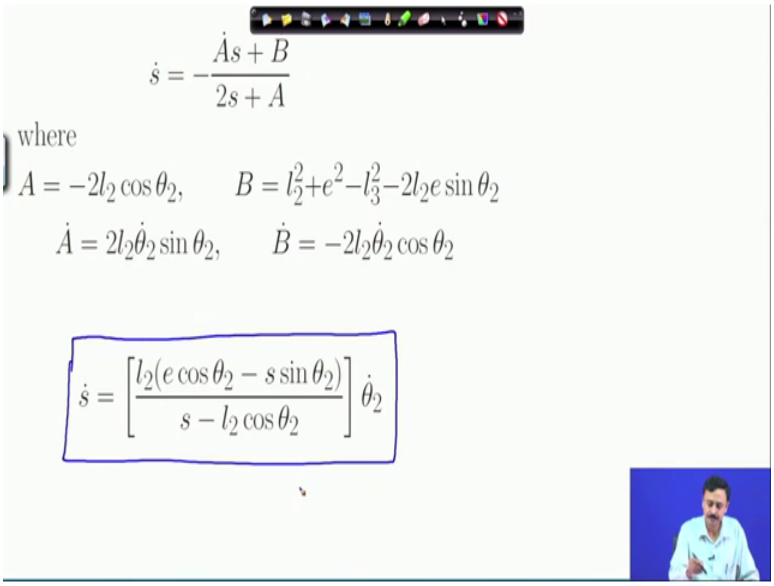

So, here I have the after simplification you will obtain this relation between s dot and theta 2 dot again you see that the velocity relations are linear.

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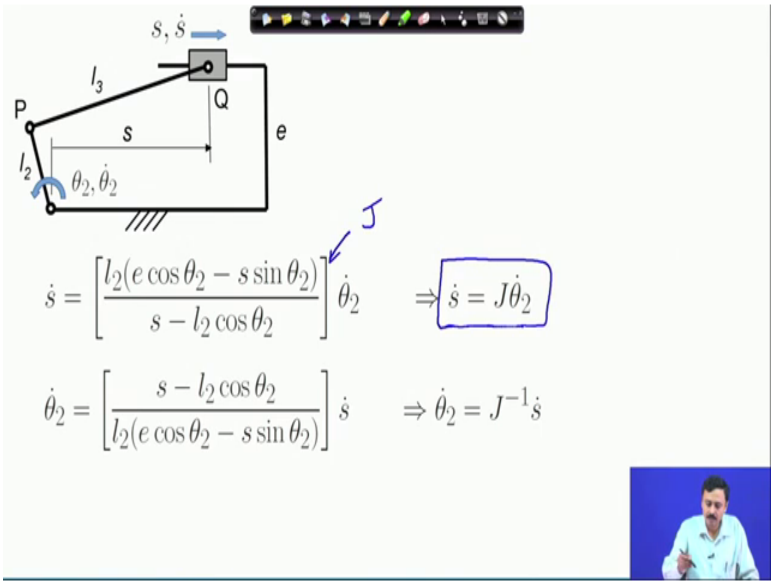

We write this expression in a compact form. As I have written on the right s dot is J times theta 2 dot, where J is the scalar Jacobian and the expression of Jacobian you have is there here so this is the Jacobian. Now if you invert this relation; that means, express theta 2 dot in terms of s dot we have the inverse Jacobian. So, these Jacobian inverse Jacobean are in terms of theta 2 s and the link parameters. So, once again we have this scalar Jacobian which we had also derived using the method of ICs if you recall that.

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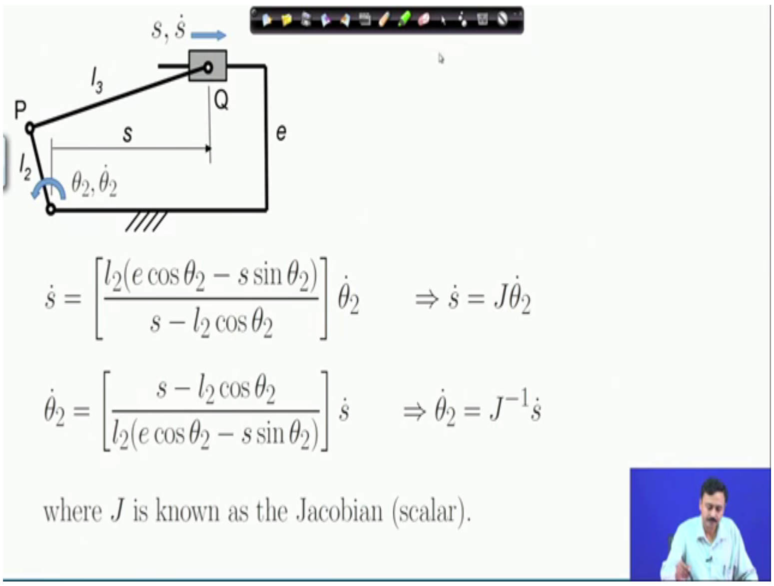

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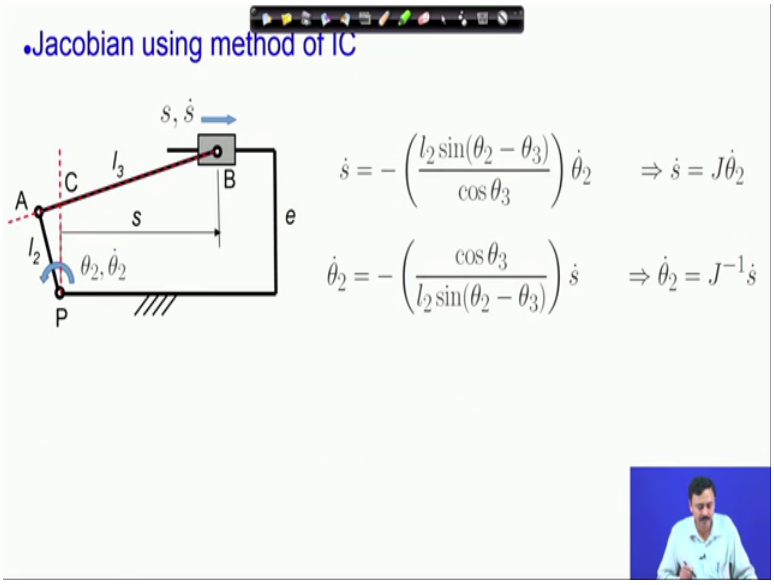

We had these expressions using the method of ICs, now if you compare here we have it in terms of theta 2 and theta 3, here the Jacobian is in terms of theta 2 and theta 3 where is in the previous case the Jacobian was in terms of theta 2 and s, but they relate to the same thing the relations remain the same.

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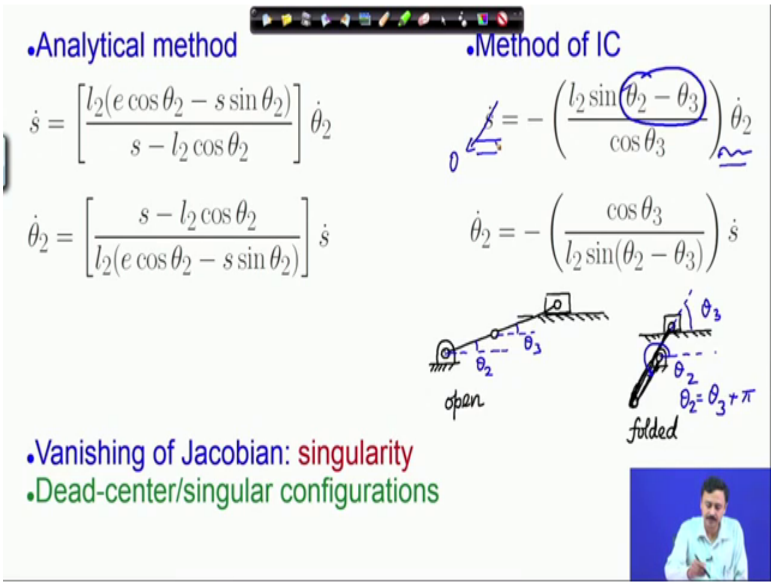

So, if you compare the expression is obtained from the analytical method and from the method of ICs. So, you have these relations between s dot and theta 2 dot. Now vanishing of the Jacobian as we have mentioned relates to the singularity of the mechanism this singularities are the dead center the singular configuration the configurations where the Jacobian is singular or non-invertible these are the singular configurations or the dead center configurations of the mechanism and we have discussed the dead center configurations we have drawn the dead center configurations. So, let me show that. So, this is one dead center configuration of the mechanism where you have this is theta 2 this is theta 3. So, where you have theta 2 equal to theta 3. So, that is dead center configuration as you can very easily see from here because if theta 2 equals theta 3 then sin of that expression in the numerator goes to 0.

So, irrespective of the value of theta 2 dot s dot must go to 0. So, it is the extreme configuration of the slider, as I have drawn out here. There was another configuration or another possibility; this was the other dead center configuration. We are names for them this is the folded configuration and this is the open configuration. So, here also the Jacobian vanishes so let us see why.

So, this angle is theta 2 and the angle theta 3 is this. You can very easily see that theta 2 is theta 3 plus 180 degree. So, theta 2 is theta 3 plus pi radian, which mean that theta 2 minus theta 3 is pi which means sin of pi comes in the numerator of the Jacobian so it vanishes. So, this is the configuration whether Jacobian vanishes so these are singular configurations of the forward velocity relations; that means, given theta 2 dot finding out s dot.

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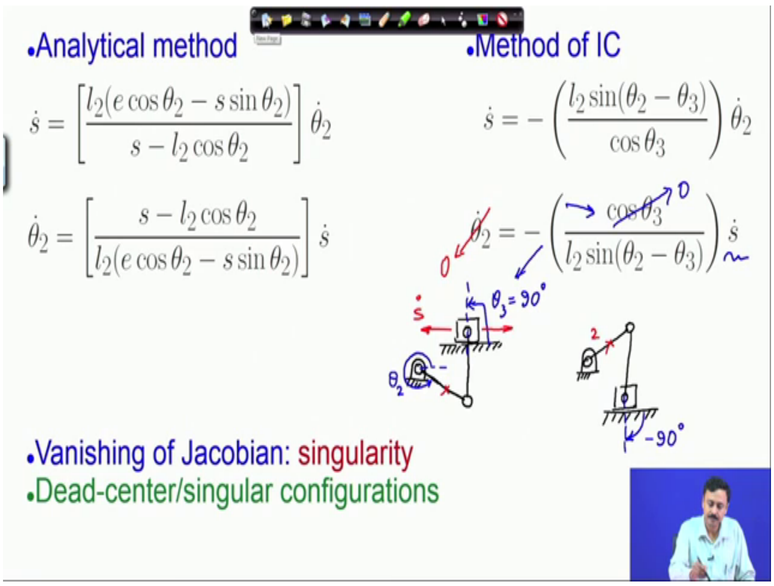

So, the Jacobian becomes singular at this these 2 configurations and therefore, these are the dead center configurations of the mechanism, there is another set of dead center configurations which we have also looked at just to reiterate the point. So, here this is theta 2 and this angle theta 3, this 90 degree what happens that when theta 3 is 90 degree as you can see here cosine of 90 degree is 0.

So, therefore, this vanishes in this configuration therefore, irrespective of value of s dot which is the slider velocity irrespective of the value of s dot, this link is not going to move theta 2 dot is 0. Theta 2 dot becomes 0 which means this link is fixed at this configuration. So, whenever this velocity goes to 0 what happens is it changes a reverses direction the link will reverse direction. So, when it reverses direction it comes to a stop and then reverses.

So, this is that extreme configuration of theta 2, and the same thing will happen in another kind of configuration where this theta 2 theta 3 is minus 90 degree this is minus 90-degree theta 3 is minus 90 degree so then also this Jacobian inverse vanishes. Once again this is dead center configuration the link to gets locked; that means, it comes to a momentary stop you cannot have any velocity of link 2 at this configuration. So, these are the dead center configurations of the 3R1P chain of type 1.

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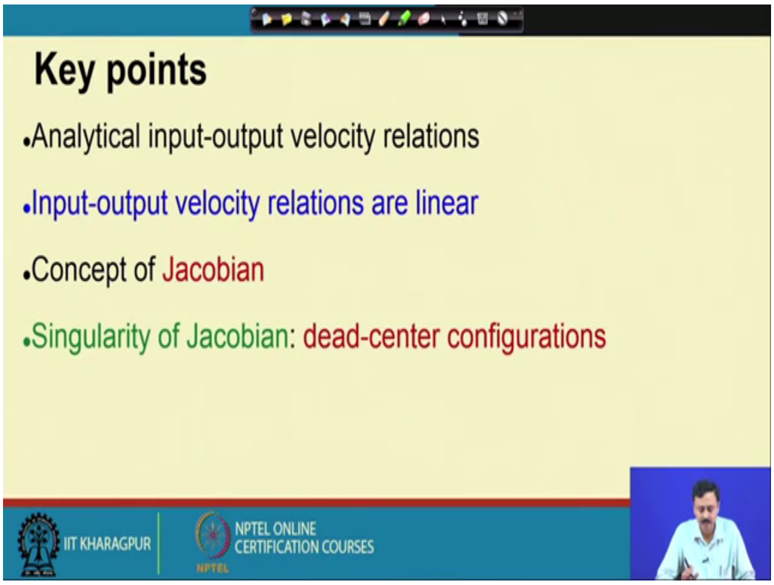

So, let us look at the key points we have derived the input output velocity relations which are linear we looked at the concept of Jacobian and the singularity of the Jacobian which gives us that dead center configurations.

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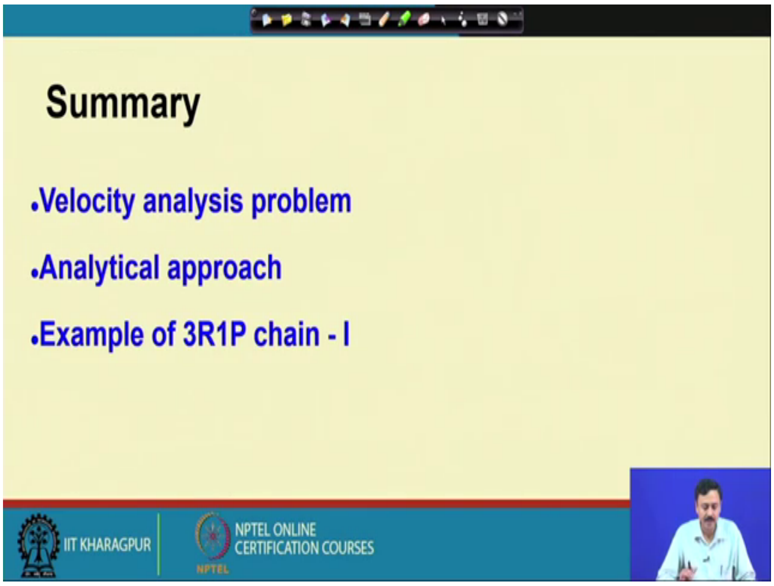

So, let me summarize this lecture we have looked at the velocity analysis problem for the 3R1P chain using the analytical approach which starts with the displacement relations of the chain. So, every time we are starting with the displacement relations of the chain differentiating that with respect to time and finally, finding out the velocity relations. As I mentioned before this is the approach which is amenable for acceleration analysis and not the one which we have derived using the method of instantaneous centers of rotation. So, we are going to follow this approach for velocity analysis as well as acceleration analysis. So, with that I will conclude this lecture.