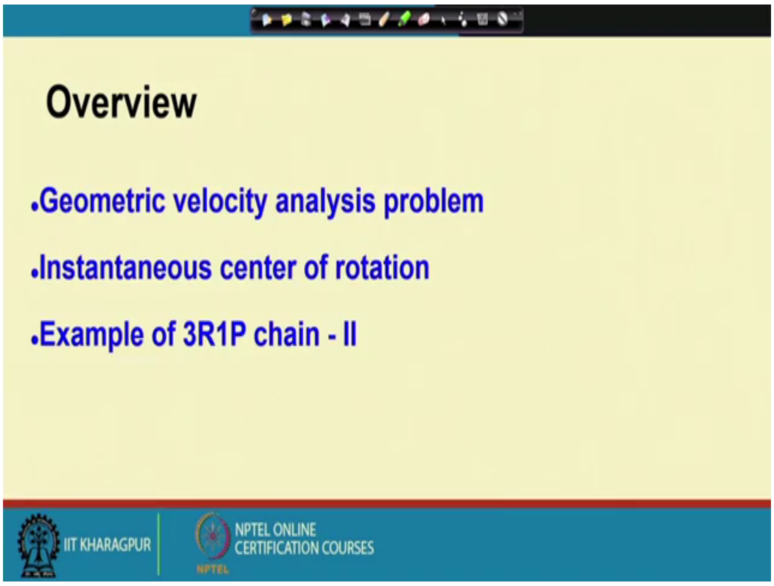
Mechanism and Robot Kinematics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 25
Velocity Analysis: Application of Geometric Concepts – III

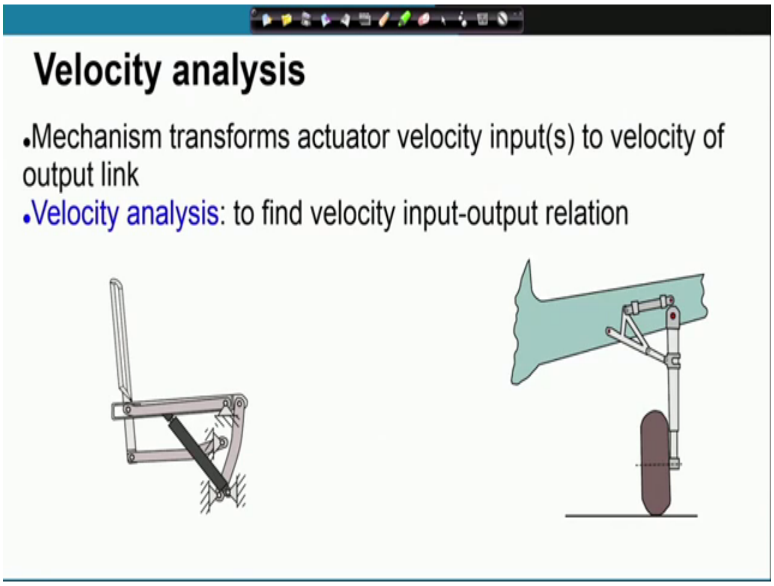
In this lecture we are going to look at the velocity analysis problem using geometric methods that we were discussing. So, today we are going to look at another kinematic chain of the type 3R1P.

(Refer Slide Time: 00:30)



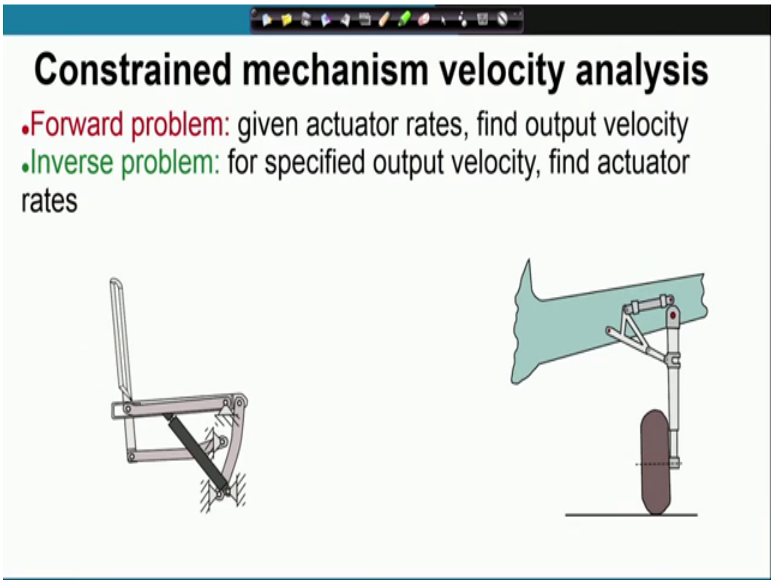
So, to give you an overview of what we are going to discuss today, we are going to continue with the geometric velocity analysis problem using the instantaneous center of rotation with the example of a 3R1P chain of type two.

(Refer Slide Time: 00:49)



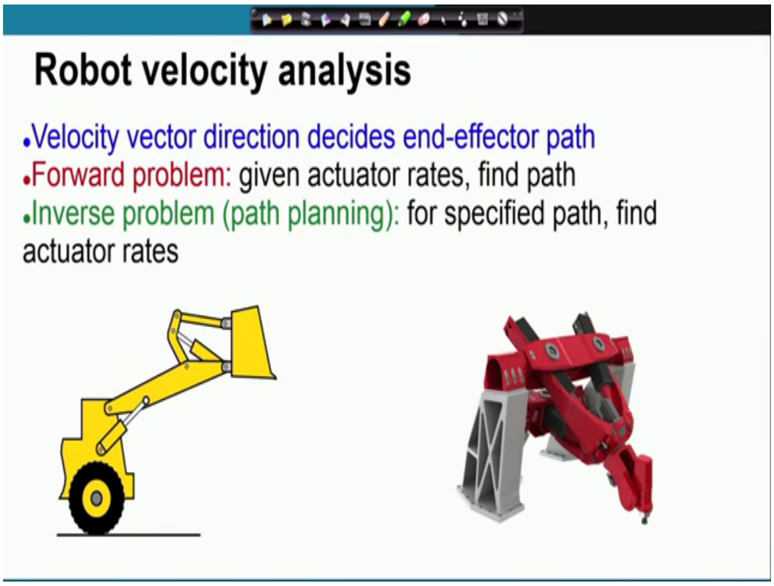
So, we have looked at the velocity analysis problem before so, will quickly go through that. So, essential goal of the velocity analysis problem is to find out the velocity input output relation, and the inputs are the actuators, and the output is the is a it can be a link or the end effector of a robotic manipulator.

(Refer Slide Time: 01:16)



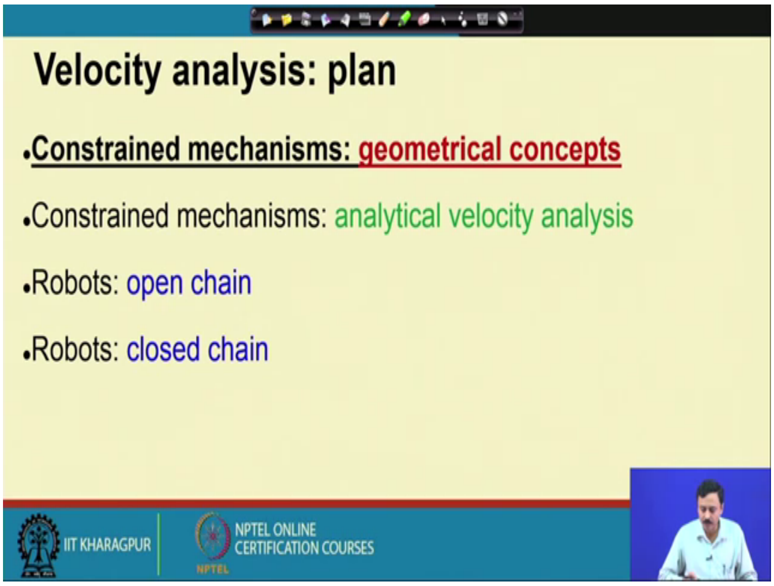
So, in the case of constraint mechanisms you have just 1 output link, and corresponding to an input velocity we have to find out the velocity of the output link. So, you have the 2 problems forward and inverse, given the actuator rates finding out the output velocity is the forward problem, and for a specified output velocity finding out the actuator rates is the inverse problem.

(Refer Slide Time: 01:49)



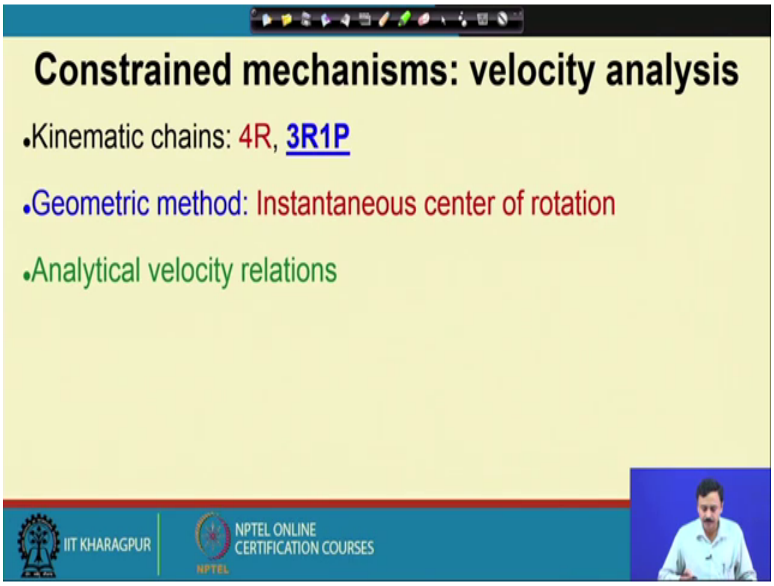
In the case of robots as we have seen before again the 2 problems are little more complicated, than the then in the case of constraint mechanisms. The reason is the velocity vector is tangential to the path. So, you if you are given a path, you can specify the tangent vectors, and you can correspondingly find the actuator rates in order to produce that velocity vector at a certain point on that path. The inverse problem is the path planning problem, in which the path is specified and you have to find out the actuator rates.

(Refer Slide Time: 02:49)



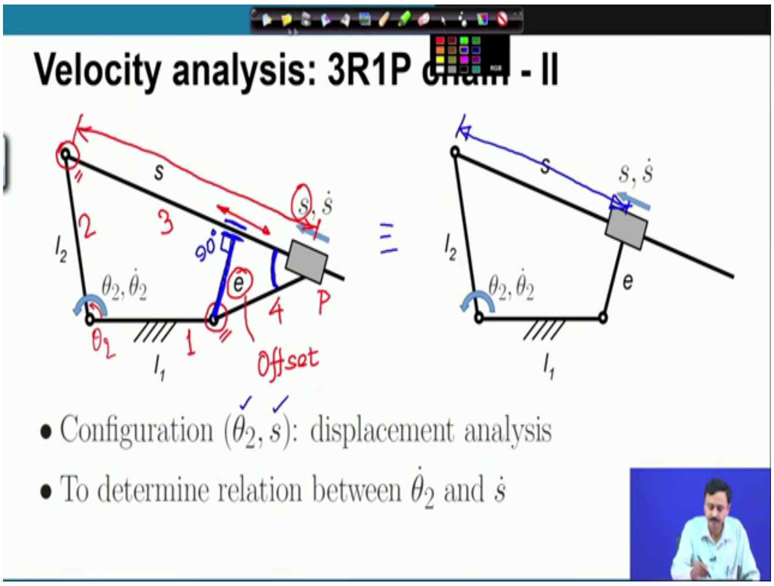
So, according to our plan we are discussing the geometrical concepts of velocity analysis, for constraint mechanisms, we will look at the analytical velocity analysis in the subsequent lectures which will be followed by velocity analysis of robotic manipulators.

(Refer Slide Time: 03:13)



So, in the constraint mechanism examples today we are going to look at the 3R1P chain of type 2, we are going to use the geometric method using the concept of instantaneous center of rotation. And we will finally, derive the analytical velocity relations based on this geometric method.

(Refer Slide Time: 03:39)



So, here is the 3R1P chain type 2 here we have the angle theta 2, and S this variable S is this length is s. So, this is the expansion of the prismatic actuator. So, here we have the prismatic actuator. So, S is the length of expansion of the prismatic actuator, previously we had defined this offset e which is measured in the direction perpendicular to the direction of the prismatic pair. So, this is the direction of expansion of the prismatic pair or motion of the prismatic pair.

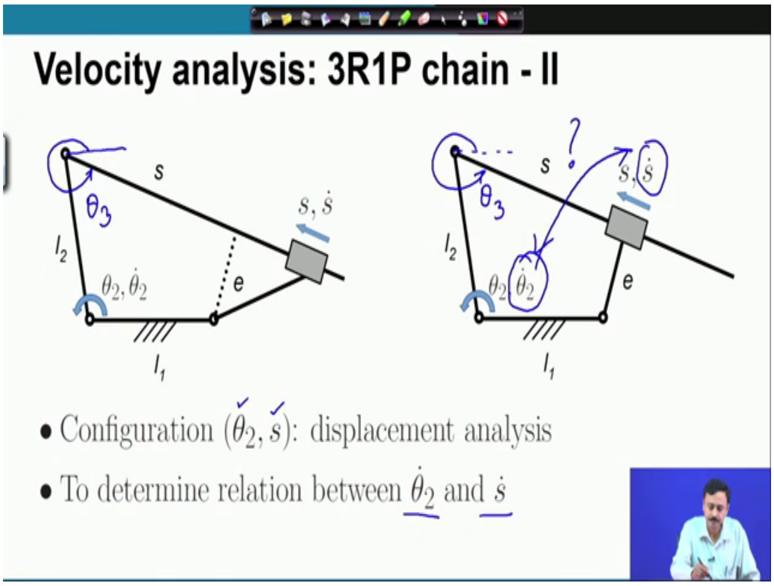
So, e is measured as the distance perpendicular to this direction of motion between the 2 hinges connecting the 2 links of the prismatic pair. So, the prismatic pair so is between so, if I number the links this is 1 the ground is 1 this link is 2 3 and this is 4. So, this prismatic pair is between links 3 and 4, and at the other end of link 4 we have this revolute pair and on the other end of the link 3 we have the other revolute pair.

So, it is a distance between these 2 revolute pairs measured perpendicular to the direction of sliding. So, that is offset. As you can see this angle is going to remain fixed this angle is going to remain fixed, because the prismatic pair does not allow relative rotation between the links 3 and 4 therefore, I can always shift this prismatic pair here. So, that this angle is 90 degree.

So, length of link 4 then I mean this, this length becomes e exactly e this simplifies the analysis substantially as we will see therefore, what we are going to do is replace this by the equivalent kinematic chain as shown on the right, these 2 are absolutely equivalent. And we redefine S as this distance that is s. So, once we have analyzed the mechanism on the right, we can always go back to the original mechanism, because these angles as I have mentioned between angles between 3 and 4 which remains fixed, we can always find the corresponding motions of the mechanism shown on the left.

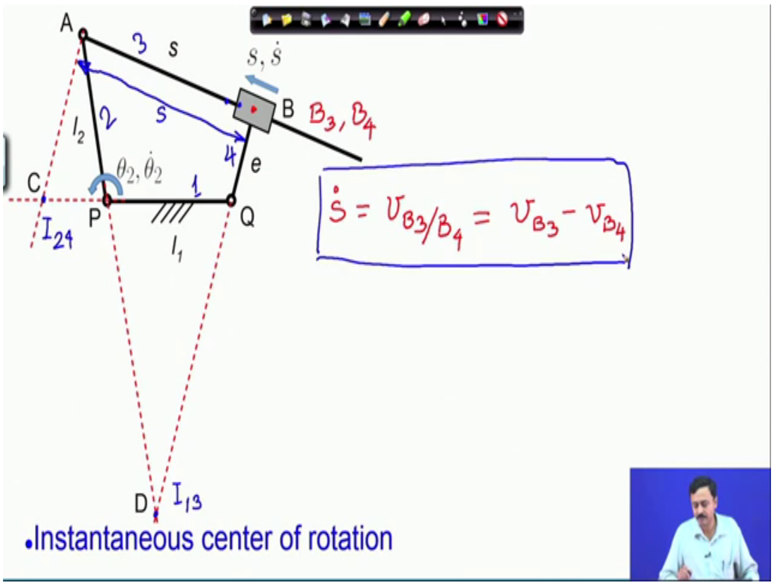
So, what is the velocity analysis problem? So, we are specified the configuration of the mechanism. Now since this is a constraint mechanism specification of theta 2 will also specify s, because we know from displacement analysis that given theta 2 I can always find s.

(Refer Slide Time: 08:46)



I can also find out this angle which is theta 3 or in this case here. So given theta 2, I can find out using displacement analysis S as well as theta 3, or I might be specified S then also I can find out theta 2 and theta 3, this we have already discussed before. So, the problem of velocity analysis is to determine the relation between theta 2 dot and S dot. So, we have here theta 2 dot S dot we want to find out the relation between these two.

(Refer Slide Time: 09:48)

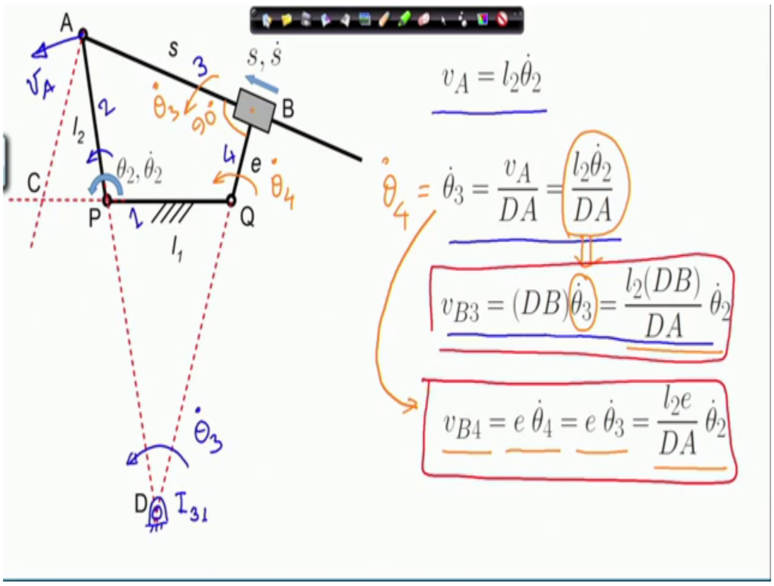


So, previously we had discussed about the instantaneous center of rotation, we had seen that we can determine these 2 instantaneous centers of rotation. So, this instantaneous center of rotation is I 1 3. So, let me again number so, this is 1 this is 2 this is link 3 and this is link 4. So, this is I 1 3 which is indicated by D, and there is another instantaneous center of rotation which is I 2 4 denoted by c. So, we are going to use these 2 instantaneous centers of rotation in this analysis, there is 1 fine point that must be remembered let me point that out this S of course, is this distance.

So, S is this distance now S dot is the velocity of a point belonging to link 3 with respect to a point belonging to link 4. Since S is this expansion of the prismatic actuator therefore, S dot is the relative velocity between a point on link 3 and a corresponding point on link 4 to make it more specific let me consider a point here. So, this point is denoted by B, now B can have 2 locations. So, coincident points B 3 and B 4. So, B 3 belongs to link 3, and B 4 belongs to link 4. So, is the relative velocity between B 3 and B 4 that is S dot.

So, S dot should remember is the velocity of B 3 relative to B 4 in other words this is equal to v B 3 minus v be 4. So, you have to remember this that S dot is the relative velocity between B 3 and B 4, which is given by v B 3 minus v B 4.

(Refer Slide Time: 13:50)



Now here I have written out all the velocity relations let us go through them 1 by 1, the first velocity relation says that v A is l 2 times theta 2 dot.

So, what is theta 2 dot theta 2 dot is the angular speed of link 2, times l 2 is the velocity of point a yes we know. So, this is v A, then I can find out the angular velocity of link 3 so, this is link 3 I can find out angular velocity of link 3, which I have written out as theta 3 dot is v A divided by this distance D A. So, velocity of point a divided by this distance D A, because d is the instantaneous center rotation remember for link 3.

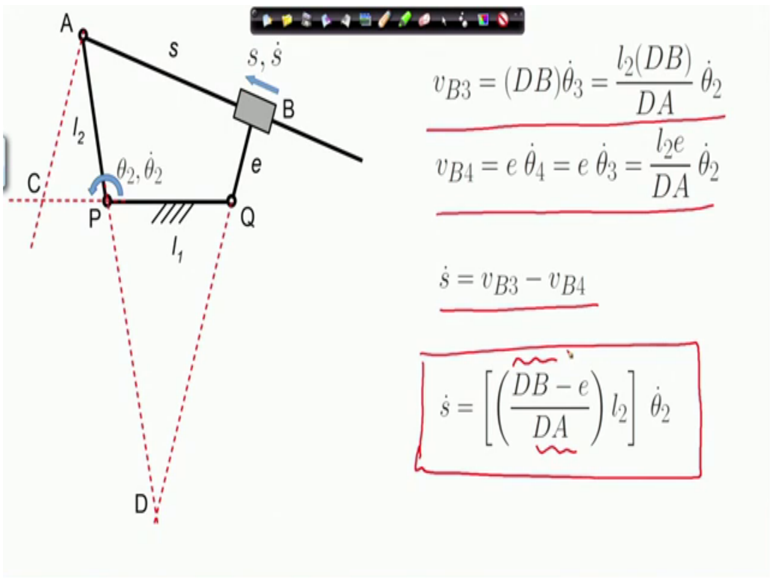
So, this is I 1 3 or 3 1. So, as if link 3 is hinged at this point. So, if I know velocity v A I can find out the angular speed of link 3, using the angular speed of link 3 I can relate velocity of B 3. So, what I have done here, I have multiplied theta 3 dot so, theta 3 dot. So, this you can consider that this is the angular speed of link 3. which is this whole triangle A D B. So, and B 3, A D B 3. So, therefore, velocity of B 3 is D B, this radial distance D B times theta 3 dot.

Now, here I have just replaced this theta 3 dot in terms of theta 2 dot, this I have replaced to get this expression. Now there is one more interesting observation this angle, this is 90 degree we know, and this angle remains fixed whatever be the relative motion of 3 with respect to 4. So, the sliding motion of 3 with respect to for whatever happens this angle 90 degree is going to remain 90 degree.

So, therefore, the angular velocity of link 3 is same as the angular velocity of link 4. So, this is theta 3 dot will be same as theta 4 dot. Why suppose I have this link? Which maintains this angle, if my palm has angular velocity theta 3 dot let us say then you can see my thumb also has the same angular velocity, because this angle is remaining fixed. So, for that same reason here theta 3 dot is also equal to theta 4 dots.

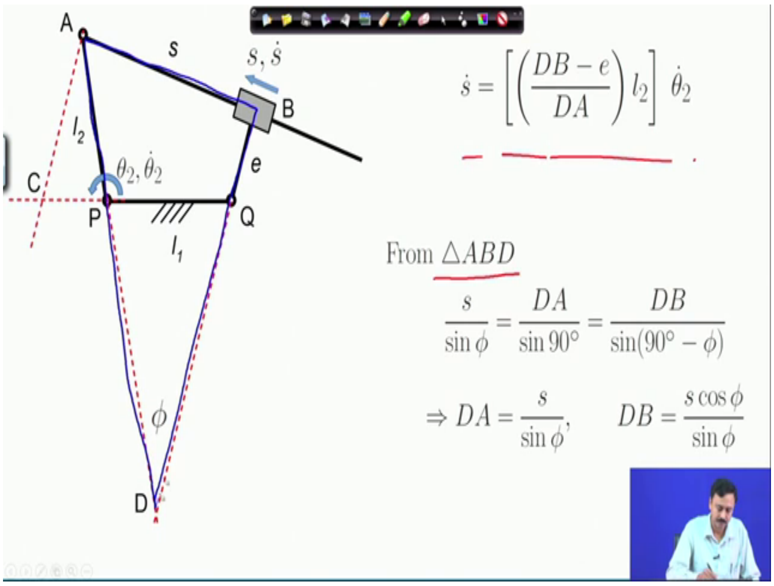
Now, I relate v B 4 which is a point on link 4, v B 4 is a well is B 4 is a point on link 4 and v B 4 is the corresponding velocity of that point on link 4, that must be e times theta 4 dot, e being the offset, as you can see from the figure, Now theta 4 dot is equal to theta 3 dot so, I can replace this. And since I have that expression of theta 3 dot, I use it here and I get v B 4. So, I have expressions of v B 3 and I have expressions of v B 4 S.

(Refer Slide Time: 18:51)



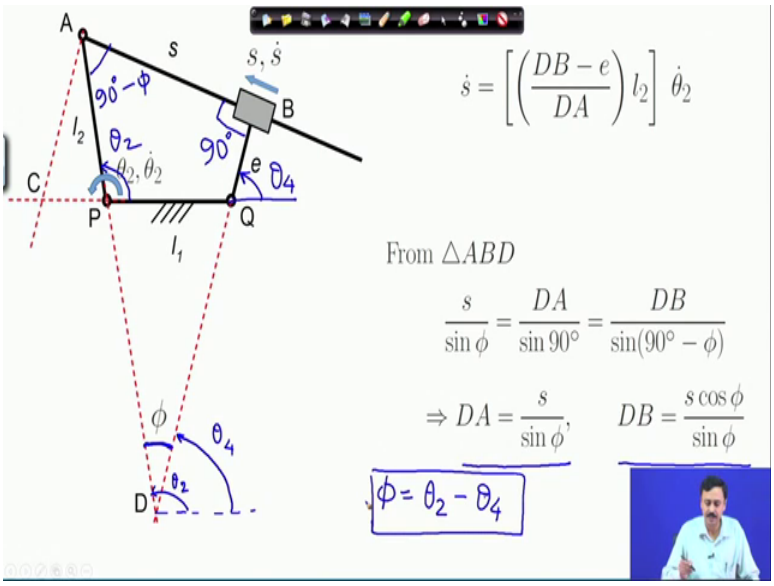
So, now, it is just a matter of taking the difference to relate S dot which I am going to do now. So, here I have rewritten these expressions of v B 3 and v B 4, and just few slides back I showed you that S dot is equal to v B 3 minus v B 4. So, therefore, S dot I can write like this in terms of theta 2 dot. Now here we have these 2 as yet unknown lengths D B and D A, which we are going to now determine from geometry.

(Refer Slide Time: 19:43)



So, I have again written out that expression of S dot relating the theta 2 dot as I mentioned D B and D A are as yet unknown. Let us look at this triangle A B D. So, A B D so we are looking at this triangle A B D so, using sin rule I can easily write S divided by this angle sin phi sin of this angle.

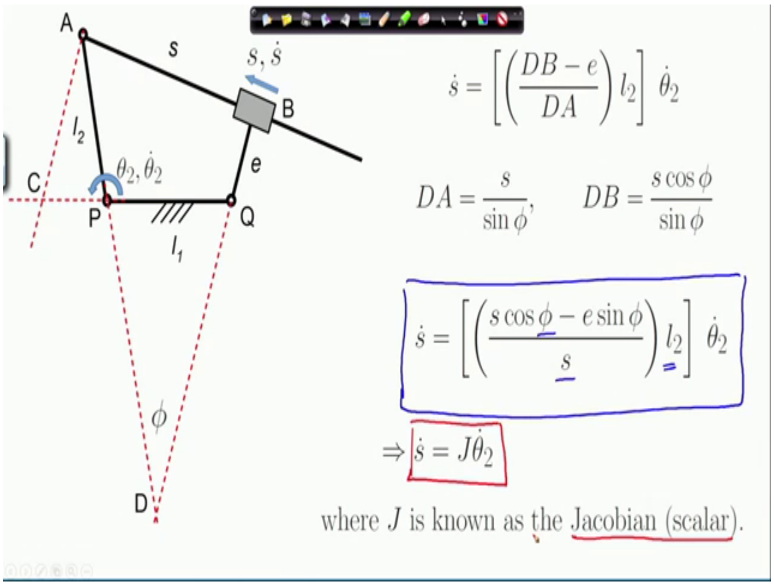
(Refer Slide Time: 20:38)



So, S divided by sin of this angle phi is equal to D A divided by sin 90 degree, this angle is 90 degree. And also equal to D B divided by sin of 90 minus phi. So, if this is phi this is 90 so, this must be 90 minus phi. So, that implies D A is equal to S divided by sin phi and D B is S cosine phi divided by sin phi.

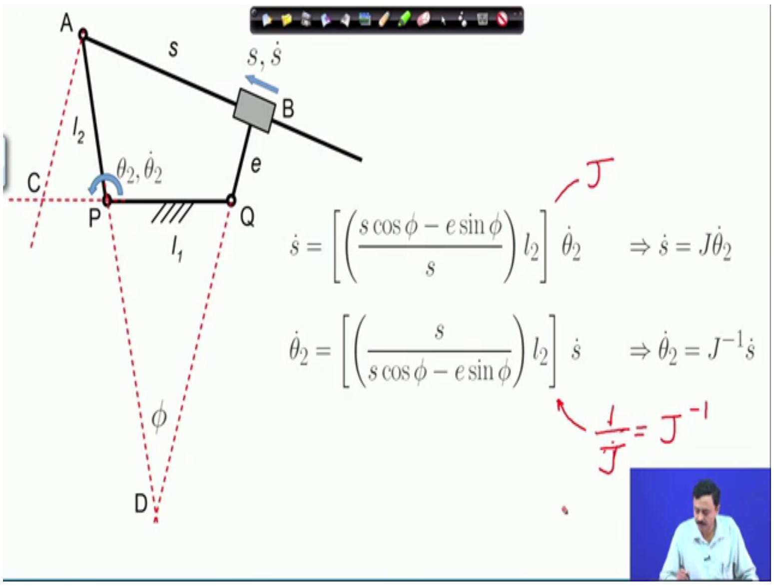
Now what is phi you can see that this angle is theta 2. And this angle is theta 4. So, in other words this angle is theta 2, and this angle is theta 4. So, therefore, there is a relation between phi theta 2 and theta 4, in this case it turns out that phi is equal to theta 2 minus theta 4. So, you can relate phi using theta 2 and theta 4, which we can find from the displacement analysis.

(Refer Slide Time: 22:32)



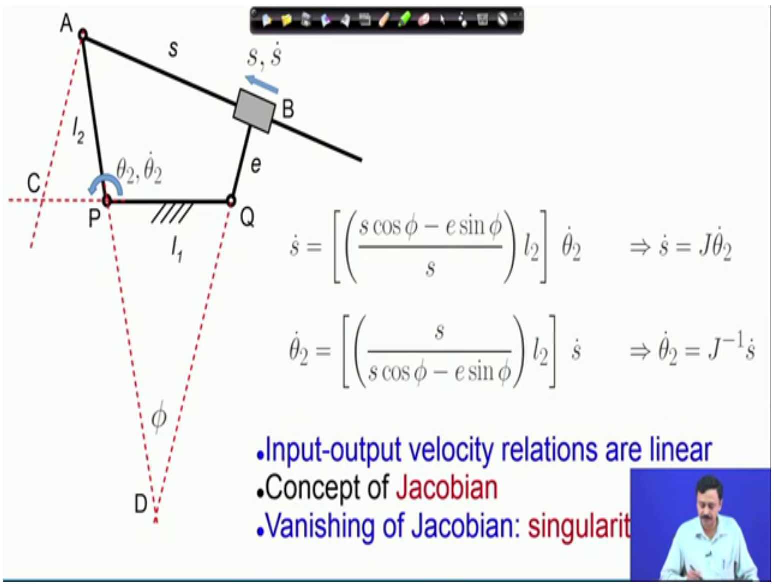
So, let us move on so, we have these expressions of S dot D A and D B, we have just now derived. So, when we substitute and simplify finally we have this expression, which uses S phi and others are the linked parameter. So, S l 2 so we have S phi l 2, l 2 is fixed S can be determined, if we are given the configuration and, similarly if I can also be determined from the configuration. Now we write this in a compact form this relation in a compact form using the Jacobian, which we are we have introduced here the Jacobian is a scalar once again it is a number. So, given S and phi you can write down the Jacobian, you can determine the value of the Jacobian.

(Refer Slide Time: 23:48)



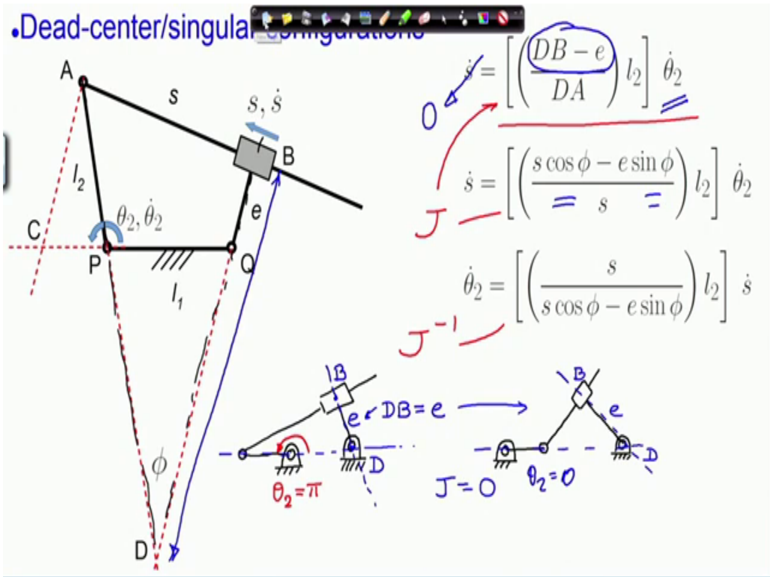
So, therefore, we have these 2 relations, now in the second relation I have inverted. So, this is the Jacobian, and this is the Jacobian inverse this is 1 one over the Jacobia so, which I am writing as Jacobian inverse. So, once again we are faced with the same question is the Jacobian invertible or is Jacobian inverse invertible, whether they can go to 0.

(Refer Slide Time: 24:50)



So, let us look at that issue so, we have already observed that the input output velocity relations are linear, and they are related through the Jacobian. So, we come to this question of vanishing of the Jacobian or its inverse, which indicates singularity in the configuration of the mechanism.

(Refer Slide Time: 25:19)

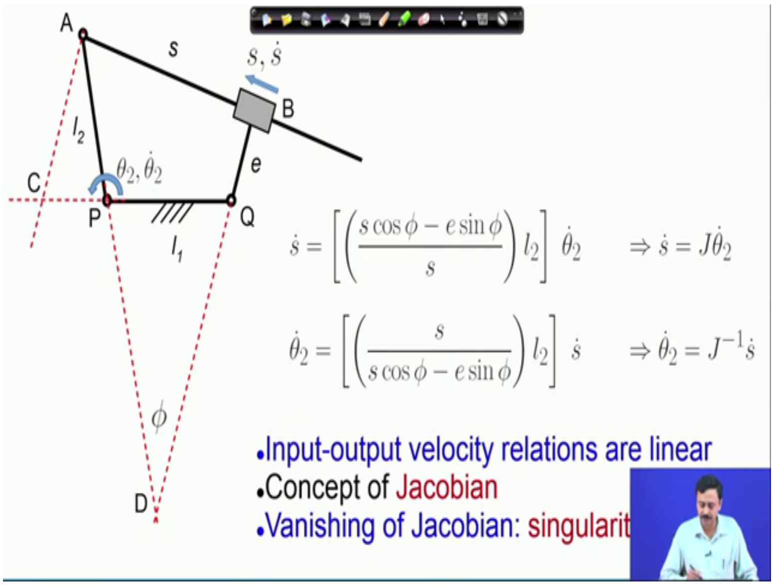


So, here I have written out the relations both direct and inverse. So, you can see very easily that so, this is the Jacobian, and this is the Jacobian inverse. So, when is this is also the Jacobian, the question is when is the Jacobian 0 it is very easy to see from the first expression, that the Jacobian goes to 0 when the numerator vanishes which means D B is equal to e. So, D B so this distance becomes equal to e. Now how can this happen, this can happen, in this way remember D is located by the intersection of the line through the point Q in the direction perpendicular to the sliding direction and through line through A and P. So, what happens in this case?

Let us see what happens in this case so, the line through this is passing like this, and the other line is passing like this so, where is d the intersection. So, that is d and where is B this point is B. You can very easily see now that D B is equal to e. So, this is the configuration where the Jacobian vanishes. So, therefore, theta 2 is equal to pi or 180 degree, there can be another configuration.

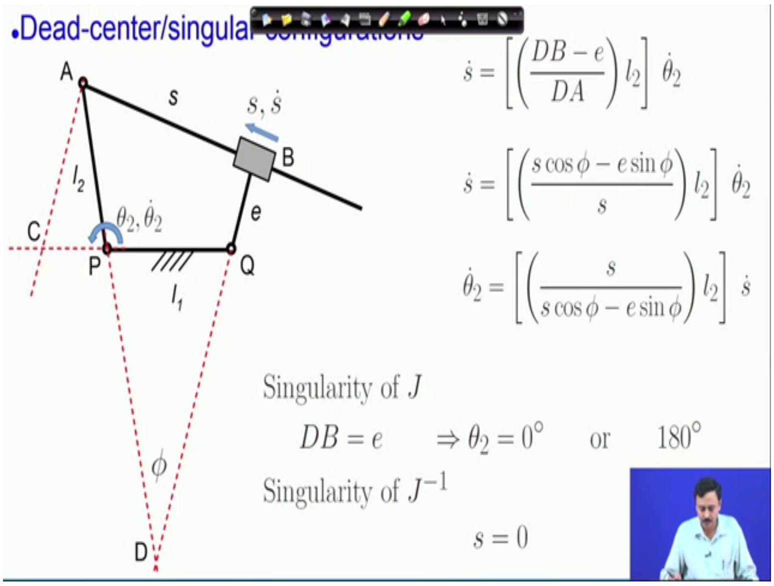
Let me draw that this is a configuration where again D B is equal to e. So, this is also a configuration where the Jacobian vanishes. So, in both these 2 configurations the Jacobian vanishes. So, it is not invertible. So, which means whatever be theta 2 dot S dot is 0 because the Jacobian is 0, irrespective of value of theta 2 dot. So, which means the prismatic actuator cannot expand at this these 2 configurations. So, these are singular configurations here, theta 2 is equals to 0. You can find these relations in terms of phi as well using this expression.

(Refer Slide Time: 29:30)



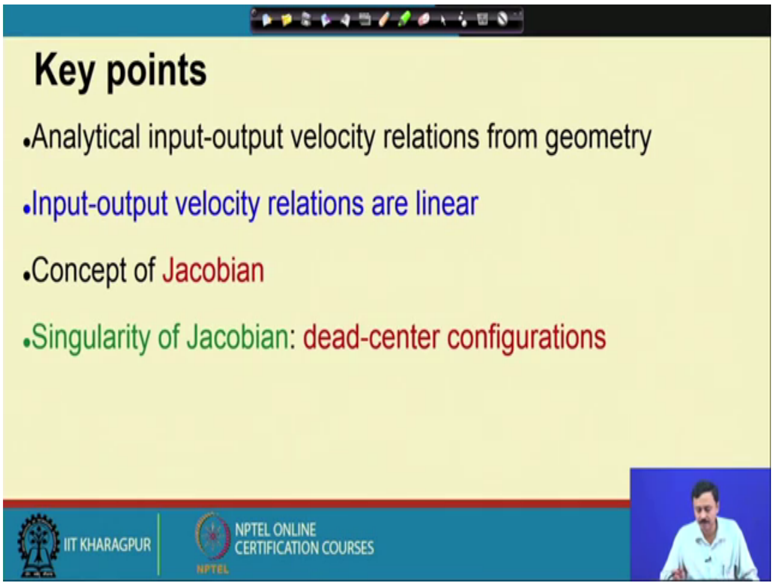
What is the other situation? The other situation is the other singularity is when the Jacobian inverse. So, this is Jacobian inverse so, Jacobian inverse vanishes when S equal to 0, and when can that happen at this configuration something like this. So, here S equal to 0, and that is the singularity where theta 2 vanishes irrespective of value of S dot. So, these are the 2 singular configurations, which I have written out for you.

(Refer Slide Time: 30:55)



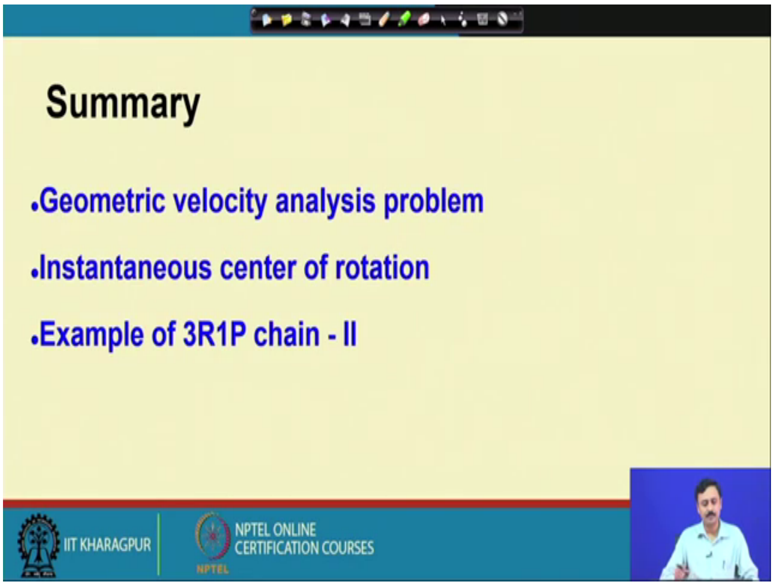
So, what are the key points we have looked at the analytical input output.

(Refer Slide Time: 30:57)



Velocity relations from geometry which are linear, the input output velocity relations are linear. We introduced the concept of Jacobian and we have looked at the dead center configurations of the mechanism.

(Refer Slide Time: 31:12)



So, with that let me summarize, we have looked at the geometric velocity analysis problem for the 3R1P chain of type 2. And we have looked at the singularities of the kinematic chain.

So, with that let me close this lecture.