Mechanism and Robot Kinematics
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Lecture – 24
Velocity Analysis: Application of Geometric Concepts – II

We will be discussing the velocity analysis problem for constraint mechanisms, which we have started. So, in this lecture we will consider the same problem, we will go through this the velocity analysis problem based on geometric methods and we will look at a different example today. So, to give you the overview, we are going to look at this velocity analysis problem using the instantaneous center of rotation concept for a 3R1P chain of type 1.

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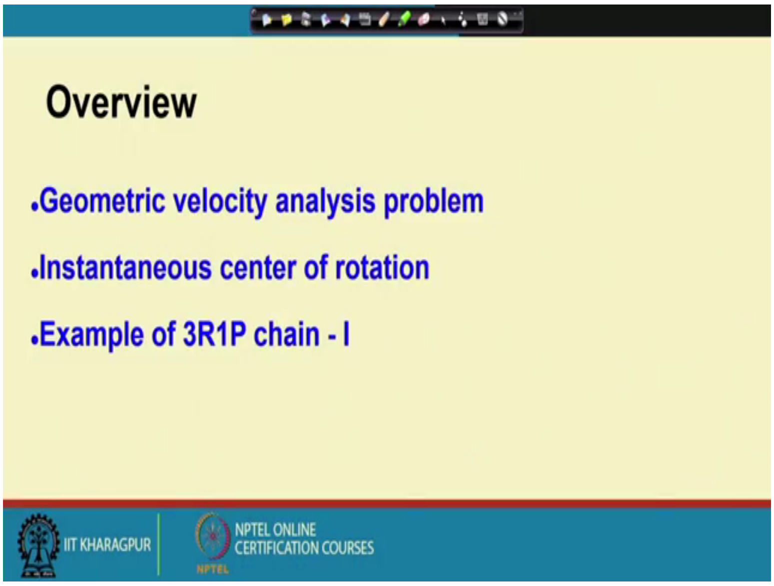

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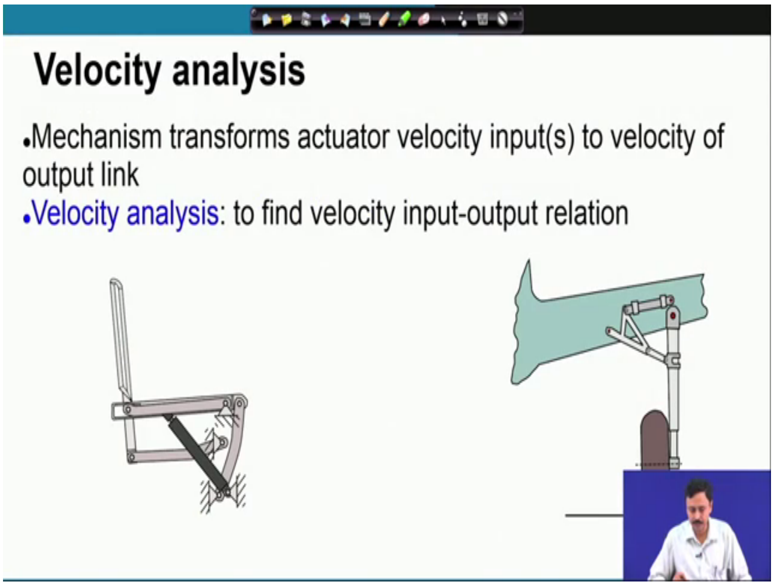

So, we have looked at what velocity analysis problem is about, it is about finding the input output velocity relations and we have the 2 problems forward and the inverse problems, which are very easily found, if you have solved 1 the other is very easy to find.

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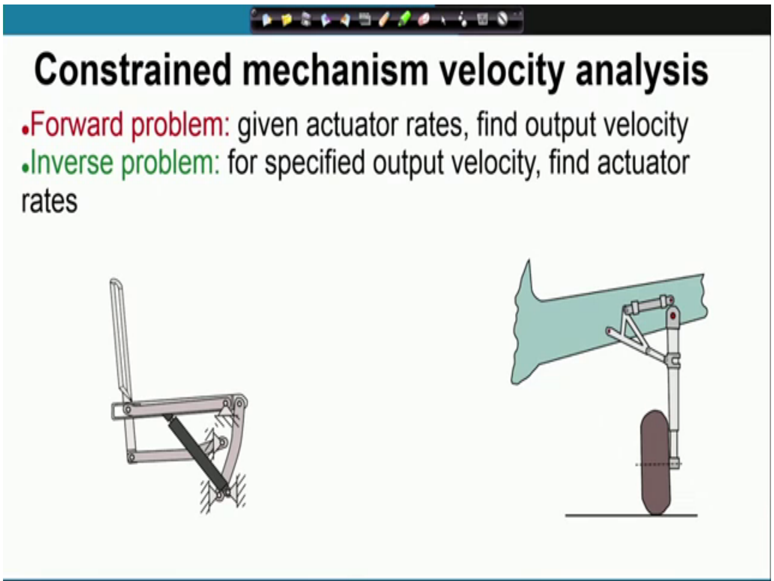

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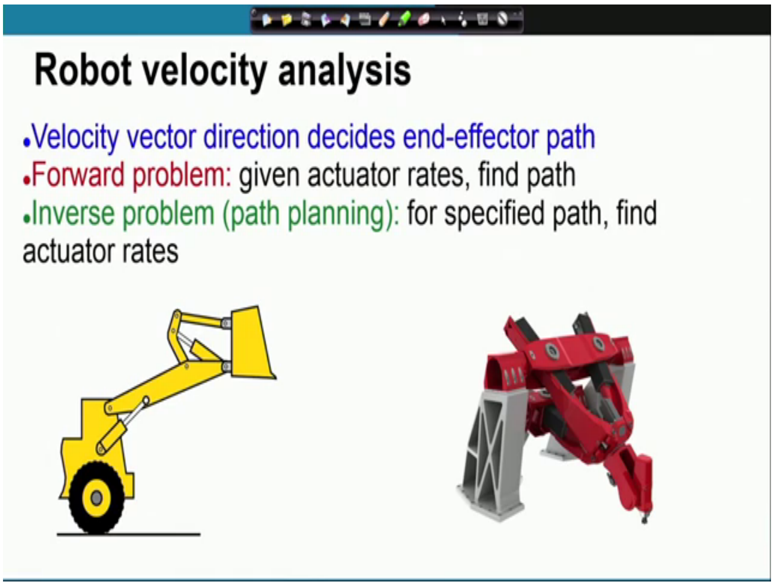

In the case of robots, we have seen that velocity analysis problem is little more complicated, where we need to specify the velocity along a path because the velocity vector will always be tangent to the path and corresponding to that velocity vector, we can find out the actuator rates through the velocity analysis by solving the velocity analysis problem. So, this inverse problem in which the path is specified, we have to find out the actuator weights, this is known as the path planning or path generation problem.

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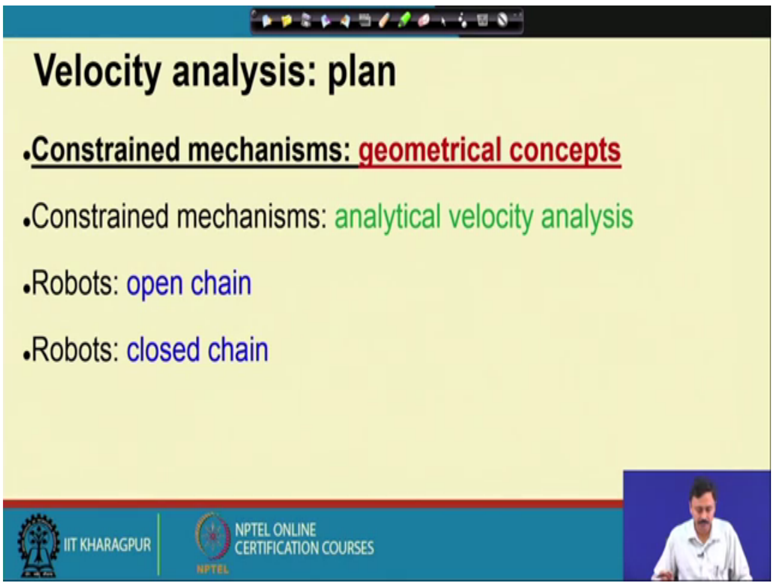

So, we are going to continue our discussions on constraint mechanisms, velocity analysis through geometric concepts, later on we will look at analytical velocity analysis and also for robots with open and closed chain configurations.

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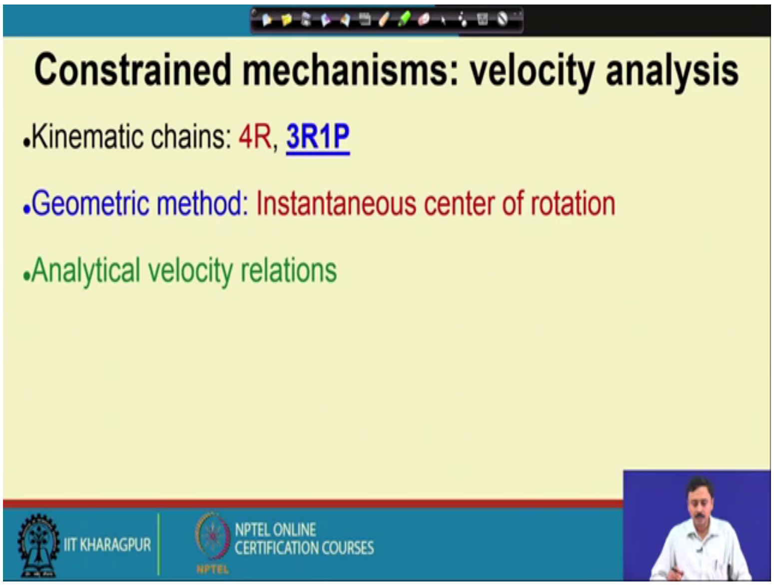

In this lecture, we are going to discuss the 3R1P chain, using the geometric method of instantaneous center of rotation and we are going to derive analytical relations for the velocity.

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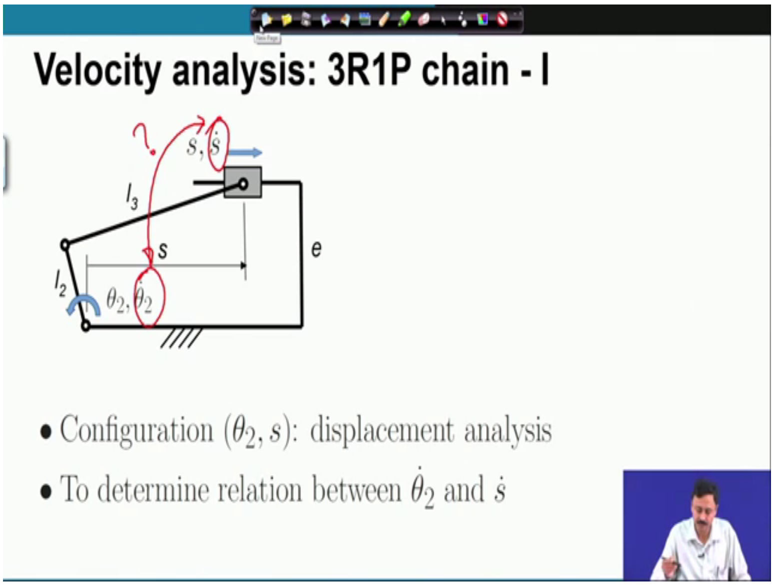

So, this is our 3R1P chain type 1. Here, we have this angle theta 2 and the displacement of the slider S, now the configuration of the mechanism is specified either by specifying theta 2 or by specifying S and through the displacement analysis problem by solving the displacement analysis problem I can solve the other. So, when I say configuration is given, then I will, I was mean that everything like theta 2 S and even this angle, this angle theta 3 are known to me. So, when I say the configuration is known to me, I would mean that theta 2 S theta 3 these are all known to me and this is ensured by the displacement analysis problem.

If I saw the displacement analysis problem, I have solved for these relations. So, I know the configuration of the mechanism. The problem is, now to determine the relation between theta 2 dot and S dot. So, I want to find out the relation between these 2, given the configuration I would like to find out the relation between theta 2 dot and S dot that is the velocity analysis problem.

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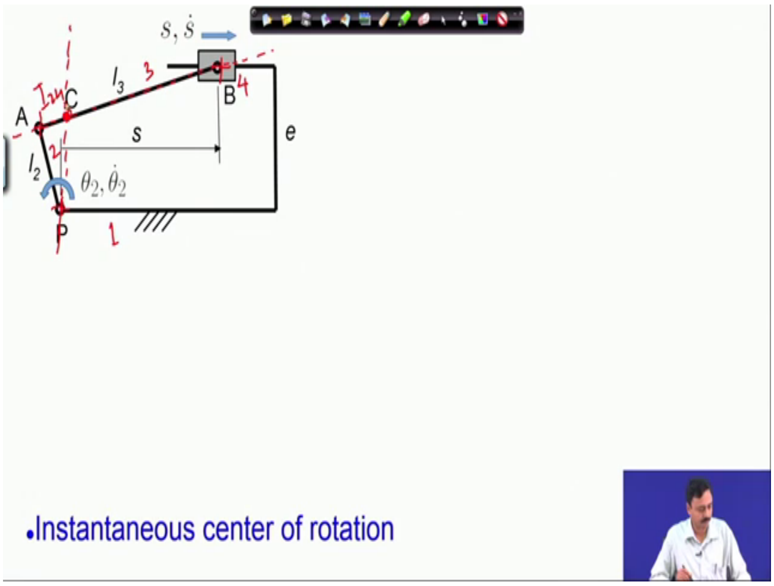

So for this, as I have mentioned we are going to use the geometric concept of instantaneous center of rotation. So, the instantaneous center of rotation that I will use for this analysis is I 2 4. So, this lies on this line passing through I 2 3, let me mark out this is 1, this is 2, this is 3 and the slider is length 4. I want to find out I 2 4, so I 2 4 must lie on, the line connecting I 2 3 till this point and I 3 4 and it must also lie on the line connecting I 1 2, which is this ground hinge P and I 1 4 which is at infinity, remember and this is obtained by drawing this line perpendicular to the direction of sliding through the point P, so this point is I 2 4.

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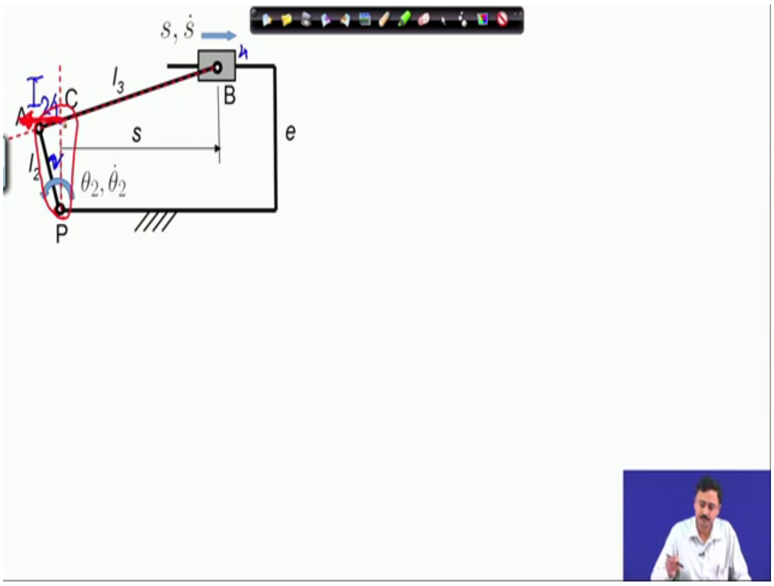

So, formally I have done this for you, so C is the instantaneous center of rotation I 2 4. So, this is I 2 4, what is the significance of I 2 4? Let us recollect, it is the point on extensions of bodies 2 and 4, which have the same velocity at that point. So, the coincident point belonging to link body 4, which is the slider and link to which is this link. So, the coincident point belonging to links 2 and 4 at this point at the point C has the same velocity and which must of course, is something like this. So, this velocity must also be the slider velocity, that means, velocity of body 4 and it also must be the velocity of a point, which is on the extension of link 2 that is the concept of I 2 4.

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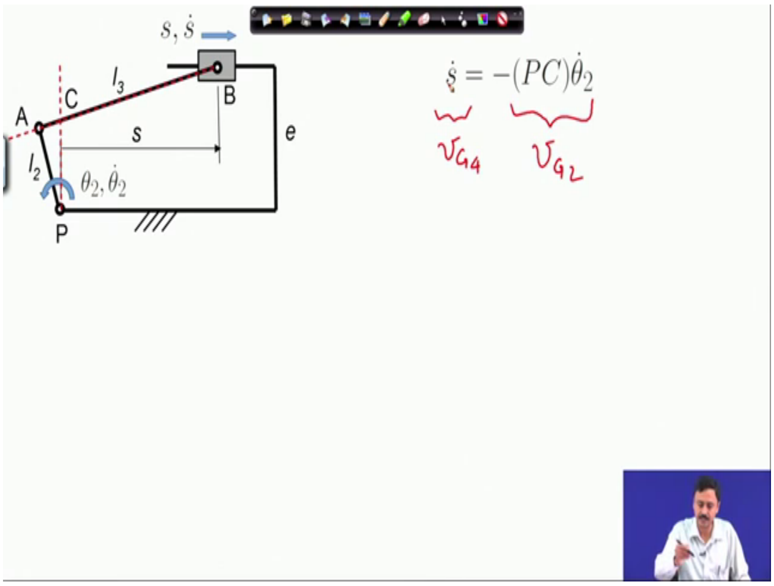

So, what do we do? With the sign convention that I have taken, that theta 2 dot is counter clockwise positive, while S dot is positive to the right, I must write S dot is equal to minus the distance PC times theta 2 dot has to be this is the relation, this is what the point C stands for. The velocity at that point belonging to the link 2 is same as the velocity of the slider. So, in other words, this is nothing but velocity of G 2 and this is also equal to then, velocity of G 4 which is also S dot, so this is the idea.

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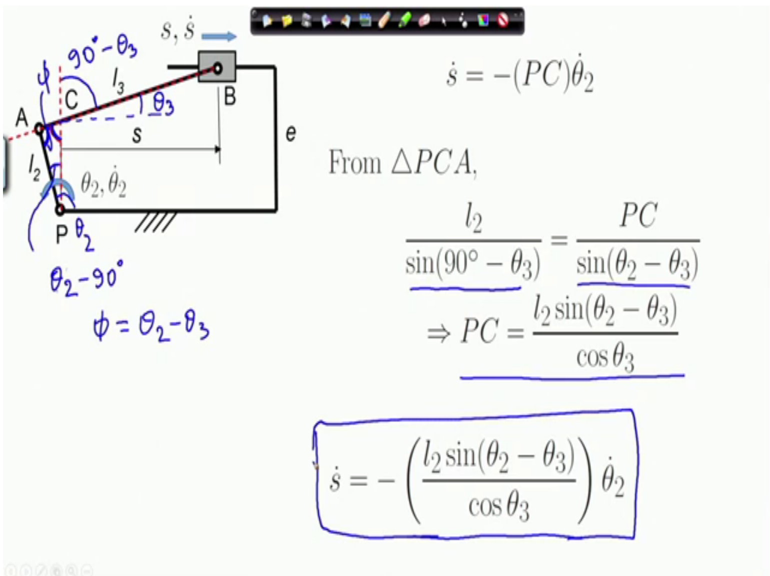

Now, we have to find out PC. To do that, we look at triangle PCA. So, you can very easily find out PCA and use the sine rule. You can see that l 2 divided by this angle, now what is this angle? Remember that this is theta 3, so the angle that length 3 makes with the x axis of the horizontal axis is theta 3. So, therefore, this angle must be 90 degree minus theta 3 and that should also be equal to then this angle. So, therefore, l 2 divided by sine 90 degree minus theta 3, must be equal to PC, PC is this distance, this arm of the triangle divided by the sin of the angle opposite to it. So, this is the angle, which you can very easily relate as theta 2 minus theta 3. How this angle is theta 2? This angle is theta 2; so therefore, this angle is 90 degree minus theta 2.

So, this angle is 90 degree, so theta 2 minus 90 degree. So, this angle, so that will be theta to minus 90 degree and if I call this angle as phi, which I want to determine, then you can very easily see that phi must be equal to theta 2 minus theta 3, so that is what I have written out. So, from this I can solve for PC as l 2 sine theta 2 minus theta 3 divided by cosine theta 3. Now, there is 1 point to be noted here, I have drawn the mechanism in a certain configuration and for that this is the calculation. If the mechanism is in a different configuration, then the procedure remains the same; the procedure will remain the same, the relations might change or the triangle might appear in a different manner and finding out these angles might be slightly different. So, these angles might turn out to be slightly different, but the basic approach remains the same. So, we are going to look at such it will going to find such a triangle and we can relate, we can find out this distance PC geometrically.

So, that is our objective, so we are finding out PC by geometry, the distance PC by geometry. So, therefore, if I substitute this expression, then S dot is equal to minus of this expression, this PC time's theta 2 dot. So, we have found the relation between S dot and theta 2 dot.

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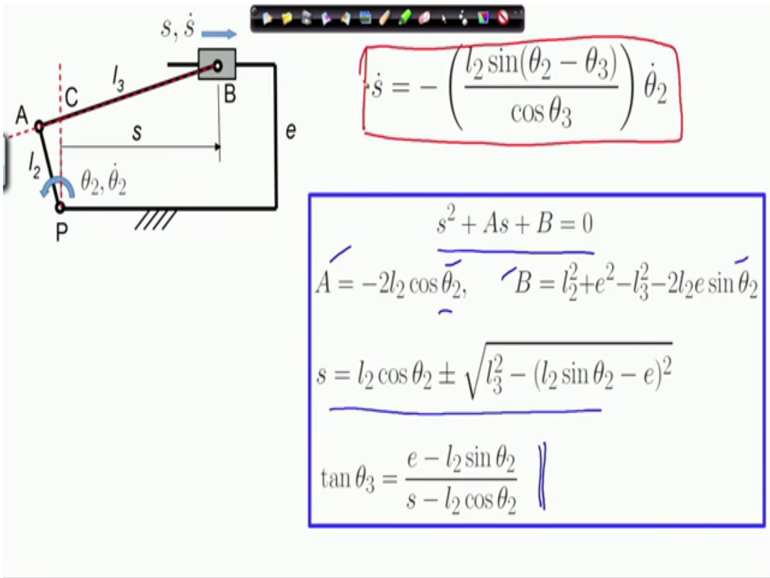

So, this is our relation, now remember our displacement analysis. So, given theta 2 let us say, if I am given theta 2 then through this displacement analysis, we had found S. Here, ABC, A B are completely known because theta 2 is given, so these are known, so A B unknown. Hence, we could solve for S and we call also we could also solve for theta 3. So, therefore, in this relation of velocities I know everything.

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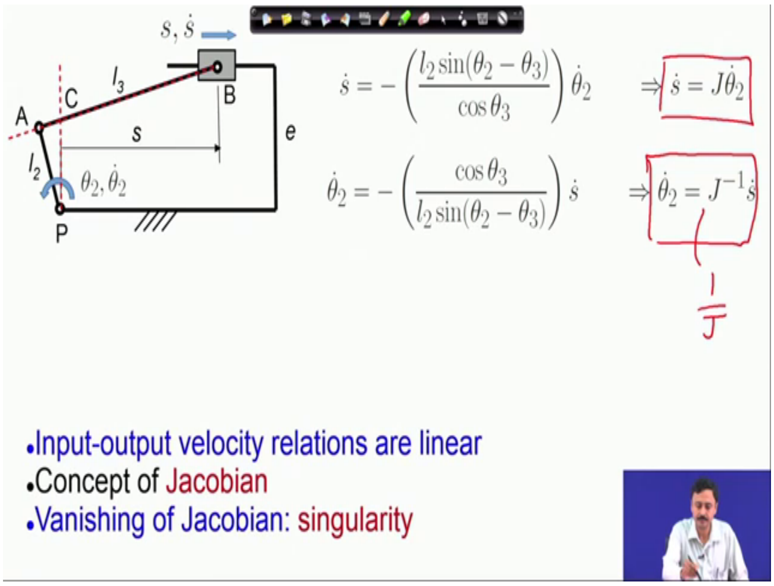

So, I will write this in a compact form as I have done here. So, S dot is the Jacobian times theta 2 dot and by inverting the Jacobian I can write theta 2 dot is Jacobian inverse, which is again 1 over J, Jacobian inverse here it is a scalar, so Jacobian inverse is 1 over J times S dot. Again, we find that the input output velocity relations are linear and we have introduced this concept of Jacobian and when I vanishing of the Jacobian implies singularity and this is what we are going to study next.

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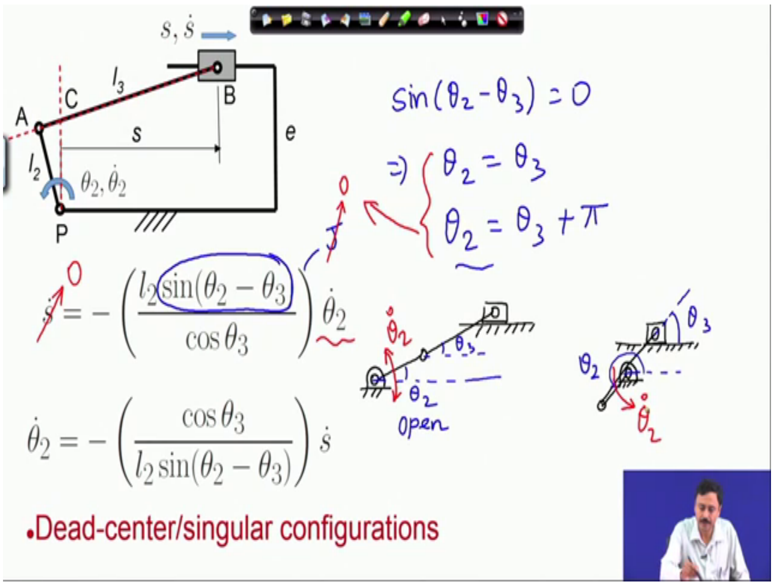

So, singularity is again take us to the dead center configurations. So, let us understand these singularities, so this is our Jacobian, when is Jacobian singular, that means, non invertible, when it becomes 0. Jacobian is singular or non invertible when it becomes 0. So, in other words sine of theta 2 minus theta 3 is 0, that implies either theta 2 equal to theta 3 or theta 2 is theta 3 plus pi radian. So, both these 2 conditions we have the singularity or Jacobian.

Now, what does that imply, Jacobian goes to 0, at such configurations the Jacobian goes to 0, which means that irrespective of value of theta 2 dot S dot goes to 0, that means, the slider is not going to move. So, slider is in the dead center configuration, mechanism is in the dead center or singular configuration. So, let us see these configurations, so there are 2 solutions, so corresponding to these 2 solutions we have the singular or dead center configurations, this is 1 where as you can see theta 2 is equal to theta 3.

So, this is the open configuration, there is another configuration corresponding to the second solution here, where we have a situation like this. So, theta 2 is this angle and theta 3 is this angle, now you can very easily see that theta 3 plus 180 degree is theta 2. So, this is the second dead center configuration and at these configurations irrespective of how theta 2 dot is specified or how theta 2 is changing, irrespective of the value of theta 2 dot we will have the slider at the static position, so thus the slider velocity will go to 0. So, this is corresponding to the singularity of J, now let us look at the other situation. This is J inverse or 1 over J, now when will this go to 0.

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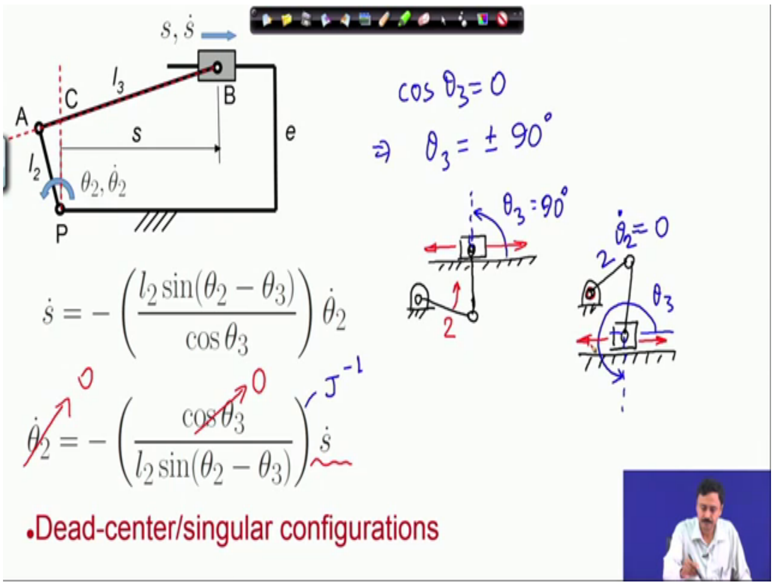

So, this j inverse goes to zero, when cosine theta 3 goes to 0 that implies theta 3 is equal to plus or minus 90 degree then cosine theta 3 is 0. So, again there are 2 configurations let us look at these configurations first, if you have you can have a situation like this in a mechanism in a 3R1P chain, where you see this angle, this angle is theta 3 is 90 degree. Then, what this says is, the numerator of the Jacobian inverse goes to 0. So, therefore, irrespective of the value of S dot, theta 2 dot goes to 0, you can realize, when I try to move the slider left or right at this configuration this link 2 cannot rotate.

If I move to the right, if I move the slider to the right this will move up, if I move to the left also this will move up, but exactly at this configuration it cannot move any further it has to come to a stop, this is the 0 velocity configuration of the dead center configuration of the link 2. In a similar manner I can have the other configuration of minus 90 degree in you a situation like this. Remember, that angles are being measured like this, so either you say that this is 270 degree or minus 90 degree, so both are same. So, this is another singular configuration, where link 2 cannot move.

So, theta 2 dot must necessarily be 0, it has to be 0 irrespective of the motion of the slider. So, this is another configuration, which is a singular configuration or the dead center configuration. So, I have written out these velocity relations again.

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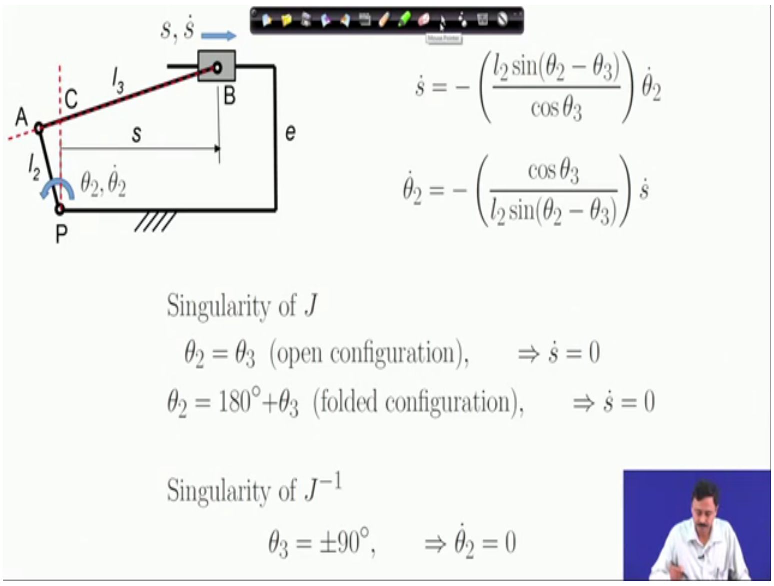

So, singularity of the Jacobian occurs when theta 2 equals equal to theta 3 the singularity is an open configuration 1, where S dot is 0 or you can also have a singularity when theta 2 is 180 degree plus theta 3 in which we have the folded configuration, in which also S dot is 0. If you look at the singularity of J inverse, then we have it at theta 3 equal to plus or minus 90 degree and here theta 2 dot is 0, that means, the link 2 cannot move at exactly at that configuration, it must have 0 velocity irrespective of the velocity of the slider.

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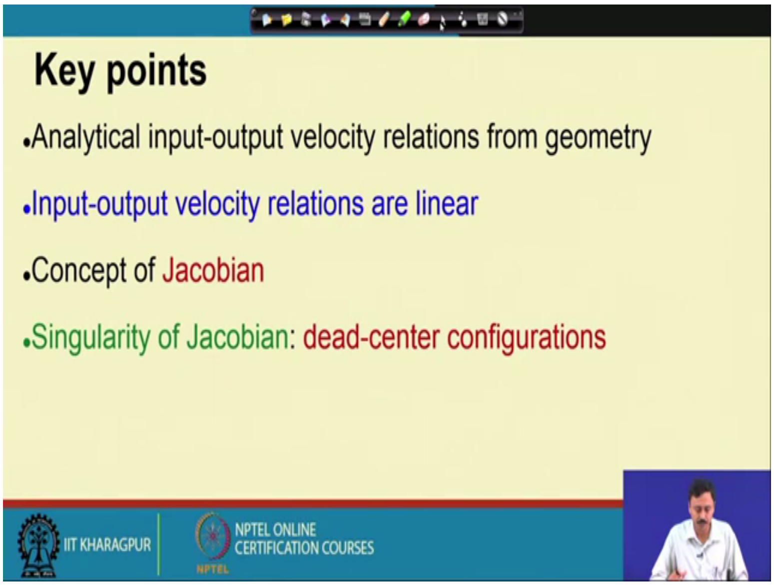

So, let us recapitulate the key points, we have found the analytical input-output velocity relations from geometry. Now, here I must mention again that we have used the concept of instantaneous center of rotation of the mechanism and then from geometric relations we have tried to find out certain lengths.

Now, if the configuration of the mechanism changes, the procedure remains the same, but the triangle may look different, but you have to device a method of finding out these distances like here we had this distance from the ground hinge to the instantaneous center. So, that we determined geometrically. So, that geometry 1 has to do, irrespective of the configurations mechanism and a every time you will find that this can be done. So, we have found the analytical input-output the velocity relations from geometry, which were found out to be linear.

We have introduced again, brought in the concept of Jacobian and the singularity of the Jacobian gave us the dead center configurations or the singular configurations of the mechanism. So, these dead centre configurations, singular configurations they are synonymous with the singularity of the Jacobian or the singularity of the inverse of the Jacobian.

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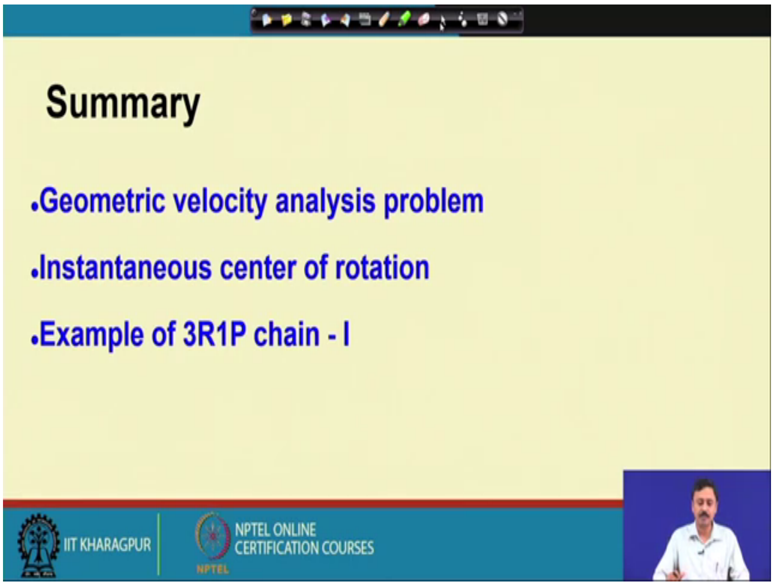

So, let me summarize, we have discussed the geometric velocity analysis problem using the concept of instantaneous center of rotation and we have considered this example of the 3R1P chain of type 1, we have looked at the singularity of the input-output velocity relations and we have discussed about the dead center configurations of the mechanism.

With that I will conclude this lecture.