Mechanism and Robot Kinematics

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Lecture – 23
Velocity Analysis: Application of Geometric Concepts – I

So, we will be discussing the velocity analysis problem in this lecture. We started our discussions on velocity analysis with certain geometric concepts and we are going to look into the application of these geometric concepts in velocity analysis of mechanisms. So, to give you an overview of what we are going to discuss in this lecture, we are going to discuss the geometric velocity analysis for mechanisms, using the concept of instantaneous center of rotation and in this lecture we are going to look at the 4R chain.

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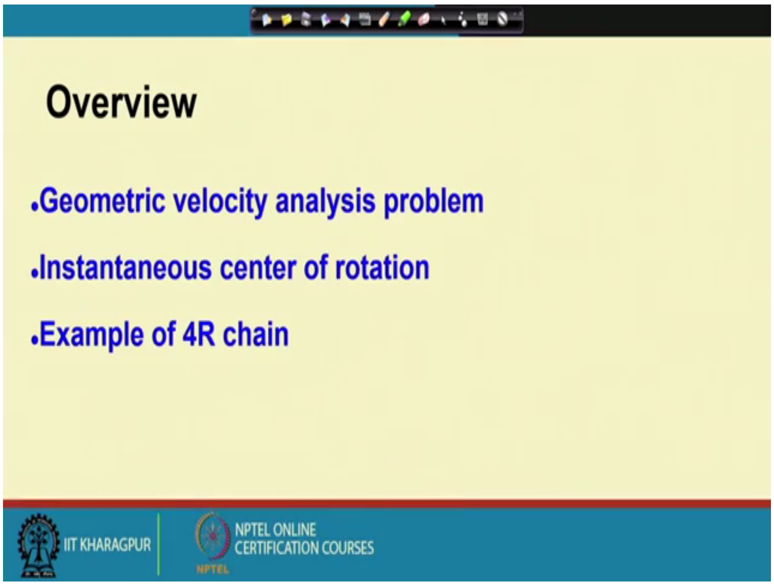

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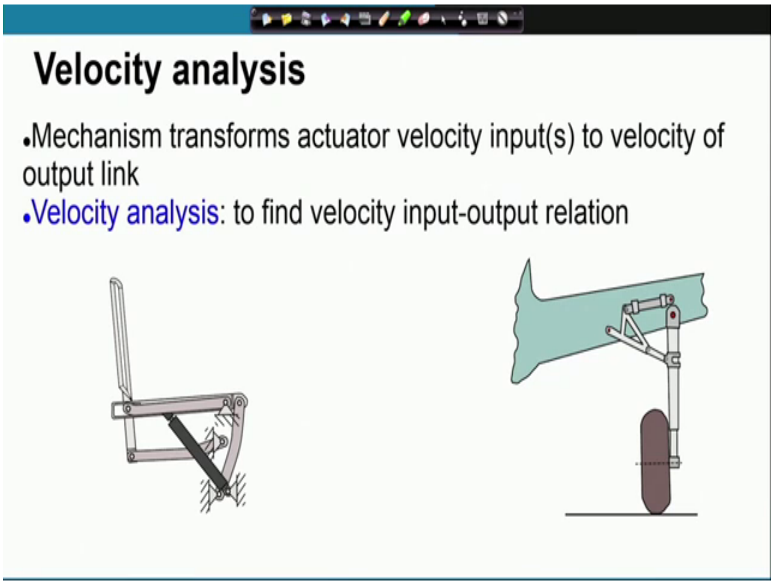

So, we have already discussed, let me just review what we discussed about velocity analysis. So, mechanisms transform the actuator inputs there can be multiple actuator inputs, velocity inputs to the velocity of the output link. Now, this output link could be, having a translatory motion or a rotary motion or it may be motion in space on in the plane or in space, it may also have orientation etcetera.

So, the problem of velocity analysis is to find the velocity input-output relation, input-output means from the actuator to the output and the reverse.

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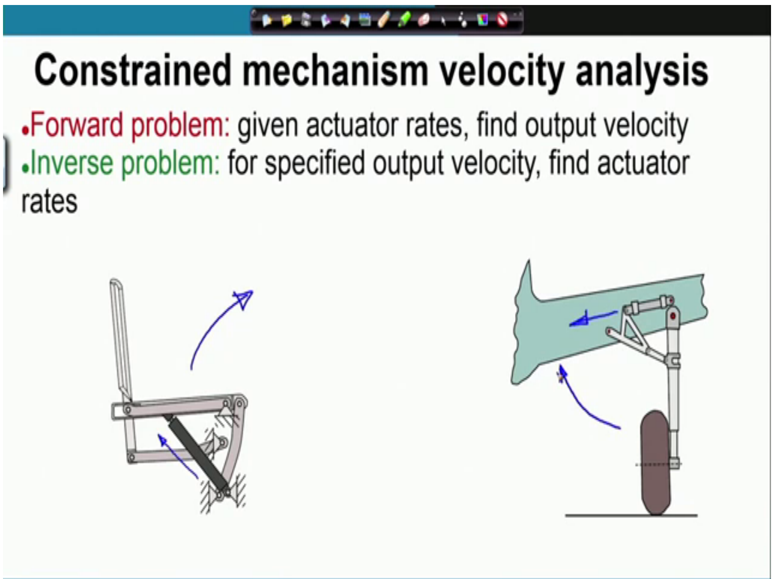

So, in the constraint mechanism velocity analysis as you know we have defined this, forward and inverse problems. The forward problem in the forward problem we are given the actuator rates and we want to find out the velocity of the output link and is it just the reverse for the inverse problem where the output velocity or the desired output velocity is given to us and we want to find out, how the actuator should move? What should be the actuator rates? So, that I can meet the output velocity specification, so here are 2 examples that we had considered, this is the transfer device where I need to find out the act the hydraulic or the pneumatic actuator expansion rate in order to provide a certain output rate for taking a person from the sitting position to the standing position.

So, that relation we need to find out or for in this case of the landing gear, we may have to find out what should be the actuator expansion rate. So, that I can fold the wheel at a certain desired rate.

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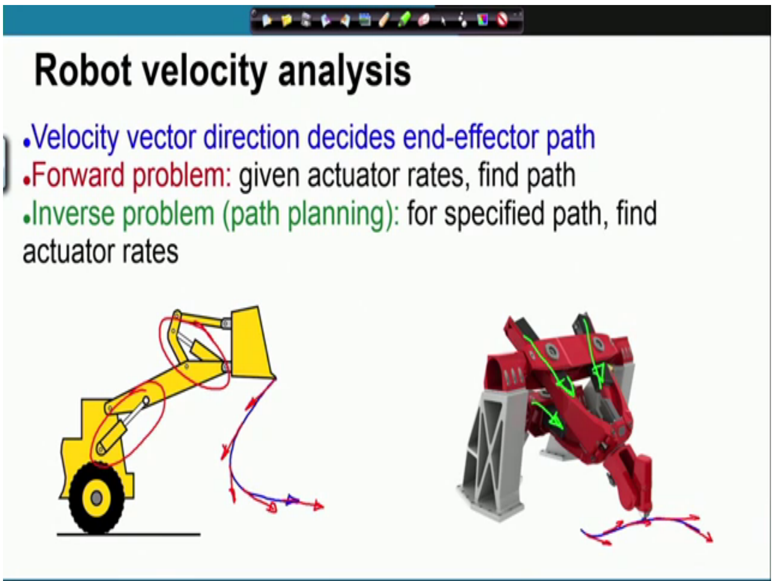

In the case of robot velocity analysis, the problem is little more complicated. The reason is now we are moving a body in space, we may have a certain path in which this body needs to be moved and therefore, I would like to find out how I should move, so that this path can be traversed, 1 way is to specify the velocities at various points on this path. So, I should be able to specify the velocity at each point on the path and corresponding to that I should be able to find out the expansion rate of the actuators, the same case in this second example of the robot manipulator.

If I am able to specify the velocity along the path and determine what should be the actuator expansion rate, corresponding to that velocity that I desire at the output then I should be able to follow the path.

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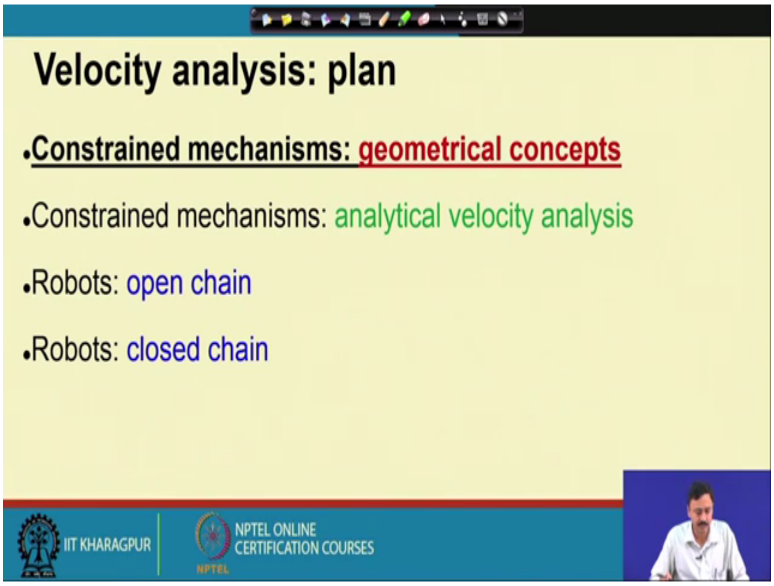

Now, our plan to study velocity analysis we are continuing with our constraint mechanisms and geometric concepts, later on we will move to analytical velocity analysis and subsequently to robots.

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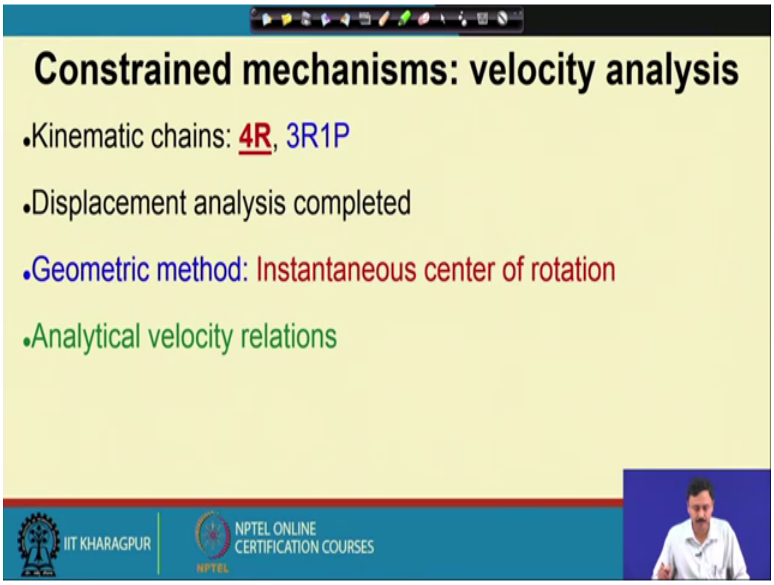

So, in this lecture we are going to look at the 4R kinematic chain, we will assume that the displacement analysis is completed, in other words, I know the configuration of the mechanism, this is a very important point. So, before I embark upon the velocity analysis problem, I must have the displacement analysis problem completely solved, in other words, I should be able to find out the configuration of the mechanism or I should know the configuration of the mechanism.

We are going to pursue the geometric method based on the instantaneous center of rotation and derive analytical velocity relations. So, this is the plan for the current lecture.

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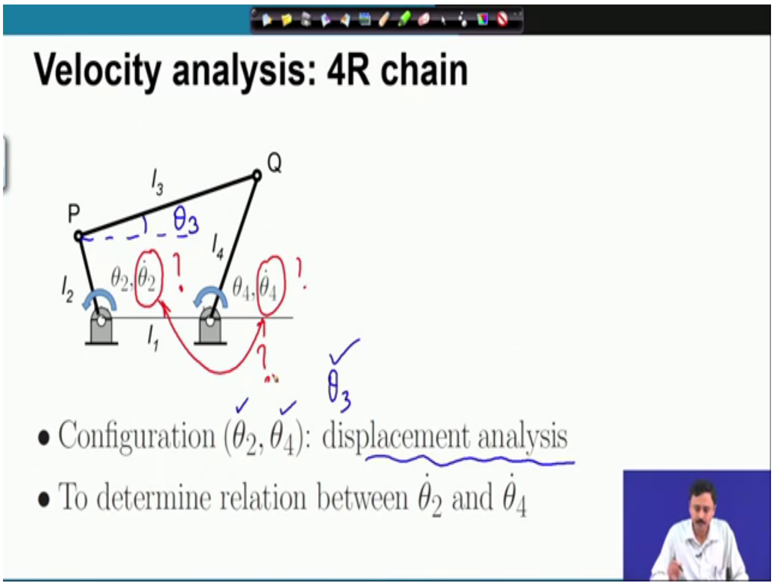

So, let us look at the problem of velocity analysis for a 4R chain. So, configuration is specified, here I have written out theta 2 and theta 4. Of course, theta 3 is also to be included, so this is theta 3. So, if I am given theta 2, I will definitely know theta 4 and they will also know theta 3. So, that is the displacement analysis problem, which I have assumed is completely solved. So, all these angles are completely known to me, configuration of the mechanism is completely known to me. I want to find out the relation between this theta 2 dot and theta 4 dot, so the relation between the 2. So, this relation is what I want. So, that is the velocity analysis problem.

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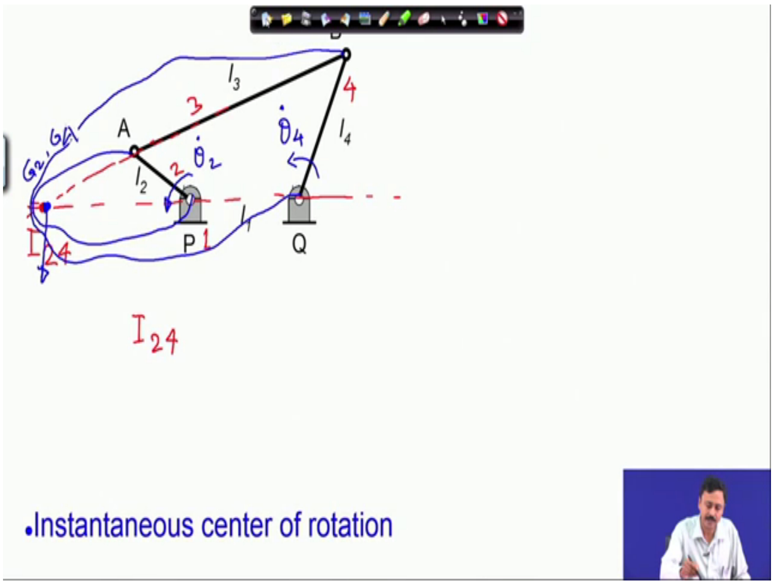

Now, we have discussed a location of the instantaneous center of rotation. So, let me just tell you, how we located the, I 2 4. So, the ground is 1 this is 2, this is 3 and this is 4 these is are the link numbers and how we located I 2 4, it lies on the line of I 1 2 and I 1 4 and it also lies on the line I 2 3 and I 3 4, so this is the line. So, here we have I 2 4, now this is what we need. Now, why do we need this? We just briefly explain, we have already gone through this. So, if I have an angular speed theta 2 dot for link 2 and theta 4 dot for link 4, remember that this is the point this I 2 4 is the point where the velocity of an extension of link 2 and an extension of link 4 has the same velocity. So, velocity of a point G 2 and G 3, G 2 belongs to link 2 and G 3 belongs to not G 3, but G 4 in this case G 4 belongs to link 4 or an extension of link 4. So, these 2 points will have the same velocity, same magnitude, same direction.

So, they have the same velocity G 2 and G 4 have the same velocity, these are the coincident points 1 belonging to link 2 the other belonging to link 4. So, that is I 2 4 that is the significance of I 2 4. So, now, let us see how this is this information is used to find out the relation between theta 2 dot and theta 4 dot.

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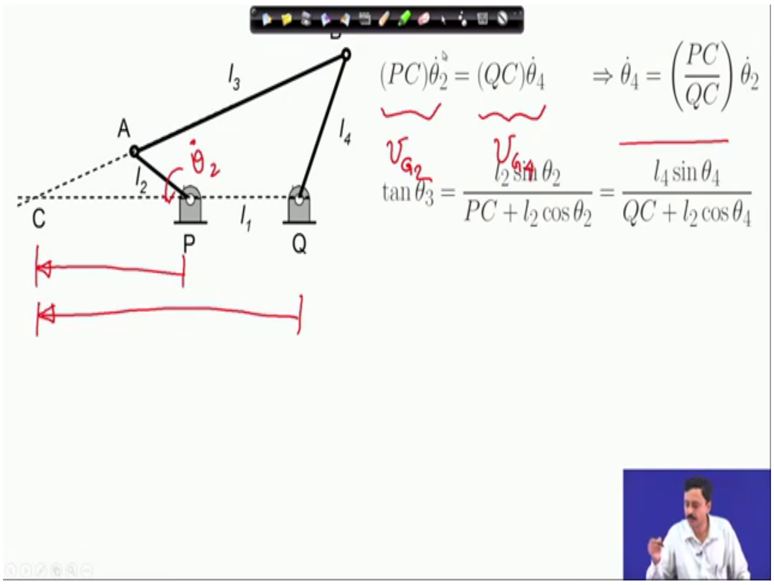

So, I have located for you, the I 2 4 which is the point C. Now, you can very easily from geometry write this expression that PC, which is this distance, PC time's theta 2 dot. What is this? This is nothing but velocity of G 2. PC time's theta 2 dot, so this is theta 2 dot must be equal to QC time's theta 4 dot this is the condition. So, this is VG 4, now this is the condition that must be satisfied at I 2 4. So, therefore, theta 4 dot must be equal to PC divided by QC times theta 2 dot, this is straight forward. Now, we are need to determine this PC and QC. So, how do we do that?

So, you can very easily write the tangent theta 3.

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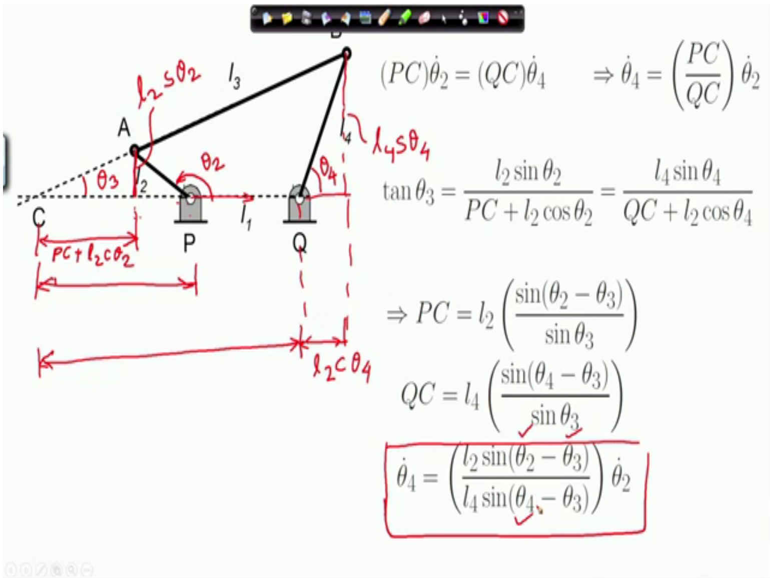

So, this is our angle theta 3, this angle is theta 3. Theta 3 tangent of theta 3 is nothing but l 2 sin theta 2, this is l 2 sin theta 2, remember that this is theta 2. I am writing S theta 2 in place of sin theta 2. So, if this is angle theta 2, then this vertical projection of the link is l 2 sin theta 2 divided by, so tangent theta 3 is l 2 sin theta 2 divided by PC which is this distance plus l 2 cosine theta 2, now that is the projection of l 2 on the x axis. Now, it happens in this case to be negative because if theta 2 is greater than 90, automatically this takes care of the sign. So, in this particular case it will be PC minus this distance, it is PC minus this distance so I will actually have this distance and you know that, this l 2 sin theta 2 divided by this distance is the tangent of theta 3.

So, this distance is PC plus l 2 cosine theta 2. So, I have written C theta 2 in place of cos theta 2 and that is also equal to l 4 sin theta 4, which is this distance l 4 sin theta 4 this is theta 4 divided by QC, this is QC plus this distance, this is l 2 cos theta 4. So, that all both of these ratios are tangent theta 3. So, from here I can very easily solve to determine PC and QC and which I have done out the, I have derived it for you. So, PC is l 2 sin theta 2 minus theta 3 divided by sin theta 3 and QC is l 4 sin theta 4 minus theta 3 by sin theta 3 and hence I have finally, this relation between theta 2 dot and theta 4 dot, in terms of you can see theta 2, theta 3, theta 4. Now, as I have said the displacement analysis is completed, so I know each of them, I know theta 2, I know theta 3, I know theta 4.

Of course, 1 of them will be specified, so if theta 2 is specified I will be able to solve for theta 4 and theta 3, if theta 4 is specified then I will be able to solve for theta 2 and theta 3. So, in whatever way I know, theta 2, theta 3, theta 4 in that case I have found the relation between theta 2 dot and theta 4 dot.

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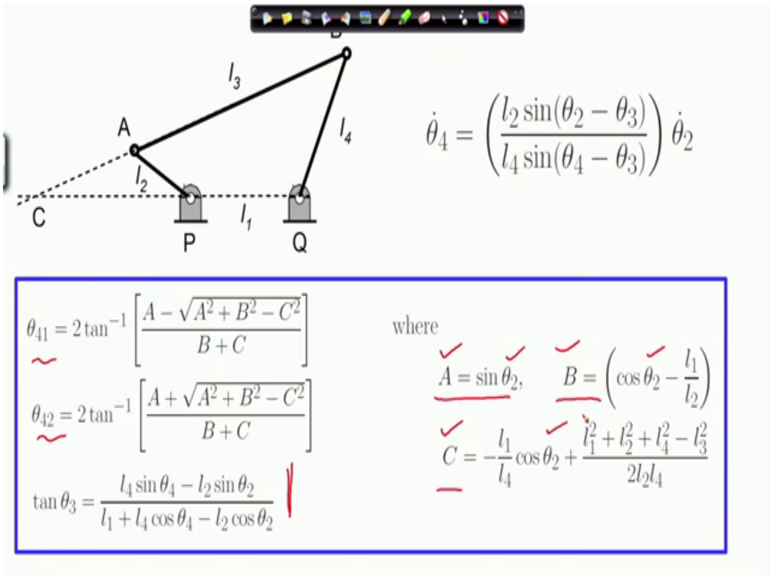

Thus, we have this relation theta 4 dot as this ratio times theta 2 dot and just to recapitulate our displacement analysis problem, just now, I was mentioning that I can solve for theta 4, given theta 2 I can solve for theta 4, there are 2 solutions of theta 4 and corresponding to those 2 solutions of theta 4, I can find out the solutions for theta 3. Here, A B and C are completely known because theta 2, I am assuming theta 2 is specified, so, this is completely known. So, once I know ABC, I can find out the 2 solutions of theta 4, from where I can find out also theta 3.

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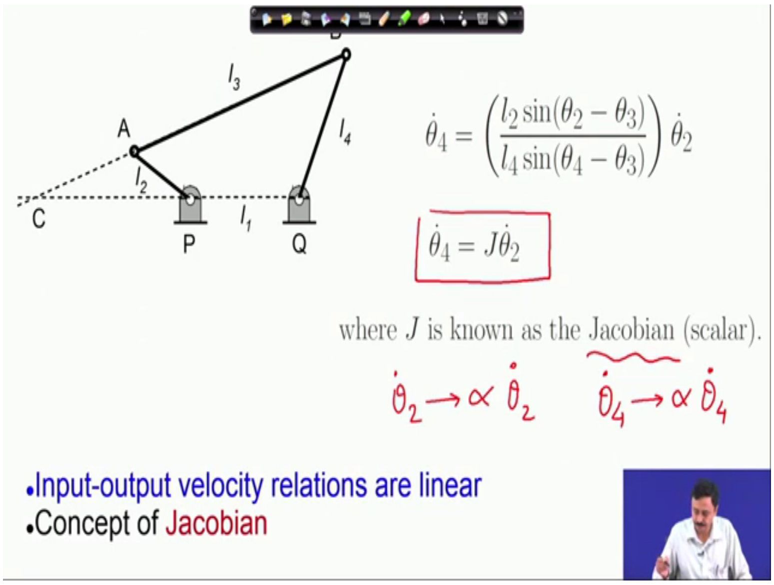

Now, this relation between theta 4 dot and theta 2 dot, I will write in a compact form as theta 4 dot is equal to J times theta 2 dot. This j is known as the Jacobian, here this is a scalar quantity; that means, it is a number. Given theta 2 or having known theta 2, theta 3, theta 4 I can exactly calculate this number which is j, this is called the Jacobian. So, the Jacobian relates the input-output velocities of the 4R chain, it is interesting to note that the velocity relations are linear between theta 2 dot and theta 4 dot, it is linear. So, if I scale theta 2 dot by a certain quantity, theta 4 dot get scaled by the same quantity. In other words, what I mean is, if I say, if I take theta 2 dot to alpha times theta 2 dot, then theta 4 dot goes to alpha times theta 4 dot.

So, this relation is linear, but remember that in displacement analysis the relation between theta 2 and theta 4 was highly non-linear, it was through those trigonometric functions, but the velocity is linearly related; the input-output velocities are linearly related and we have introduced this concept of Jacobian, which will be very useful and we will continue discussing this concept and this is very useful in the case of robotics as well, you will come across Jacobian in robot kinematics, which we will discuss a in the subsequent lectures.

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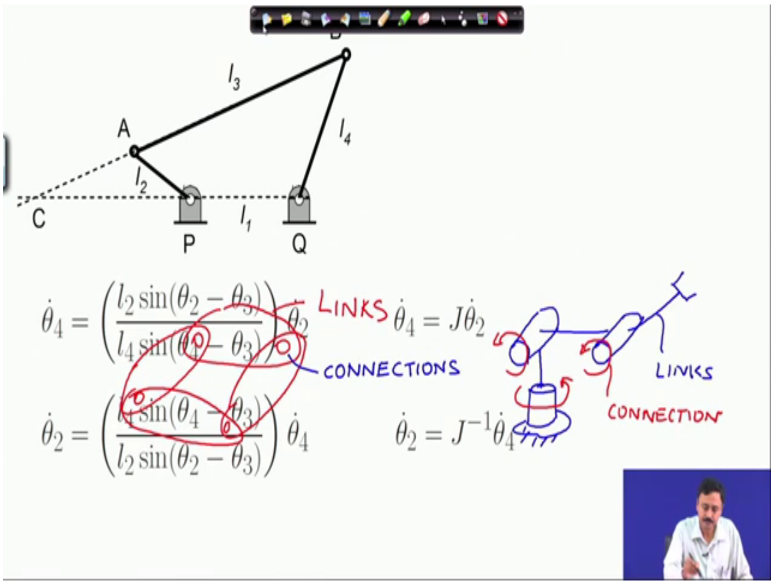

So, let me write down the relations once again. Now, what I have done here is I have written it in 2 ways I have kind of inverted.

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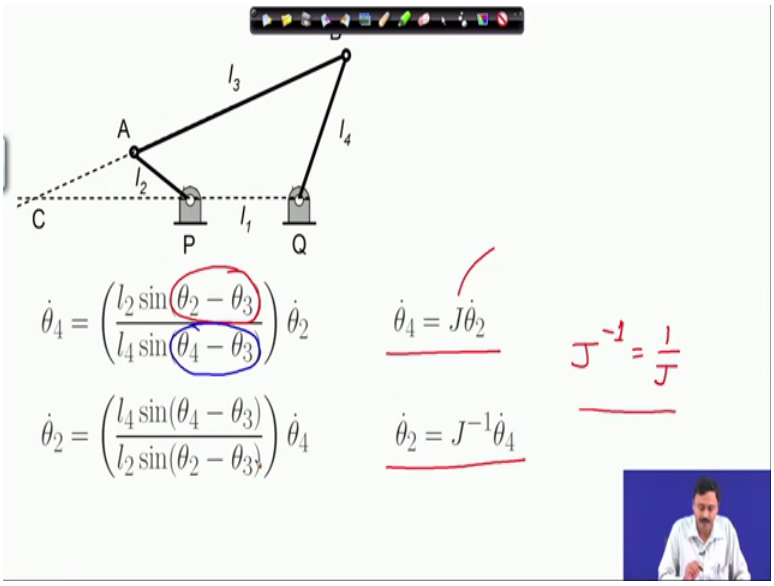

You see, I have written out theta 4 dot in terms of theta 2 dot and here I have written theta 2 dot in terms of theta 4 dot by the Jacobian inverse which is 1 by J, essentially Jacobian inverse, since it is a scalar Jacobian inverse is 1 over J. Now, this brings in some issues, as you can very well realize that for certain relations of theta 2 and theta 3 the sin of theta 2 minus theta 3 can go to 0, if it goes to 0.

Then I cannot invert the Jacobian because Jacobian has gone to 0, I cannot find out 1 by 0. It can also happen the other way, if, it so happens, that theta 4 and theta 3 are such that sin of theta 4 minus theta 3 is 0, then the Jacobian is infinity or the Jacobian inverse is 0 then it is another problem. So, what are these problem; problematic configurations as you can very well realize that, these are certain configurations at which this will happen. So, there are very special configurations at which such conditions are met.

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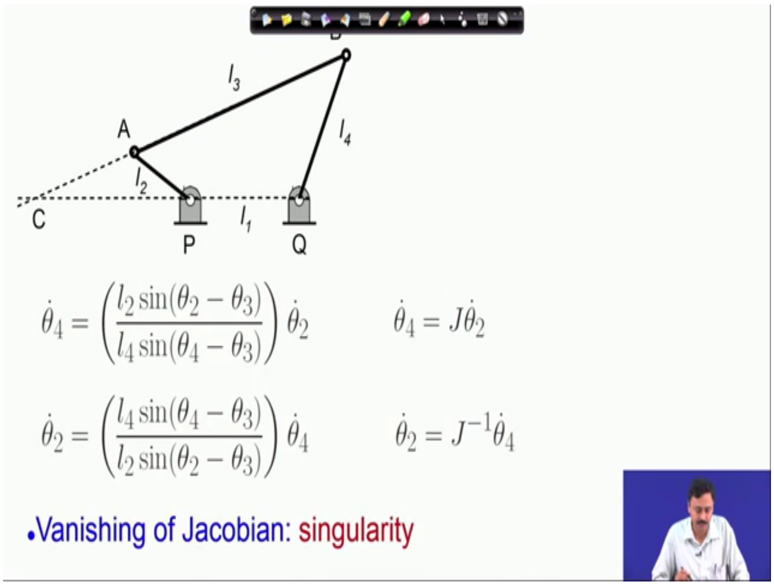

So, vanishing of the Jacobian is called the singularity of the kinematic chain of the mechanism. So, going we are will spend a minute or some minutes discussing this vanishing because this is very important and I will relate this to 1 of the previous concepts that we have used, which is the dead center configuration.

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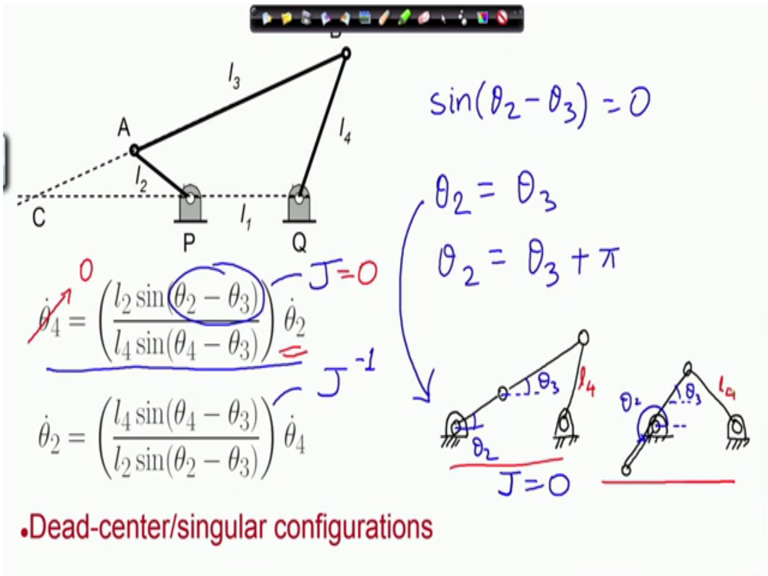

If you remember, in the previous lectures we have discussed dead center configurations, there I wrote singular configurations as well. So, the reason is this that at these configurations the Jacobian or it is inverse can vanish. So, for let us consider this situation, where the Jacobian, so this is the Jacobian and this is the Jacobian inverse. So, when can Jacobian vanish whenever sin theta 2 minus theta 3 is equal to 0, this can happen in 2 ways theta 2 equal to theta 3 and theta 2 equal to theta 3 plus pi. So, in both these cases Jacobian will vanish.

Now, what are these configurations, these very special configurations as you can really see, theta 2 equal to theta 3 let me try to draw it out. This is the configuration, where theta 2 equals theta 3, this is theta 2 and this is theta 3, as you can see theta 2 equal to theta 3 and this we have discussed is a dead center configuration. So, this is for this case, let us look at the other case theta 2 is theta 3 plus pi. So, this is the folded configuration, this is also dead center configuration and what is happening when sin theta 2 minus theta 3 goes to 0.

So, let me mark this out, so this is theta 2 and this is theta 3, so theta 2 is theta 3 plus pi, both are dead center configurations and what is happening at these dead center configurations the Jacobian is going to 0, j equal to 0 and what does it mean if I look at this relation, it means that whatever be theta 2 dot, it does not matter whatever be theta 2 dot, theta 4 dot is 0. So, if J goes to 0 theta 4 dot goes to 0, whatever be the value of theta

2 dot and indeed these are the configurations, where the output link which is l 4, the 4th link goes to 0 velocity.

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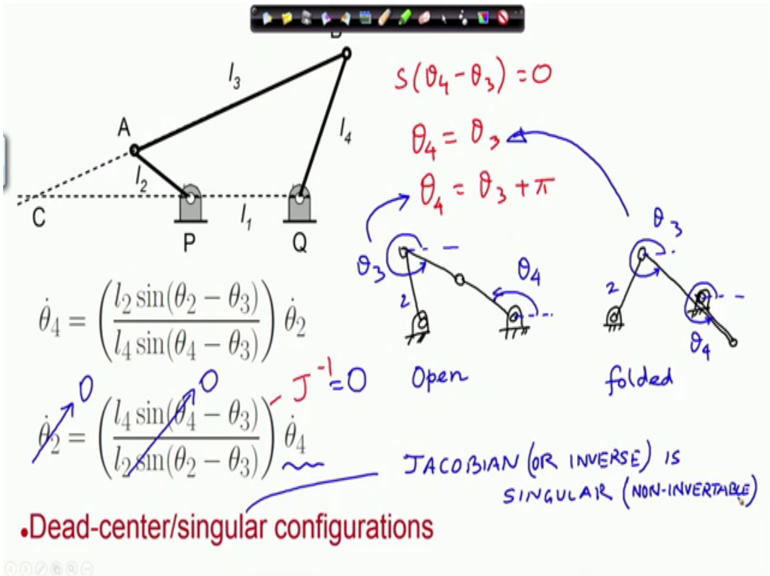

Now, let us discuss the other case which is the Jacobian inverse case. So, this is J inverse, now when will J inverse go to 0, when sin theta 4 minus theta 3 equal to 0. So, the situation is absolutely same, so these are the 2 situations theta 4 is equals to theta 3 or theta 4 is theta 3 plus pi. So, the configurations are also very similar, only thing it occurs on the other side, so this is the case, this is theta 4 and remember, this is theta 3. So, this is the situation in this case theta 4 is theta 3 plus pi.

So, this is the open configuration, there is another configuration corresponding to the first case. So, here this is theta 4 and this is theta 3 and you can see that both are equal. So, this corresponds to the first solution theta 4 equal to theta 3, so this is known as the folded configuration. Now, what happens when these 2 configurations occur, J inverse goes to 0, so this goes to 0. So, whatever the theta 4 dot, theta 2 dot goes to 0. So, there is no motion of link 2, link 2 must come to a stop it cannot move whatever with theta 4 dot.

So, these are the dead center configurations which we have discussed before and these are also singular configurations for the reason that now you realize that these are the singularities of the Jacobian. So, singular configurations always imply that, these are the singularities of the Jacobian so or it is inverse. So, singularity sometimes we define in terms of non invertibility, so non invertible. So singular, in that in the sense that it is not invertible, so these are the dead center or singular configurations, where the Jacobian or it is inverse, is singular which means they are not invertible.

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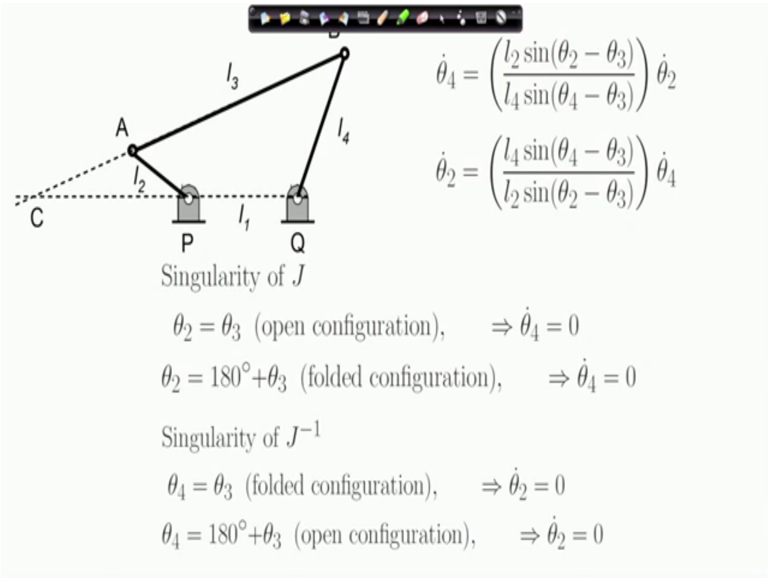

So, this is what I have discussed, singularity of J, when theta 2 equal to theta 3, then J is singular, theta 4 dot goes to 0 this is the open configuration and the other solution theta 2 is 180 degree plus theta 3 is a folded configuration, there also theta 4 dot goes to 0 and the other situation where singularity of J inverse, that means, J inverse becomes 0 or non invertible, where theta 4 is equal to theta 3 which is folded and theta 4 is 180 degree plus theta 3 which is open, both cases theta 2 dot is 0.

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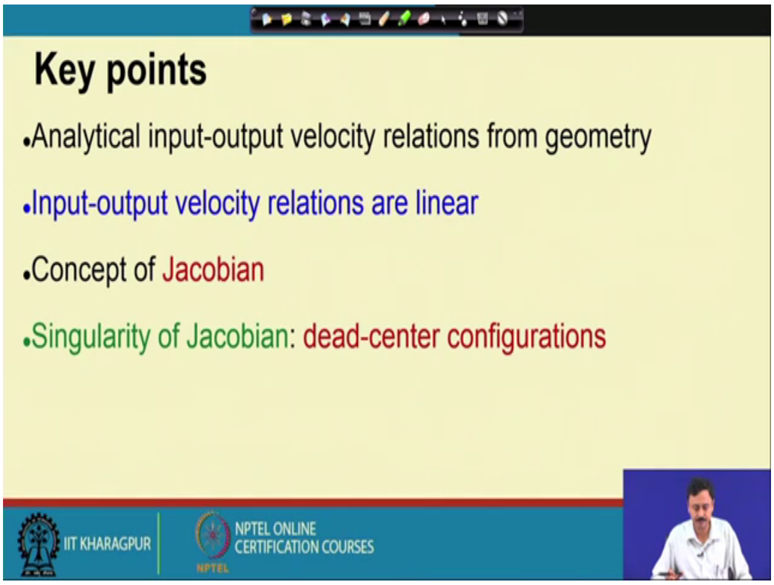

So, let us review the key points that we have discussed in this lecture, we have derived the analytical input-output velocity relations from geometry, purely from geometric considerations. We have found that, the input-output velocity relations are linear, we have introduced the concept of Jacobian and the singularity of Jacobian that means, non invertibility of Jacobian or the inverse of the Jacobian, they correspond to the dead center configuration or the singular configurations of the mechanism.

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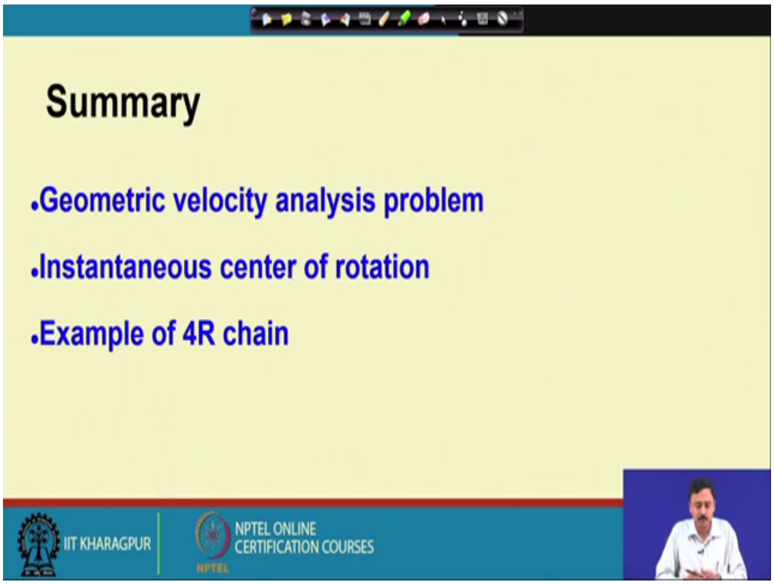

So, to summarize we have looked at the geometric velocity analysis problem based on the instantaneous center of rotation, we have derived the analytical relations between the input-output velocity of 4R chain, so with that I will conclude this lecture.