Mechanism and Robot Kinematics
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Lecture – 22
Velocity Analysis: Geometric Concepts - III

We will continue our discussion on the velocity analysis problem, we have been looking at the geometric problem in velocity analysis, we have looked at certain concepts of some of the certain geometric concepts like instantaneous center rotation, and relative instantaneous center rotation and based on that we have discussed an important theorem which is known as the Aronhold Kennedy theorem.

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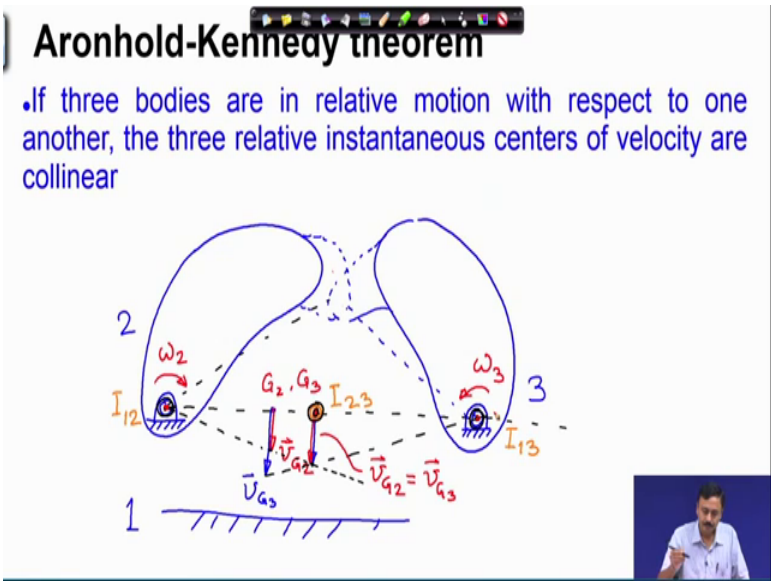

So, we are going to start by just reviewing the Aronhold Kennedy theorem which states that if 3 bodies are in relative motion with respect to 1 another the 3 relative instantaneous centres of velocity are collinear.

So, just to explain this once again let me consider 3 bodies. So, 2 and the third 1 I will define like this that, suppose this is the instantaneous center of rotation of let me call it body 2. So, this point is the relative instantaneous center of body 2 with the ground which I call body 1, and let me assume that this is the relative instantaneous center of body 3 with respect to the ground. In other words these 2 points are hinged to the ground at this instant of time; I can consider that these 2 points are hinged to the ground at this point of time.

So, as per our nomenclature we can say that this is I 1 2, this is I 1 3. So, relative instantaneous centres of rotation of body 2 with respect to the ground is I 1 2, relative instantaneous center of rotation of body 3 with respect to the ground is I 1 3. Let me consider this line joining I 1 2 and 1 3, now I ask the question what is the relative instantaneous center of rotation of body 2 and 3, let me also make 1 more consideration that this is omega 2, this is omega 3 I have taken arbitrarily, now I ask this question where lies the relative instantaneous center of rotation of body 2 related to body 3.

Now, as we have seen before this point may lie outside the domain of the physical domain of the body, this relative instantaneous center of rotation I 2 3 may lie outside the physical domain of the bodies. Let us consider a test point we will begin with a test point. So, if I consider a test point that is a G then I know that velocity of G 2 because this point remember, this point I can always bring into the extensions of body 2 and body 3. So, G 2 and G 3 are correspondingly points on body 2 and body 3.

So, velocity of point G 2 if I mark it with red it must be perpendicular to this line joining I 1 2 and G. Similarly the velocity of point G 3 must be perpendicular to this line. So, this is V G 3 and this is V G 2, now they do not match. So, definitely G cannot be the relative IC between of 2 and 3, then we had seen that in order to have a point a feasible point, which can be the relative IC it must be a point on the line joining I 1 2 and I 1 3, consider this is our test point now.

So, here we have again G 2 and G 3 they are coincident points on the extensions of bodies 2 and 3. Now once again if I mark by red the velocity of point G 2 it must be perpendicular to this black line. So, V G 2 and velocity of G 3 must also be perpendicular to this black line. Now their directions match, but their magnitudes do not, now how to determine then the exact point what we do is since the red vector belongs to body 2 so, from the centre of instantaneous centre of body 2, I draw this black dashed line and instantaneous centre of body 3. I draw this black line wherever they intersect you can very easily realize you must have velocities as same these 2 velocities must be equal. So, here I have V G 2 equal to V G 3. So, this must be the relative IC I 2 3. So, this is I 2 3, here is I 1 3, and here is I 1 2, and you can see that I 1 2 I 2 3 and I 1 3 lie on a single straight line.

Now, here I have fixed the ground, but you can also make the ground move the situation will not change; the situation will not change even, if I let the ground move because remember I 1 2 is the relative instantaneous center of body 2 with respect to the ground that will remain, the same I 1 3 is the relative instantaneous center of body 3 with respect to the ground that also will remain same, even if I let the ground move, but keeping the relative motions the same. So, nothing is going to change.

So, therefore, we have this theorem known as the Aronhold Kennedy theorem which states that if 3 bodies on relative motion with respect to 1 another the 3 relative instantaneous centres of velocity are collinear. Now we are going to look at the implications of this theorem.

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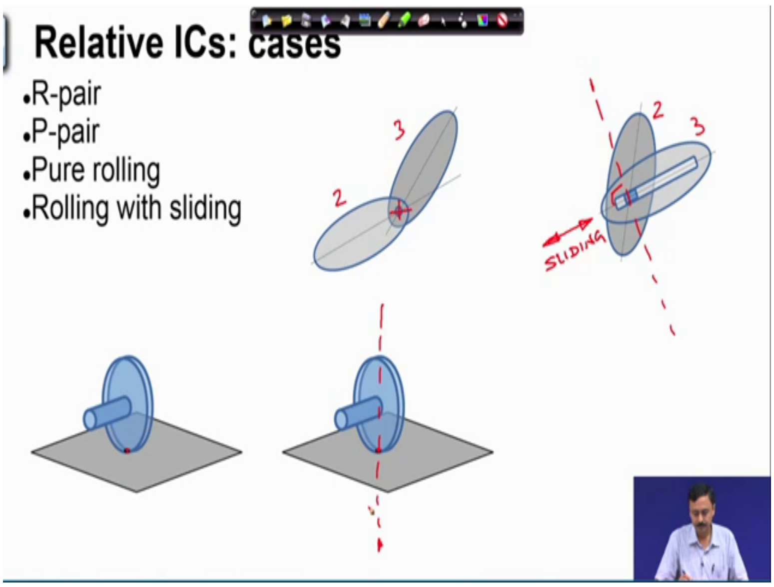

After looking at some examples which you have seen before so, this is the relative IC between bodies 2 and 3 the revolute pair for the r pair, the revolute pair itself is the relative IC between 2 3 in the case of a prismatic pair the relative IC between the bodies 2 and 3 lie on this line which is perpendicular to the direction of sliding. So, this is the sliding direction and the relative IC lies on this line perpendicular to the sliding direction.

So, I 2 3 lies on this red dashed line somewhere at infinity so, it lies at infinity on this red dashed line. In the case of purely rolling disc as you know is the velocity of at the point of contact is 0 for pure rolling case therefore, this is the relative IC between the ground and the wheel, in the case of rolling with sliding we cannot exactly locate, but it must lie on this line why because when it is purely rolling it must be at the point of contact, when it is purely sliding when it is only translating and sliding it must be at infinity either in the upward or downward direction. So, in between if it is rolling and sliding it will lie on this line somewhere now let us apply these concepts in mechanisms.

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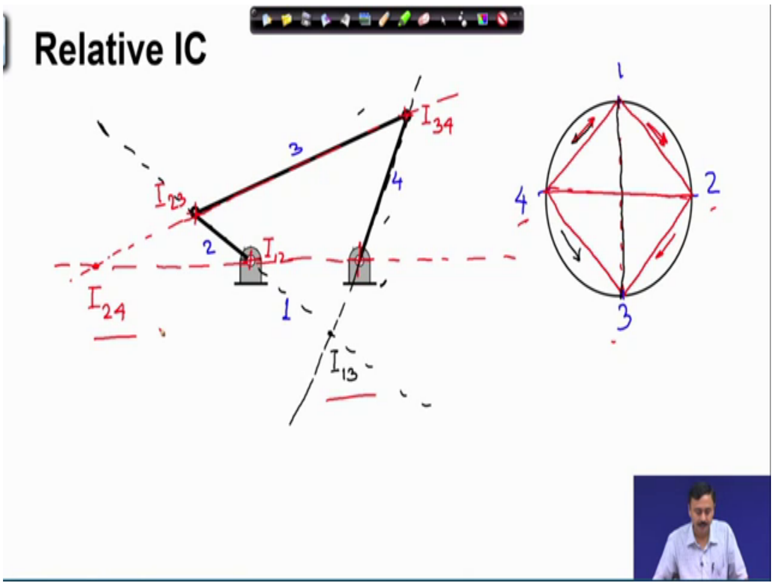

So, we will try to locate the relative ICs for this mechanism let me number the links first. So, ground is 1 this is 2 3 and 4, now to locate all the relative ICs we have what is known as the circle diagram. So, I will mark the number of links so, 1 2 3 4. So, we have 4 links. So, I will mark 4 points on this circle, I will join the 2 points if I exactly know the relative IC between those 2 bodies.

So, on the circle as you realize I have marked out exactly the number of links I have and the relative ICs between 2 links, if it is known if the relative IC between 2 links is known, then I will mark that on that circle diagram I will join. So, for example, I 1 2 is this point the relative instantaneous center of rotation between 1 and 2 is this ground hinge. So, I know that so I will join 1 with 2 I also know the relative instantaneous center of rotation, this is I 1 2 this is I 2 3 I also know that. So, I know I 2 3 this is I 3 4 the relative instantaneous centres of rotation of links 3 and 4. So, I know I 3 4 so I will join 3 4 and I also know the relative IC of 4 link 4 with respect to link 1 so, I will join 1 and 4, now what I do not know that the circle diagram immediately tells me for example, I do not know what is the relative IC between 1 and 3 this has not come out as yet, but can we find it the answer is yes we need to find out 2 independent paths starting from 1 or between 1 and 3; between 1 and 3, we need 2 independent paths why we need it will get cleared very soon. So, 1 path is I 1 2 and I 2 3 so, I 1 2 and I 2 3.

So, I 1 3 must lie on this line as per the Aronhold Kennedy theorem, I 1 3 must lie somewhere on this line because I 1 2, I 2 3 and I 1 3 these are the 3 relative ICs of 3 bodies 1 2 3. So, 3 bodies 1 2 3 are in relative motion. So, therefore, their relative ICs must lie on a line now of the 3 relative ICs I exactly know 2 of them I 1 2 and I 1, I 2 3 these are known I 1 2 and I 2 3 are known. So, I 1 3 must lie on this line so this is the black line dashed line. Similarly 1 4 and 3, 1 is the ground 4 and 3 these are 3 bodies.

Again which are in relative motion so, their relative ICs must lie on the line of which I 1 4 is known, and I 3 4 is known I 1 4 is here on the ground range and I 3 4 is here. So, I 1 3 must also lie on that line. Now when the point has to lie on both these lines is the intersection point this which must be I 1 3. So, I have located now I 1 3 so, I will join it by a solid line. Next is I 2 4 which is as yet unknown, but then I have 2 independent paths I 1 4 and I 1 2. So, I 2 4 must lie on that line why because 2 1 and 4 the links 2 1 and 4 they are 3 bodies in relative motion.

So, the 3 relative ICs must lie on that line of which I 1 4 and I 1 2 unknown to me so therefore, I 2 4 must lie on this red dashed line. Similarly 2 3 4 are 3 rigid bodies are in relative motion. So, therefore, the 3 relative ICs must lie on that line of which I 2 3 and I 4 3, are known to me. So, therefore, I 2 4 must also lie on this line. So, the intersection point gives me I 2 4. So, I have found this as well and that completes all the relative ICs. So, I have found I 1 3 and I have found I to 4 which were unknown using the Kennedy Aronhold theorem and this circle diagram circle diagram helps you to keep track of the bodies of the links, and finding out searching for paths which connect 2 points or 2 links.

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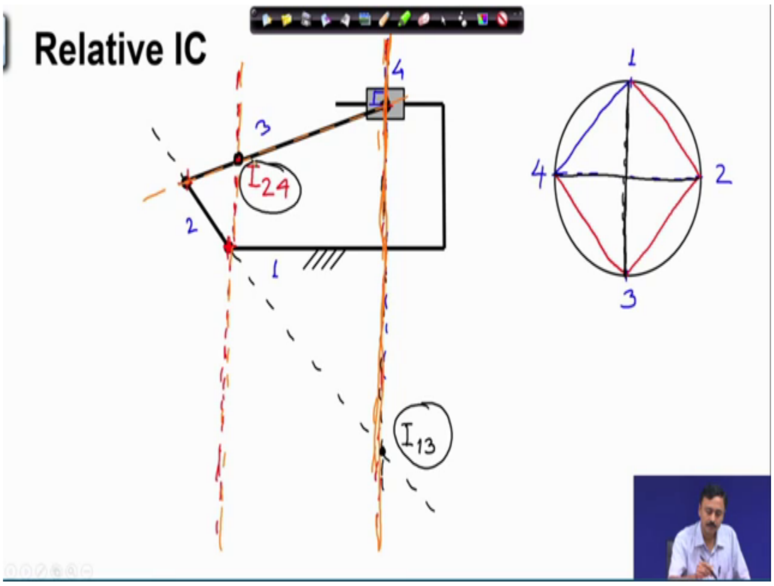

So, let us proceed further so, here we have a 3 r 1 P chain ground is 1 2 3 the slider is 4. So, again 4 bodies, now what do we know we know I 1 2, we know I 2 3, we know I 3 4, now I 1 4 this is a prismatic pair prismatic pair between links 1 and 4.

So, I 1 4 must lie at infinity along this direction which is perpendicular to the direction of sliding. So, I 1 4 must lie at infinity so, I know this it is at infinity. Now let me ask the question whether I can find I let say 1 3. So, I have 2 paths I 1 2 is known and I 2 3 is known so therefore, I 1 3 must lie on this line, the other path I 3 4 is known and I 1 4 is that infinity somewhere.

So, therefore, I 1 3 must also lie on this line the intersection is this so this must be I 1 3 so, now, I also know I 1 3. Next I ask the question whether I can find out I 2 4. So, I 2 4 must lie on the line of I 1 4 and I 1 2, I 1 4 is at infinity and I 1 2 is this ground hinge and I 1 4 is that infinity. So, through this point through this ground hinge I must draw a line to infinity. So, this line meets this dashed line this line at infinity.

So, these 2 lines are parallel so, what I have drawn is a line through I 1 2 to meet I 1 4, now I 1 4 is at infinity and as I have said I 1 4 lies on this I 1 4 lies on this line I 1 4 lies at infinity on this line therefore, I must draw a line parallel to that line through I 1 2 to meet at infinity to meet I 1 4 at infinity. So, that is 1 the other is other path is I 2 3 which is here, I 2 3 and I 3 4 which is here so, which means it is this line. So, I 2 4 must lie on this line as well so this point as you can see this point is I 2 4. So, this intersection point is I 2 4 so now, I also know I 2 4. So, I have located all instantaneous centers of rotation.

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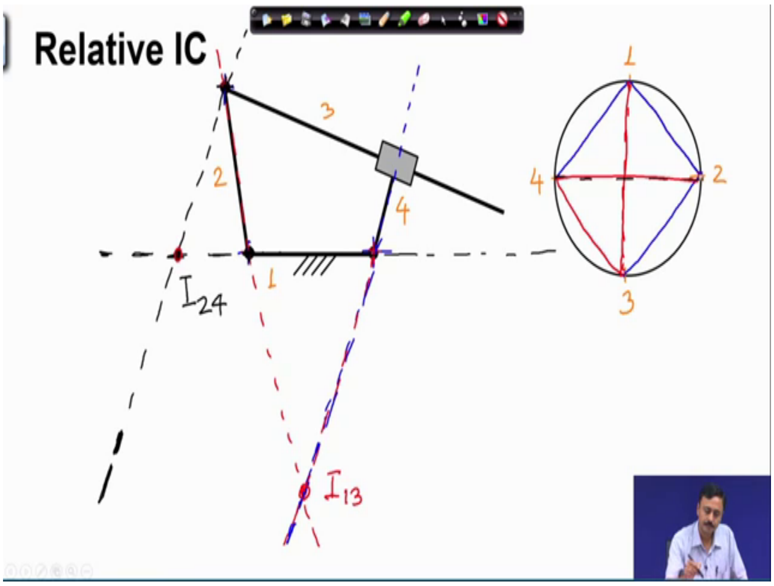

Let us move to the next mechanism. So, let me number 1 2 3 and 4 so, there are 4 links so 4 points on our circle diagram I know the IC between 1 and 2. So, I 1 2 is known, I 2 3 is also known I 1 4 is also known. Now I 3 4 must lie on this line which is perpendicular to the sliding direction I 1 4, I 3 4 must lie on this line at infinity because this is a prismatic pair.

So, this also I know is at it is at infinity I 3 4 is that infinity on this blue dashed line. Now I ask this question whether I can find out I 1 3. So, I 1 3 lies on the line of I 1 2, this is I 1 2 and I 2 3. So, which means this line I 1 3 lies on this red dashed line, and I 1 3 also lies on this I 1 4 which is here and I 4 3 which is at infinity on this line. So, this point this intersection point must be I 1 3. So, I know I 1 3 next I asked the question whether I can locate I 2 4.

So, I 2 4 it must lie on I 1 2 and I 1 4, and I 1 2 so, on this line. So, I 1 2 and I 1 4 so, I 2 4 must lie on this black dashed line, and I 2 4 must also lie on I 2 3, and I 3 4, I 2 3, I 2 3 is this and I 3 4 is at infinity along this blue dashed line remember.

So, therefore, I must draw a line through I 2 3 which is parallel to this blue dashed line. So, as to meet I 3 4 at infinity so, this black dashed line, and this blue dashed line, they are parallel. So, where do these 2 lines meet they meet here so, this must be I 2 4. So, we are now located both I 2 4 and I 1 3. So, all the relative ICs are now known.

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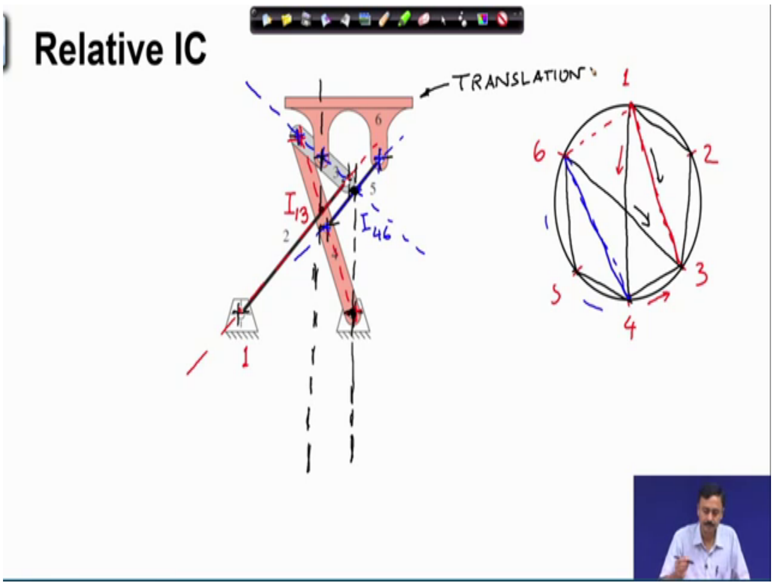

Let us look at this mechanism, now this has got 6 links 1 2 3 4 5 6 so, six links. Now, we are going to again see what we know. So, I 1 2 is known, I 2 3 is known, here I 3 4 is known, I 4 1 is known, I 4 5 is known, and I 5 6 is also known, I also know I 3 6. Suppose I want to know I 1 6. So, I am asking this question whether I can find out I 1 6, now you will find that there is that there are no 2 paths between 2 independent paths between 1 and 6.

So, I must have a path I define a path has 1 in which there are 3 rigid bodies, because the then only I can apply Arnholt Kennedy theorem. So, a path with 3 rigid bodies and I have none, but then if you see if I can find out I 1 3, then I have 1 path involving 3 rigid bodies, 1 will be 1 3 and 3 6. So, let us try to find out 1 3, I 1 3, now for I 1 3 I have 2 independent paths involving 2 rigid bodies or 3 rigid bodies. So, 1 1 is 1 2, I 1 2 and I 2 3, 1 2 and 2 3. So, this must be 1 line, and the other path is 1 4 and 4 3, I 1 4 and 4 3. So, this is the other path. So, this point intersection of this is I 1 3. So, I have now located I 1 3 let me now ask the question whether I can locate I 6 4, in that case I will have 2 independent paths in the involving 3 bodies each on these paths between 1 and 6. So, I have this question of I 4 6. Now I 4 6 must lie on I 3 6 so, 3 6 is here, and 3 4, 3 4 is here.

So on this line must lie I 6 4, similarly I 6 4 must also lie on I 5 6 which is this and I 5 4, I 5 4 is this so therefore, this intersection point is I 4 6 so, I have located I 4 6. Now I can ask the question whether I can find out I 1 6, now I have 2 independent paths involving 3 rigid bodies so, I 4 6 and I 4 1 or 1 4, I 4 6 and I 1 4. So, let me use this so, here I have this as my path I passing through I 4 6 which is this and I 1 4 which is so this line. So, I 1 6 must lie on this line and I 1 6 must also lie on 1, I 1 3 and I 3 6, I 1 3, I this is I 1 3 and I 3 6 is this. So, therefore, it must also lie on this line.

Now I 1 6 therefore must lie on the intersection of these 2 black dashed line, now these 2 lines are actually parallel. So therefore, they meet at infinity what it means is that body 6 is in pure translation, because the instantaneous center of rotation I 1 6; that means, 6 with respect to ground is that infinity. So, this body 6 is in translation, body 6 is in translation with respect to the ground, and this mechanism is indeed a parallel transfer device where body 6 is parallely transferred so it is in your translation.

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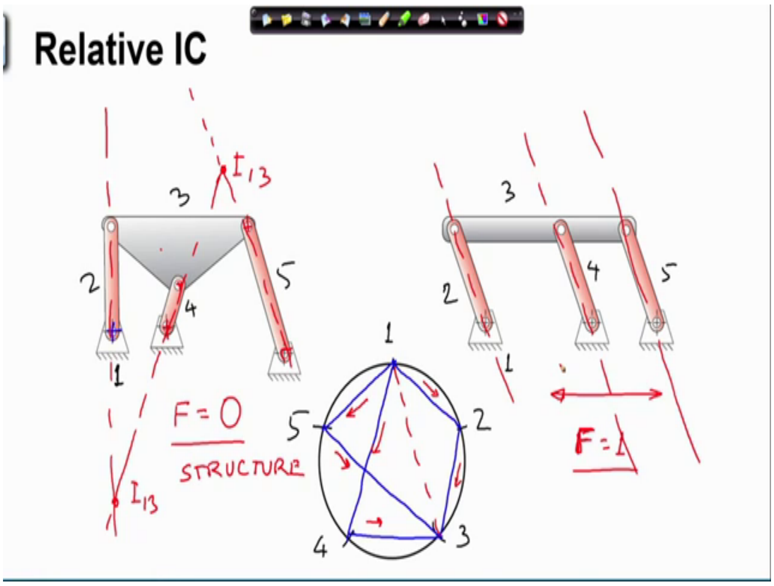

Let us look at this case of again 6 link mechanism so 3 4 this is a 5 link mechanism. So, 1 is ground 2 3 4 and 5, on the right also I have another 5 thing mechanism which is only different because of the dimensions. Now let us look at the relative ICs so, I know I 1 2, I know I 2 3, I know I 3 4, I know also 3 5, I know 4 1, and I know 5 1. Suppose I want to find out what is I 1 3, now there are 2 sets of independent paths as you can see I 1 3 must lie on I 1 2 and I 2 3 I 1 2 and I 2 3 which is this line, and this must also lie on I 1 4 and 4 3, 1 4 is this, and 4 3 is this, so it must lie on this.

So, this is the intersection, but then it also must lie on 1 5 and 5 3, I 1 5, and I 5 3. So, it meets somewhere. So it also meets this, now these are 2 independent paths as well I 1 4, 4 3 and I 1 5, and 5 3. So, this also is a possible possibility for I 1 3 this is also I 1 3, and there will be another I 1 3 that I will obtain using I 2 3 and I 1 2. Now a single body 3 cannot have 3 relative instantaneous centres of rotation.

Then it cannot move and if you remember the degree of freedom what we have calculated for this mechanism was 0. So, this is not a mechanism it is actually a structure, because there are 3 possibilities for relative instantaneous centres of rotation of body 3 with respect to 1, but then the mechanism on the right is actually a mechanism it has got 1 degree of freedom, on the right the kinematic chain on the right has 1 degree of freedom because of very special dimensions.

And if you again calculate or find out the relative IC is I 1 3 they must lie on this 3 lines, as we had done here and all these lines meet at infinity. So, 3 is in translation with respect to the ground. So, in the case of the mechanism on the right you have motion, because the relative IC of 3 whatever way you calculate comes at infinity. So therefore, this can move.

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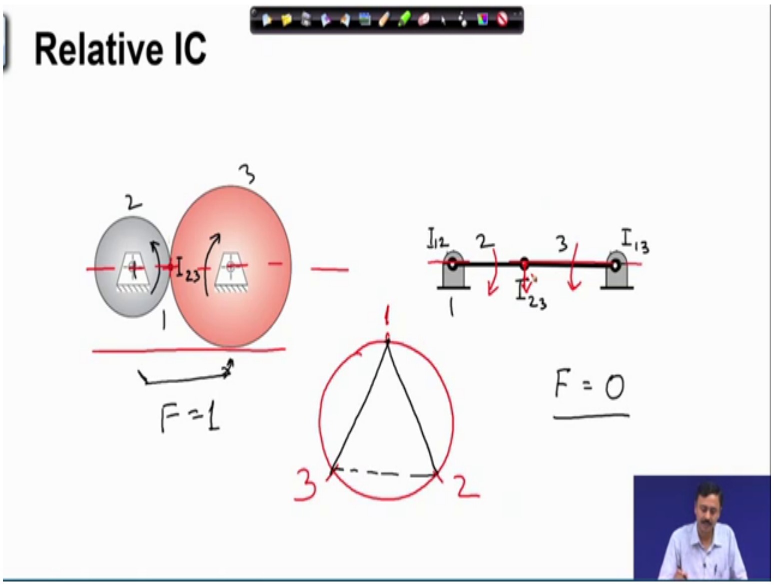

Here is the final example; here I have a chain with 3 links. Now I so, this is 2 this is 3 ground is 1 I know I 1 2, I know I 1 3, and in both cases I also know I 2 3.

So, this is in pure rolling so this is the IC of 2 relative to 3. So, this is in the case here it is I 1 2, this is I 1 3, and this is I 2 3, but then you know this mechanism in which you have 2 friction discs they can continuously roll, this has 1 degree of freedom whereas, the chain on the right is a structure with 0 degrees of freedom, the reason is very simple the IC 2 3 on the on this gear always remains at this point when these 2 move whereas, when these 2 links if you try to move this I 2 3 is going to shift out of this line, and once it shifts out of this line Aronhold Kennedy theorem does not hold, but it must hold therefore this cannot move any further whereas, here it the Aronhold Kennedy theorem is always satisfied the Aronhold Kennedy theorem is always satisfied.

In the case of this 2 friction disks rolling because I 2 3 always lies on this line whereas, for the kinematic chain on the right, this has a tendency to move out, and that breaks the Aronhold Kennedy theorem which must be satisfied it is a theorem, it must be satisfied if it is not satisfied then the thing cannot move. So, that makes the kinematic chain on the right as a structure.

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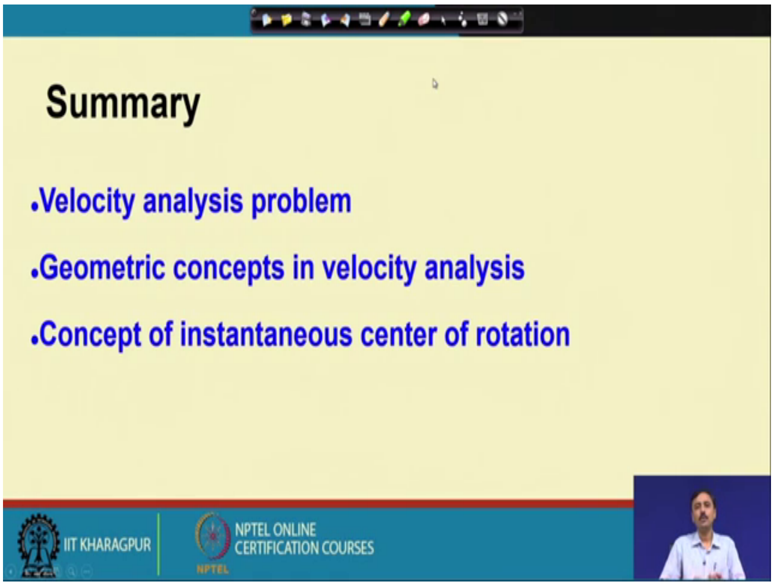

So, I leave you with the summary of what we have discussed today.