Mechanism and Robot Kinematics
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 21
Velocity Analysis: Geometric Concepts – II

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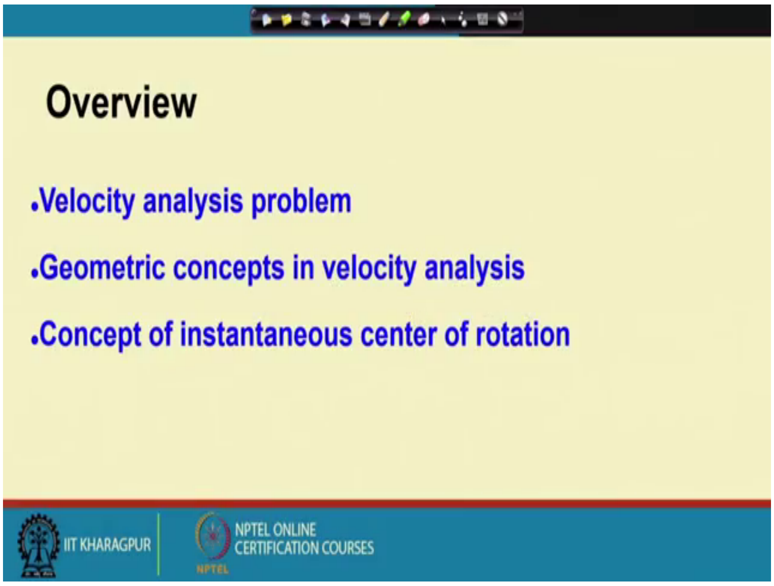

In this lecture, we are going to continue our discussions on velocity analysis, we are going to look at the geometric concept that we had started off with. So, to give you an overview of what we are going to discuss today, we are going to continue with the geometric concepts in velocity analysis. And the concept of instantaneous center of rotation and see how these are useful in velocity analysis problem of mechanisms.

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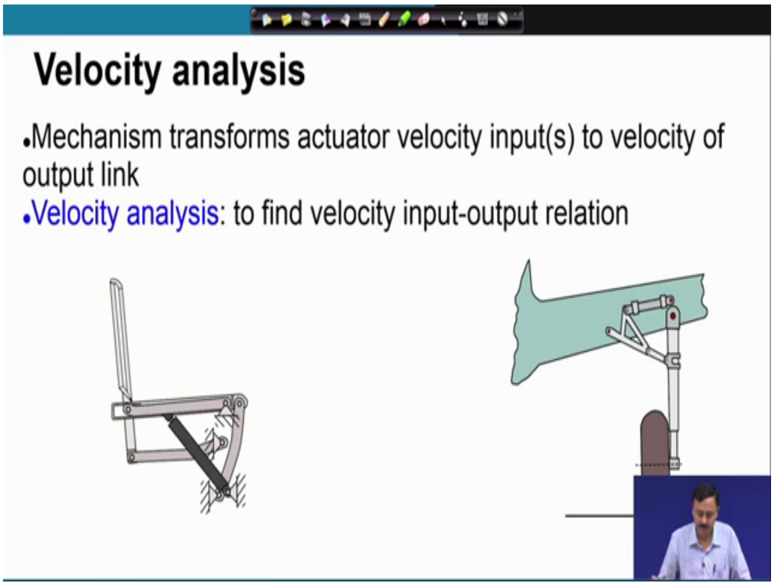

So, we have looked at the velocity analysis problem; so, as you know that mechanism transforms actuator velocity inputs to velocity the output link. And the velocity analysis problem is to find out the velocity input output relations and we have discussed these examples before.

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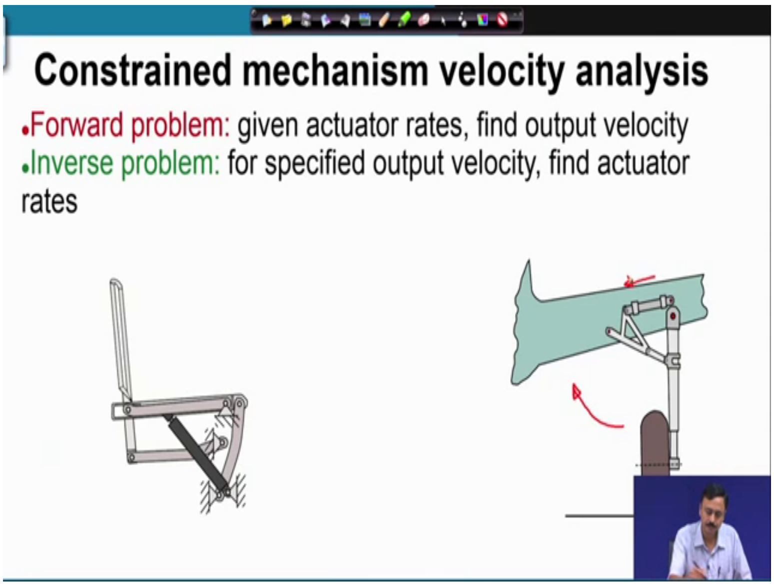

In the case of constrained mechanisms, that means mechanisms with one degree of freedom; we have one actuator input, one output. And given the actuator velocities finding out the output velocity is the forward velocity analysis problem; and the inverse velocity analysis problem is just the reverse; which means the given the output velocity you have to find out the actuator rates.

So, we have looked at this problem of transfer device as well and also for the aircraft landing gear. So, if I am given the velocity of retraction for example, if I am given the velocity of retraction; what should be the expansion rate of this actuator is the inverse velocity problem.

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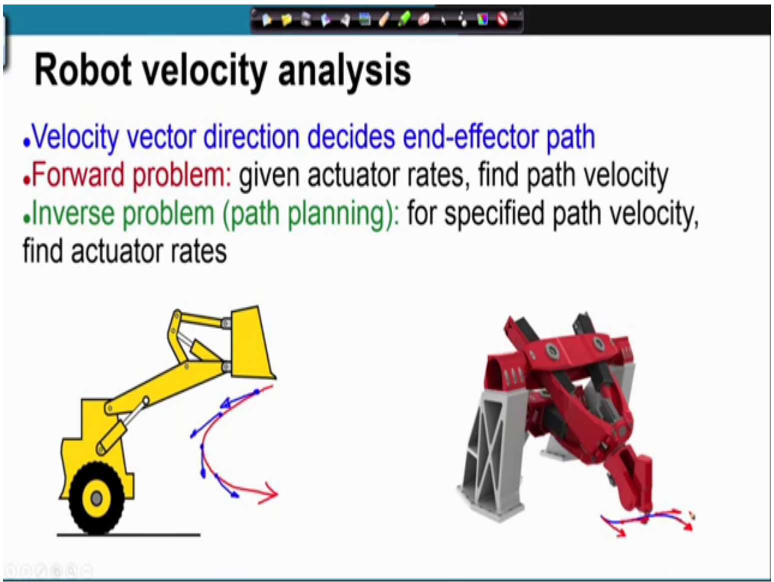

In the case of robots, as you know that the velocity analysis problem is little more complicated; because this velocity vector direction decides the end effector path. So, given the end effector path, the velocity vector direction must be tangential to this path. So, the forward problem is given the actuator rates; find the path velocity and hence find the path. If you know the path velocity of the end effector at each point of the path, if you know the velocity vector; then the path must be the tangent curve to all these velocity vectors.

So, given the actuator rates; finding path velocity and hence the path is the forward problem, the inverse problem is also called path generation or path planning is just the reverse; that means, for a given path velocity; you have to find out the actuator rates. Now, the path velocity can be determined if the path is specified. If the path is specified, I take tangent vectors along the path and hence I can find out.

So, this if I am specified the path; so, I can find out the tangent vectors and these must be the velocities along the path. Also for this parallel kinematic machine, if I am given the path; I can find out the velocities along the path. And hence I can find out; if I know the input output velocity relations, I can find out the actuator rates.

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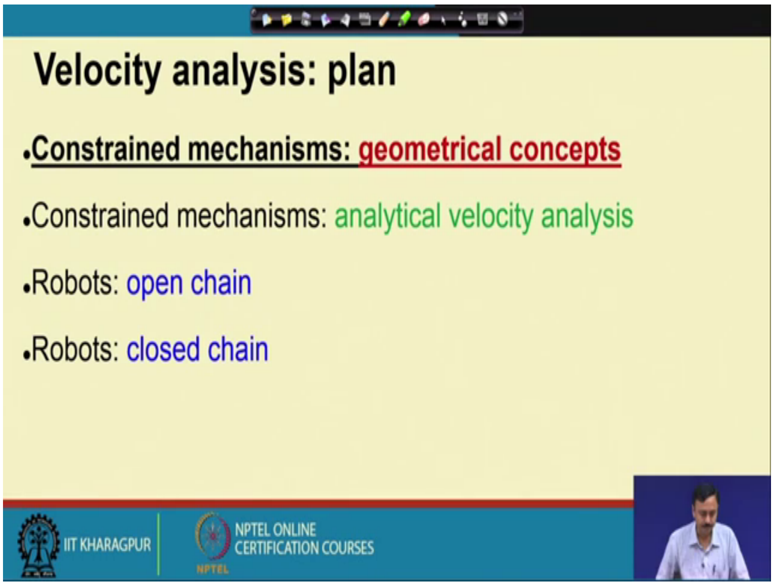

So, our plan was to discuss constraint mechanisms first and we are discussing geometrical concepts now. We will subsequently discuss analytical velocity relations; robots with open chain configurations and robots with closed chain configuration.

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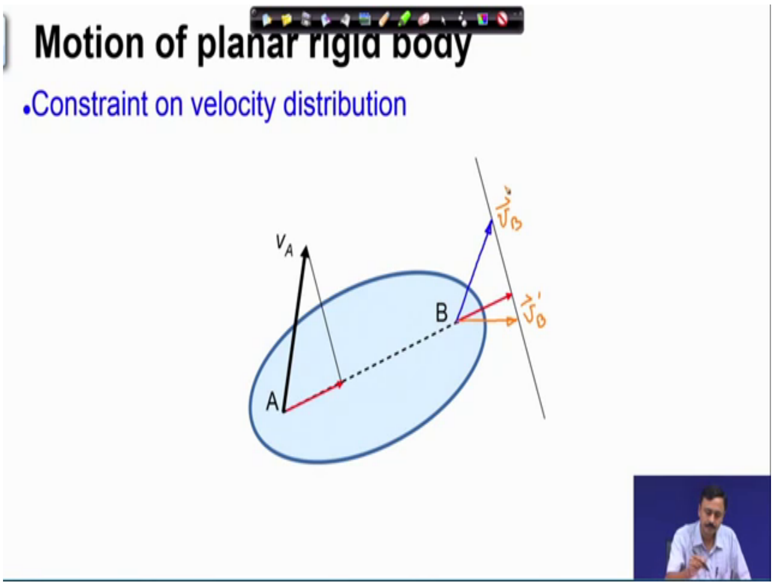

So, we have looked at the constraint on the velocity distribution of a rigid body on a plane. So, if V A is the velocity of point A; then the projection of V A along A B is this red vector and B also must have the same velocity in the direction A B. And therefore, any velocity at B; the actual velocity of point B; must be such that its projection along A B is this red vector; so, these are valid velocity vectors at point B.

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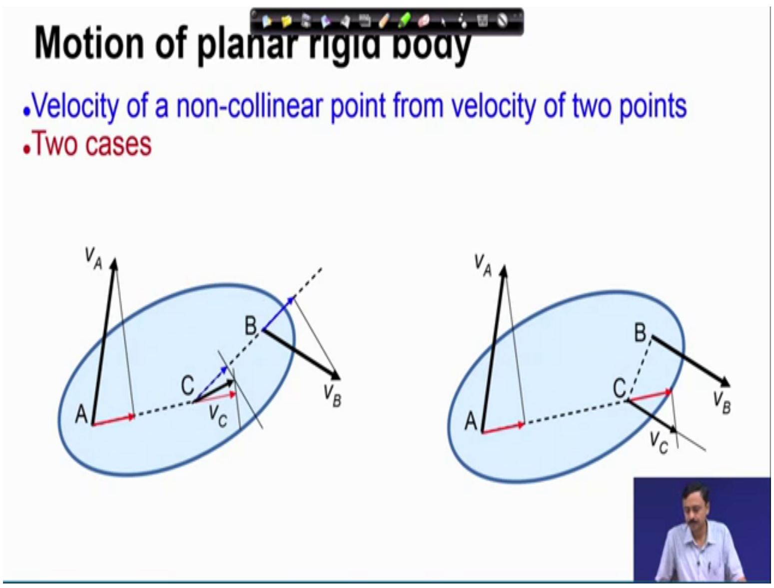

Then we looked at this problem of determining velocity of any other point from the velocities of 2 points. So, here the two points are A and B, whose velocities are specified; then I can show that, I can find out velocity of any other point; say C; the velocity of point C.

So, I project V A along A C; take the projection vector, transfer the projection vector to see; velocity of C must be such that the projection of that vector along A C must be this red vector. Similarly, velocity of point B projected along B C is the blue vector which I transfer to C again. Again, I claim that the velocity of point C must be such that its projection along C B must be this blue vector.

So, how do I determine the velocity of point C? I take 2 perpendiculars to these red and blue vectors as I have done here. And then the intersection gives me the velocity of point C and you can very easily see that the projection of V C; along C B is the blue vector velocity, the projection of velocity vector V C along A C is the red vector.

In the other case, we have taken C such that; C B is perpendicular to the velocity of point B. Now to find out actual velocity of point C; I must project V A along A C, take the projection vector, transfer it to C. Now V B does not have any projection along B C; so, that is 0. So, velocity of C along C B must be 0; now how to locate? How to find out velocity of point C? I drop perpendiculars onto this red vector and the 0 vector; through point C; along C B; I drop a perpendicular; that means, it will now pass through C, perpendicular to C B. And this intersection of these two perpendiculars must be V C and you can very easily check that projection of V C; along A C is the red vector and projection of V C; along C B is 0.

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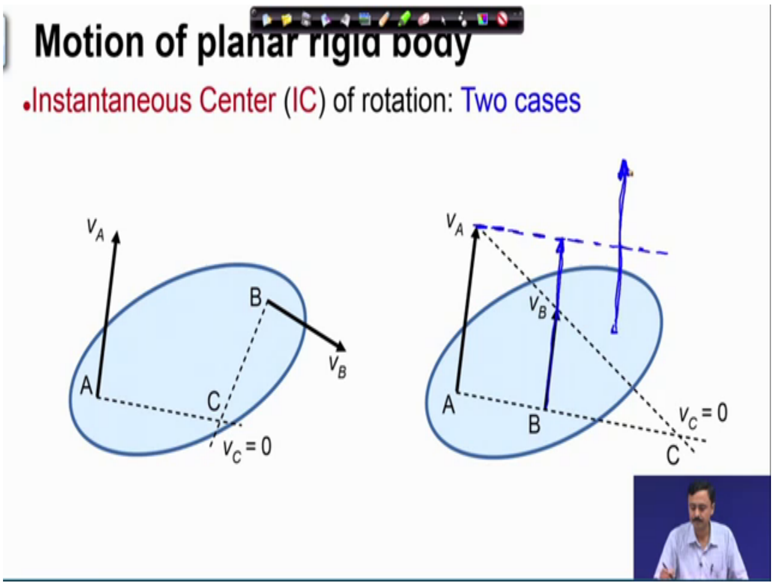

Then we had discussed the concept of instantaneous center of rotation. So, if I take perpendicular C and then on the intersection of the two perpendiculars of the velocity vectors at A and B, then the velocity at C must be 0 and I can also locate the instantaneous center of rotation; when the velocity distribution at A and B is parallel as shown here.

Now, if I ask this question that; what would happen if the velocity of point B were to be the same as velocity of point A; the magnitude and directions are the same? Then what happens? This line and this line; A C or A B; the line A B and this blue line; blue dashed line, they are parallel; what it means? Is that they will meet at infinity, thus this body is in translation; pure translation.

There is no rotation because these two lines meet at infinity; C is at infinity. So, this body is in translation which means the velocity of every point in that case is the same; every point will have the same velocity, magnitude and direction. So, C is the instantaneous center of rotation.

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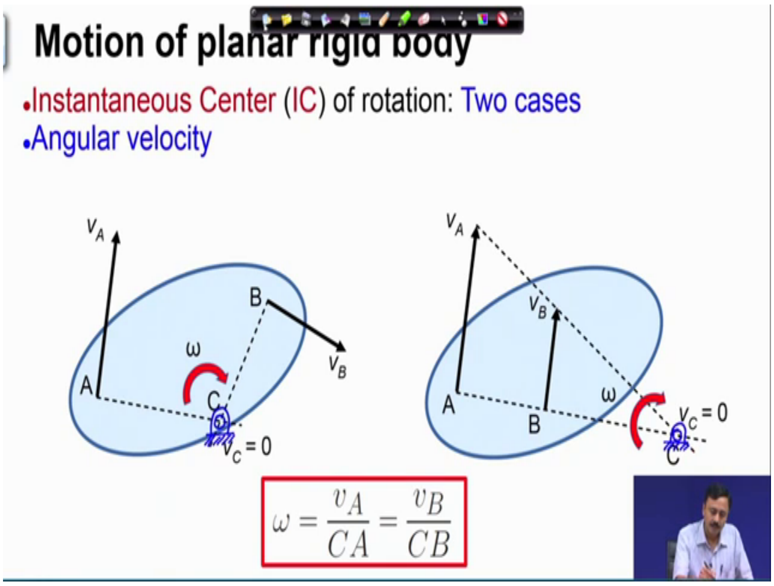

And then we had defined angular velocity, as velocity of point A divided by C A and which must also be equal to velocity of point B divided by C B. Now there is something to be understood here; when I say V C is 0 and C is the instantaneous center of rotation.

As I mentioned, you can imagine that at this instant point C is hinged to the ground; you can assume that point C is hinged to the ground; at this instant only. In other words, the velocity of point C is same as the velocity of the ground; I can also interpret this as the velocity of point C is same as the velocity of the coincident point of the ground. Now if you consider this definition of instantaneous center of rotation.

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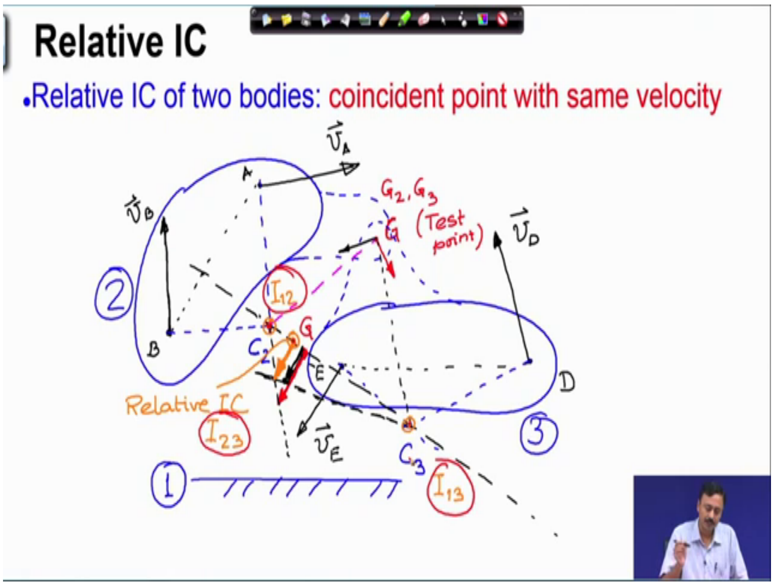

Then we can generalize to the concept of relative instantaneous centres of rotation of two bodies. So, let me draw out two bodies; so if I have a point A with velocity V A and if I have another point B, only thing I have to remember is the projection of V A along A B must be same as projection of V B; along A B. So, let me take this as V B; so, that the projection is roughly same as far as the accuracy of drawing is concerned. So, this is V A and this is V B.

Now this body can have another point let us say; let me call it D; velocity of point D and another point E; let me say that. So, this must be the projection of velocity of E along E D; so, with the hope that I have defined valid velocity distributions at A B D and E; let us proceed with our discussions. So, what I have here is a set of two bodies; whose velocities are now completely specified.

Because specifying velocities of two points means specifying velocity of all points on the bodies. So, let me locate the instantaneous centers of rotation; let me call it C 2; let me call this body as 2; this body as 1, this body as 3. The reason is I would like to reserve 1 for the ground. So, C 2 is the instantaneous center of rotation of body 2; similarly I can locate the instantaneous center rotation of body 3; this I will call C 3. So, C 2 is the instantaneous center of rotation of body 2 and C 3 is the instantaneous center of rotation of body 3.

Now, I ask this question; what is the relative instantaneous center of rotation of bodies 2 and 3? So, what is the concept of this relative instantaneous center of rotation? It means that at that point the velocity of body 2 and velocity of body 3 must be the same. Now this point may lie outside the physical domain of the bodies, for example I take this point; this is a test point let us say; this is a test point let me call it G; it is a test point. I want to know whether in the extension of body; if I extend this body 3 and I extend this body 2. So, G can now belong to body 2 or body 3; so, this is a coincident point in other words there are two points G 2 and G 3.

So, G is a coincident point it can belong to body 2, when it belongs to body 2 or extension of body 2; it is G 2, when it belongs to the extension of body 3 it is G 3. Let me ask the question whether the velocity at G as seen from body 2 and as seen from body 3 are equal or not because that is the relative instantaneous center of rotation.

As you know this point C 2; why is this called instantaneous center of rotation of body 2 because the velocity of point C 2; as it belongs to body 2 as an extension of body 2 is equal to the velocity of the ground which is 0? Similarly, the velocity of point C 3 as an extension of body 3 has a velocity same as that of the coincident point on the ground; so, that is 0.

Similarly, I asked the question whether velocity velocities at G 2 and G 3 are same or not; that means, it must have the same magnitude and it must have the same direction. So, let us test this now body 2 has instantaneous center of rotation C 2. So, therefore, point G 2 must have a velocity perpendicular to this direction C 2; G 2.

So, the velocity of point G 2 must be perpendicular to this direction; similarly the velocity of point G 3 which belongs to the body 3 must perpendicular to this direction; perpendicular to C 3; G 3, so it must be this now they are definitely not same.

Now if you ponder a little bit; you will realize that they can be same only on the line joining C 2 and C 3. So, these two velocities can be same only on the line joining C 2 and C 3; why? Because velocity of a test point on this say this is now the new G; G 2 and G 3 are coincident. So, the velocity of point G 3 must be this and velocity of point G 2 must be perpendicular to C to G 2.

So, velocity of point G 2 must be perpendicular to C to G 2, which is something like this and velocity of point G 3 must be perpendicular to C 3; G 3; so, which must be like this. So, now the magnitudes are not matching this it may not match because G is the test point right now.

So, how to find out that point at which the magnitude is also must match; see the directions will match on this line. Any point on this line, the direction of the velocities will match but the magnitudes are not matching. So, to find that out I draw this line and I draw this line. So, these are touching these vectors; so this is touching the black vector and the other line from C 2 is touching the red vector.

Red vector belongs to body 2 and the black vector belongs to body 3; now wherever they intersect, you can see that must be the point G. Where the velocity direction also matches as well as the magnitude also matches. So, this point G must be the relative IC of bodies 2 and 3; so, this point G is the relative I C.

Sometimes we will indicate it as I; 2 3, in a similar manner C 3 will be denoted as I 1 3 and C 2 will be denoted as I 1 2. The relative instantaneous centres of rotation of body 2 with respect to 1 is C 2 or I 1 2. The relative instantaneous centres of rotation of body 3 with respect to the ground is C 3 or I 1 3; the relative instantaneous centres of rotation of body 2 with respect to 3 is I 2 3.

So, this is the concept of relative instantaneous centres of rotation. Now you have possibly noticed another very interesting thing that; these three relative IC's they lie on a line, they have to lie on a line. The way we have constructed now, they have to lie on a line; I mean this naturally appears from our construction. So, these 3 relative IC's; they lie on one line; so, I 1 2, I 2 3, I 1 3. So, I 1 2, I 2 3, I 1 3; they all lie in one line, this is what we are going to now generalize.

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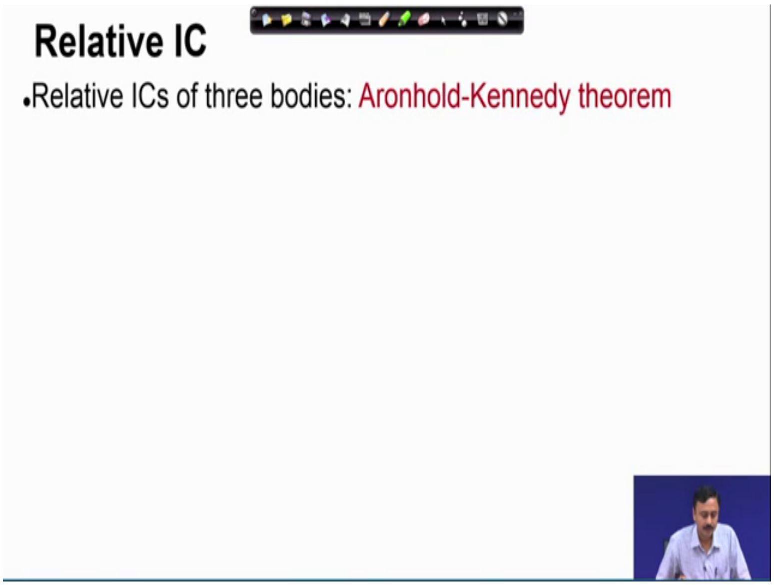

And this generalization is called the Aronhold-Kennedy theorem, which states that if three bodies are in relative motion with respect to one another.

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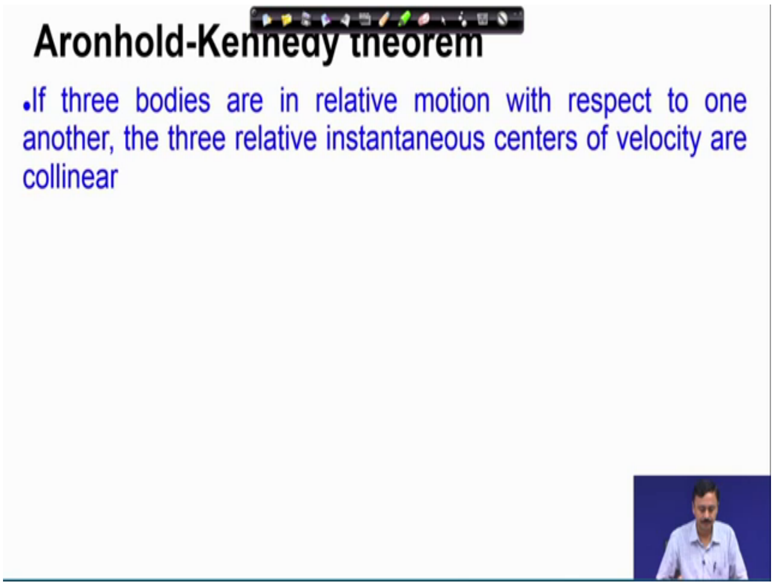

The three relative instantaneous centres of velocities are collinear; so, that is the Aronhold Kennedy theorem.

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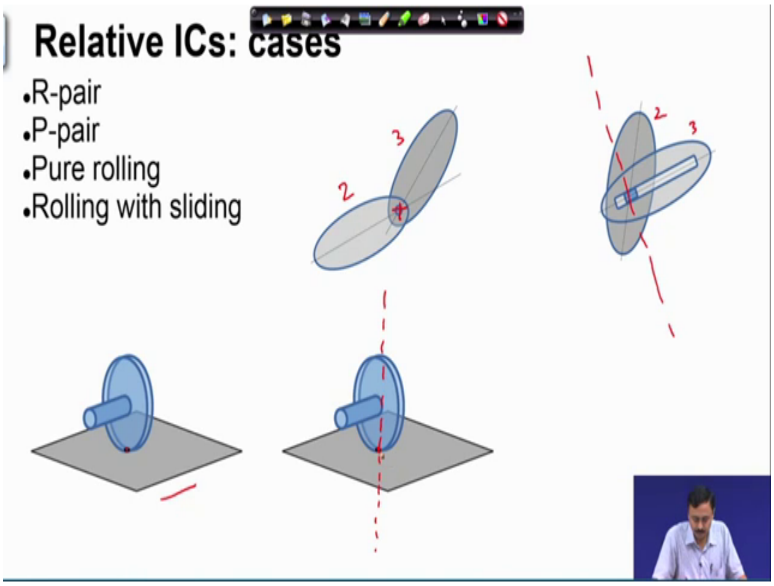

So, let us look at the relative IC's for some simple cases; for the revolute pair the relative IC of the two bodies; 2 and 3 is the R pair itself. The instantaneous center of rotation, the relative instantaneous center of rotation of these two bodies 2 and 3; this being prismatic pair is that infinity along this line which is perpendicular to the direction of sliding.

The relative IC for a wheel, which is in pure rolling as I have shown here; the relative instantaneous center rotation, so now, here it is between the wheel and the ground is the point itself the point of contact itself because we know that it is purely rolling, there is no slip. So, therefore the velocity of this point of contact is 0 and the ground also has velocity at that point as 0. So, therefore, the velocity is matched; so, this must be the instantaneous center of rotation. So, the relative instantaneous center of rotation between the wheel and the ground; in the case of rolling with sliding, now this point does not have 0 velocity; this can have some arbitrary velocity.

So, what we can at most say is the relative IC must lie on this line; which is perpendicular to the ground and passing through the point, the relative IC must lie somewhere on this line which is not known. If it is in pure translation, if this wheel is in pure translation; that means, it is absolutely sliding no rolling motion; then it is at infinity as we have seen, but if it is rolling and sliding it must be somewhere on this line.

So, the two extreme cases are now known to us; when it is pure rolling, it is the point of contact that is the relative IC; relative instantaneous center rotation. If it is purely in translation then it is at infinity on this line, if it is in between; that means, it is rolling and sliding then its somewhere on this line; between this point and infinity either in the upward direction or in the downward direction.

So, let me summarize this lecture; what we have discussed today in this lecture. We have introduced the concept of instantaneous center of rotation from the geometric concept of velocity. We have introduced the concept of relative instantaneous center of rotation and we have seen how; when three bodies are in relative motion, how their relative instantaneous centres of rotation must lie on a single straight line, which is the Aronhold Kennedy theorem. So, we have looked at these concepts; now we are going to use these concepts to find out the relative instantaneous centres of rotation in the case of mechanisms. And based on that, we are going to carry forward our discussions on velocity analysis.

So, with that; I will close this lecture.