Mechanism and Robot Kinematics
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Lecture – 20
Velocity Analysis: Geometric Concepts – I

In this lecture, we are going to begin discussions on velocity analysis of mechanisms. So, first I will define what is the velocity analysis problem, then we will look at some elementary geometric concepts in velocity analysis based on which we are going to discuss something called instantaneous center of rotation of A body, of A rigid body.

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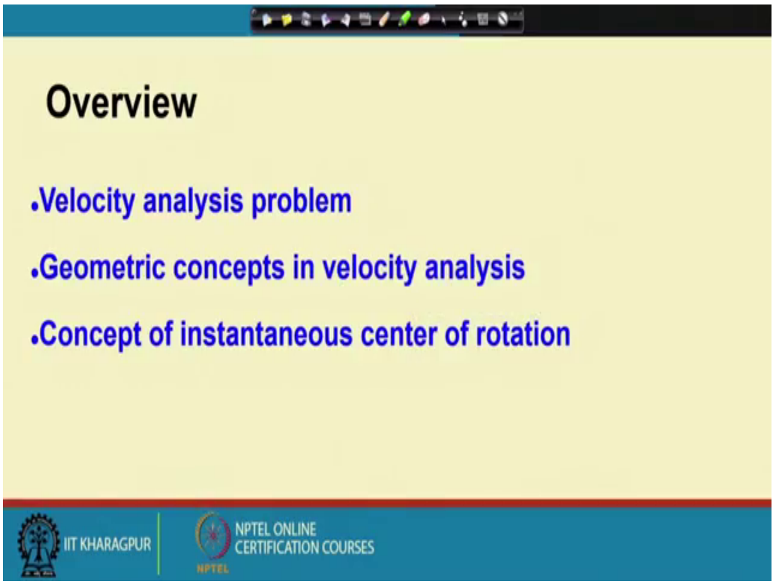

So, to give you an overview, so we are going to first define the velocity analysis problem, look at some geometric concepts as I mentioned, which will lead us to this concept of instantaneous center of rotation.

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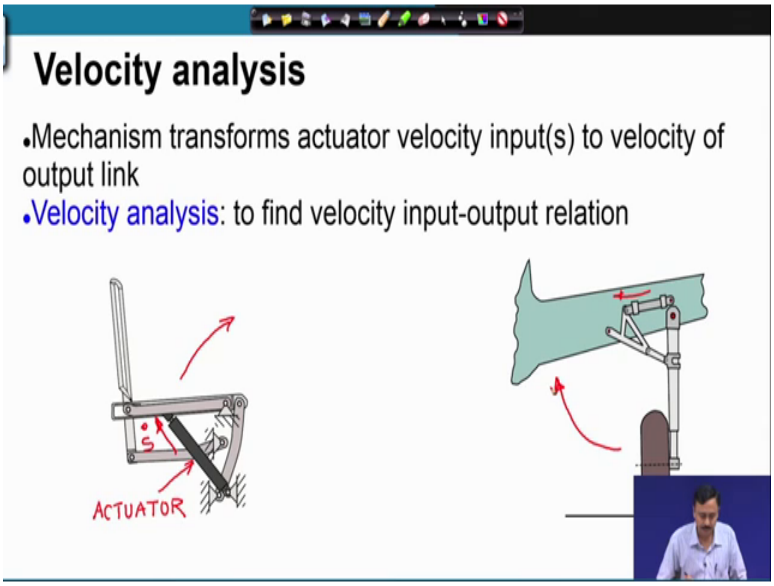

So, what is the velocity analysis problem? As you know, a mechanism transforms actuator velocity inputs, so there can be multiple actuators and these actuators provide velocity inputs you can think of actuators providing velocity inputs which are transformed by the mechanism to the output link. So, the velocity analysis problem is to find out the velocity input-output relations, let us consider this example of this transfer device.

Here, we have this actuator as we have seen before this is a constrained mechanism with 1 degree of freedom, so this is the actuator. It is a prismatic actuator, which will expand and it will take this chair from the sitting position to A standing position. Now, the problem is, if I want to have a certain velocity of transfer then what should be the expansion rate of this actuator. So, rate of change of length of this actuator, how does it relate to the rate of movement of the chair from the sitting to the standing position. Now, when the person is near about the standing position then the velocity should be less, so that he is not pushed forward. So, we need to control the velocity of the rate of expansion of the prismatic actuator.

Here, is another example of the landing gear mechanism of an aircraft. So, if I want to know at what rate mud must this actuator, this hydraulic actuator expand, so that I have A certain rate of retraction of the wheel.

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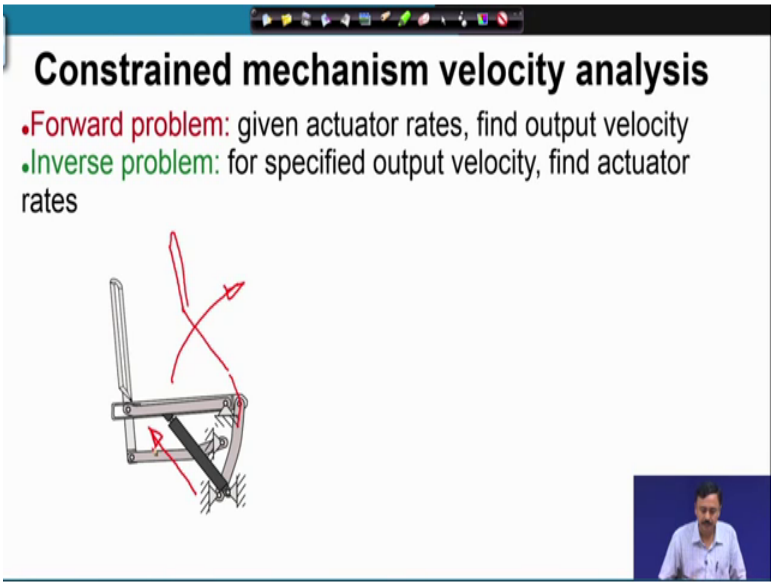

So, for constraint mechanisms, we are going to look at the velocity analysis problem we are going to start with constraint mechanisms. Now, there are 2 problems in velocity analysis as in the displacement analysis. So, the forward velocity analysis problem states that, given the actuator rates find the output velocity; velocity of the output link. The inverse problem is about finding out the actuator rates given a specified output velocity for example, in this transfer device, if I am specified a certain rate of upward movement which might also change with the configuration of the chair. So, it might also change with the configuration of the chair. So, how do I control the actuator expansion rate? So, how should I specify the actuator expansion rate? So, that is the inverse problem.

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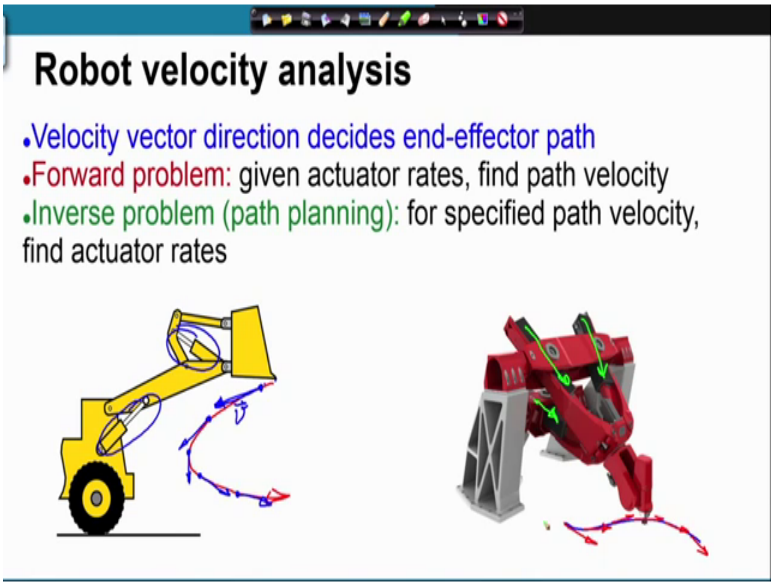

In the case of robot velocity analysis, the problem is a little more complicated, the reason being in robots as we know the degree of freedom is 2 or higher. So, therefore, the end effector velocity direction decides the path of the end effector, because as you know that the path on which the end effector moves the velocity must be tangent to that path. So, therefore, if you want to have a certain path you must also have a certain velocity direction at each point of the path.

So, here we have this forward problem in which we, are given the actuator rates which maybe angular rates or maybe expansion rate of a prismatic actuator and we to find out the path velocity; that means, the velocity at every point on the path which of course, will be tangent to the path. The inverse problem, we are given the path velocity which means we are given the tangent velocities at every point on the path, we have to find out the actuator rates which can realize that velocity and hence, it can also realize that path. So this is also a path planning problem, so this is intimately connected with the path planning problem. So, for example, in this excavator if I want to move it on a certain path, I must have the velocity is defined tangent to the path.

So this, these are the velocity vectors. At each point, we have such a velocity vector, which must be tangent to the path. So, suppose at this point if I want to have this velocity vector, what should be the expansion rates of these 2 actuators or the forward problem is given the actuator rates what is the velocity on the path, so either way you can look at this problem. Similarly, for this parallel kinematic machine, suppose I want to have this path, then I must specify the velocities something like this on that path and corresponding to these velocities I can find out what should be the expansion rate of the 3 prismatic actuators, so that will realize this path.

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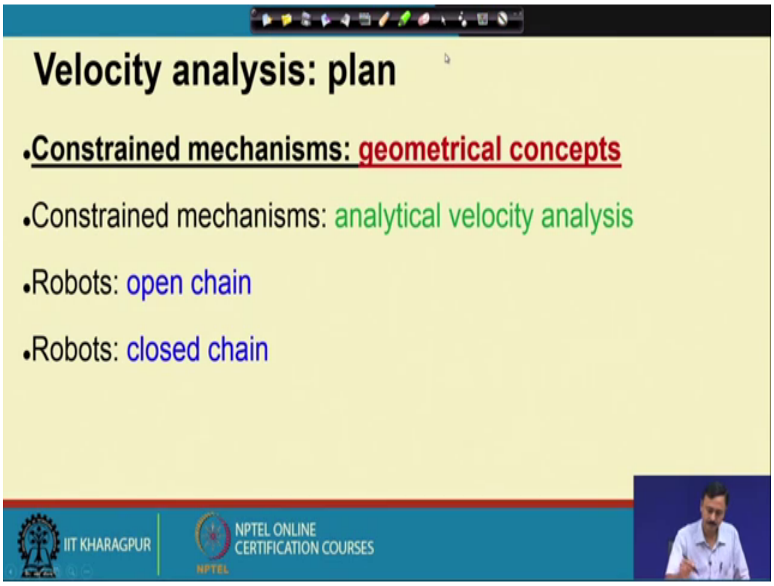

So, let us plan our discussions on velocity analysis, so we will start with constraint mechanisms, we look at some geometric concepts in this lecture. In subsequent lectures, we are going to study and the analytical velocity relations for constraint mechanisms, after that we are going to look at open chain robot mechanisms and finally, closed chain robot mechanisms.

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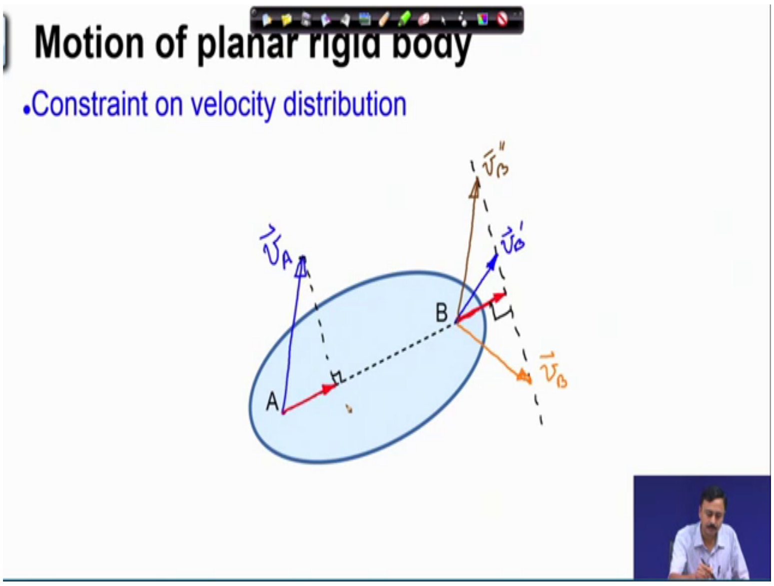

So, we will start off with some geometric concepts on velocity distribution on a planar rigid body. So, I have drawn out for you, body, which is assumed to be a rigid body, a planar rigid body and this can move only on the plane of the paper. Let us look at, what we can say about the velocity distribution on the body which means when this body moves what can I say about the velocities of different points of the body. So, here I have marked out for you 2 points A and B, suppose the velocity of point A is V A, if I ask this question, is there any restriction or constraint on the velocity of point B? So, point B is here, is there any constraint on the velocity of point B? The answer is yes and this is why, it should be because this is a rigid body, remember this is a rigid body.

If I drop a perpendicular on to this line A B, which means I am projecting V A along A B. So, I get this projection vector, this red vector is the projection of V A along A B. Now, the velocity of point B in the direction of A B must remain the same, which means, this must be the velocity of point B in the direction of A B, mind you I am not saying this should be the velocity of B, this should be the velocity of B projected along A B. Now, why this requirement? Because, this body is a rigid body, therefore distance between A and B cannot change. So, the relative velocity in this direction A B must be 0.

The relative velocity V A relative to B in the direction of A B must be 0 because the distance A B must remain fixed. Therefore, if A has a velocity of this red vector along B, then B must also have this same vector along A B. So, what can be the velocity vector at B, so it must be that whatever be the velocity at B it is projection along A B must be this red vector? So, how do I then determine that? So, if I take a direction, perpendicular to the direction A B on the tip of this red vector, then you can very easily see that velocity of B can be any vector that must end at on this dashed line.

So, this could be V B, this could be V B, so any vector starting from B and ending on this line is A valid velocity vector for point B. So, any of these vectors are valid velocity vectors at point B because their projection along A B, is same as the projection of V A along A B. So that, the distance A B cannot change. So, this is an important concept based on the consideration that this body is a rigid body.

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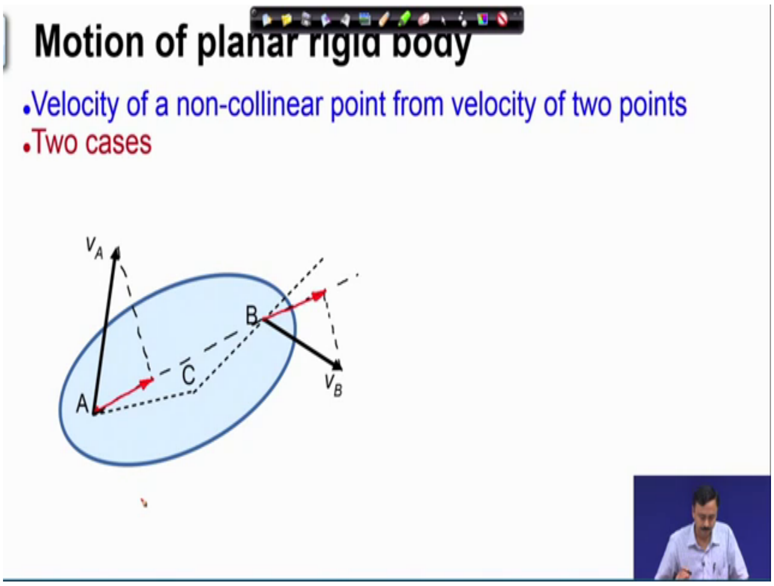

So, let us move on. Now, let us consider a rigid body again on the plane with velocities specified V A and V B which must satisfy of course, our constraint let us look at that once again. So, this is the direction A B, so the projection of V A along A B and V B, along A B, so let us see that vector. So this and this, these 2 red vectors must be equal and they are equal to the accuracy of my drawing. So, V and V B are valid velocity vectors of points A and B.

Now, consider another point C as I have shown here. I would like to find out the velocity of point C, given the velocities of point A and B. So, can I do that? Remember, using the same constraint that the distance A C cannot change and distance C B cannot change, what I must do is I must drop perpendiculars, I must project V A along A C, V B along B C look at these projection vectors.

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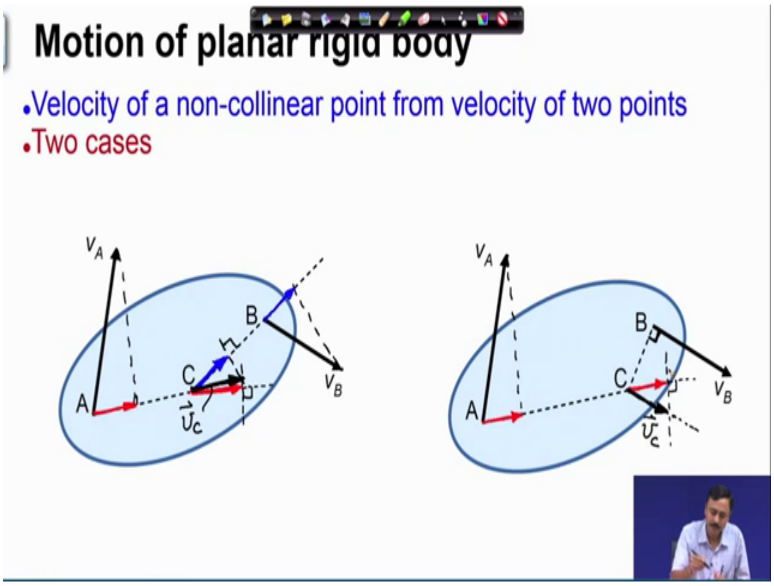

So, this is the projection of V A along A C, this is the projection of V B along B C. So, therefore, C along the direction V, along the direction C B must have this velocity. Once again, please remember this blue vector that I have drawn at C is only the velocity of C in the direction C B; it is not the velocity of point C. Similarly, velocity of point C in the direction A C must be this red vector.

So, I have these 2 projection vectors. now, I have to locate or find out velocity of point c, so remember, what are the constraints on velocity of point C, the whatever be the velocity of point C it is projection along C B must be the blue vector at C and it is projection along A C must be the red vector that I have drawn at C. So, since these are projection vectors, so I must draw perpendiculars to these directions and these 2 perpendiculars meet at this point. So, therefore, so these are perpendiculars.

So, therefore, the velocity of point C must be this, why it must be this? Because it is projection along C B is the blue vector as you consider easily and it is projection along A C is the red vector as you can see. So, this black vector must be the velocity of point C. So, please remember, that when I want, when I am given the projections along 2 directions of the velocity vector of a point then to find out the actual velocity vector I must draw perpendiculars on the heads of these projection vectors, wherever these perpendicular intersect that point joined with the point here it is C gives us the velocity of point C.

The reason is clear, this black vector must have projection there projection along A C must be the red vector and projection along C B must be the blue vector, so we have this construction. Let us move on, again I have chosen another point C, such that the angle that B C makes with the velocity V B is 90 degree as you can see here. I want to find out the velocity of point c, so I proceed as we have done. So, I drop my project V A that means, I drop a perpendicular on A C and this is the projection of V A along A C.

So, C must have the same velocity along A C, so this must be the velocity of C along A C. Now, what should be the velocity of C along C B, it must the projection of V B along C B. Since, V B is perpendicular to C B, so therefore, projection is 0. Now, because this is 0, it means that point C cannot move or does not have any velocity along C B. So, point C does not have any velocity along C B; it cannot have any velocity along C B. So, now, how do I find out the velocity of point C? So, as before I must first draw perpendicular to the direction A C. Now, since C does not have any velocity along B C, so that vector is exactly this point C, the vector along C B is exactly 0, so it is this point C itself.

So, if I consider that I to draw perpendicular to the direction C B, it must be through the point C because the vector; the velocity vector of C along C B the projection of velocity of C along C B is 0. So, that velocity along C B is the point C itself. So, if I have to draw perpendicular to that vector it must be through the point C and perpendicular to C B, which I have drawn.

So, therefore, my velocity of point C must be this, this must be the velocity of point C as you can that is easily realize that V C has A projection of this red vector along A C and V C has 0 projection along C B, V C has 0 projection along C B, V C has the projection of that red vector along A c, so this must be the velocity of point C. Now, this construction brings us to this important or interesting question that what would happen, if I had taken the point C which lies on the common perpendiculars of both V B and V A. So, that is the question we are going to address next.

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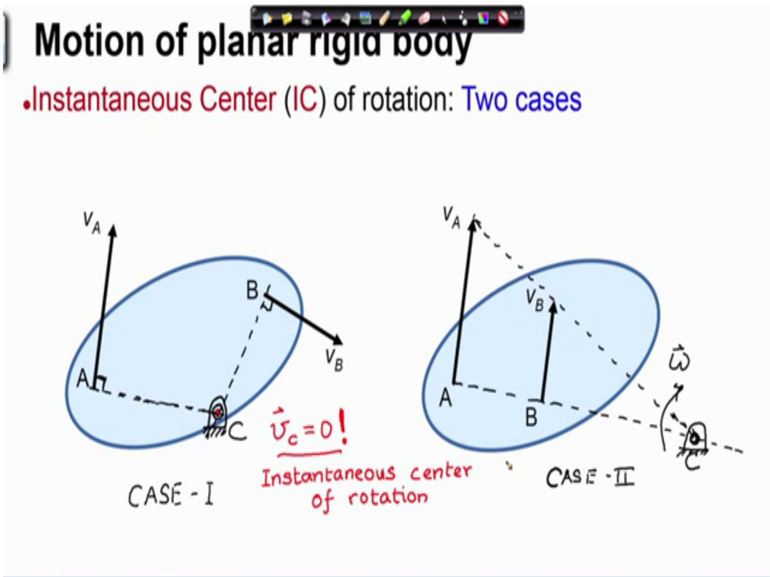

So, here I have 2 cases, let me start with case 1. So, if I ask this question, that let me take A direction perpendicular to V B and A direction perpendicular to V A and locate my point C here on the intersection of these 2 perpendiculars, so these are perpendiculars. I am locating point C on the intersection at the intersection of the 2 perpendiculars 2 V A and V B.

Now, what is the velocity of point C? From the previous constructions it is very easy to see that velocity of point C must be 0. Why, because along B C, there is no projection of V B. Therefore, the velocity of C along C B must be 0; the velocity of C along A C also must be 0 because V A also does not have any projection along A C. So, therefore, velocity of C along 2 directions; projection of the actual velocity of C along 2 directions is 0. These 2 directions are non collinear, so therefore, the velocity of C must be 0, this is known as the instantaneous center of velocity or instantaneous center of rotation, this is the instantaneous center of rotation.

So, the idea is that, at this instant with this velocity distribution that means velocity of point A given by V A and velocity of point B given by V B and hence since these 2 velocities are specified, velocity of any other point is actually specified, so the whole motion of this body is specified. So, at such an instant you can imagine that this point C is kind of hinged to the ground because it is velocity is 0 at this instant.

So as if, the whole body is rotating about point C, that is the idea of instantaneous center of rotation, but remember this is at this instant, that is why it is called instantaneous center, at this instant next instant the velocities might change and C will go to some other point, but still there will be a point C, which is the instantaneous under rotation the velocity at that point is 0 and hence it may be assumed, that at that instant it is kind of hinged to the ground. Now, we come to case 2, the velocities of the 2 points A and B might be like this.

I can of course, draw a common perpendicular direction, but beyond this I do not have a clue how to locate C, but then remember, if some point is to be hinged which is the instantaneous center, then the velocities of these points A and B must be proportional to the distance from that point . So, therefore, if I locate it this way, this point is C and you may assume that this point is hinged. This is the instantaneous center of rotation, when the velocity distribution of 2 points A and B are as shown in case 2. You can see that, velocity B is proportional to B C and velocity of A is proportional to C A and that is why it is increasing linearly with that distance because the angular velocity of this body is something which is fixed. So, that brings us to the concept of angular velocity.

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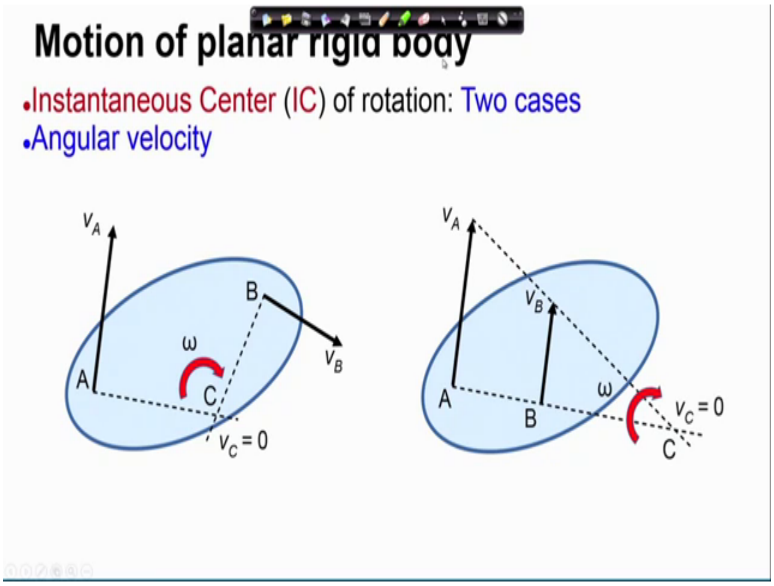

So, here I have drawn out for you the instantaneous centers of rotation for the 2 cases and we have this angular velocity. Now, how do I define angular velocity? Angular velocity is defined as V A by C A. So, the C A is the radius vector, so you know that omega cross C A must be V A and similarly omega cross C B must be V B.

So, therefore, this must also be equal to V B by C B and this is valid for both the cases that I have shown you. So, you need to remember, how to relate the velocities, say the center of rotation with the angular velocity of the body or the angular speed of the body. Now, this we are going to use, so this is how we define the angular velocity or angular speed of the body, how do we relate the angular speed with the centre of rotation. So, once we can locate the centre of rotation of a body and we can find out the angular speed of that body.

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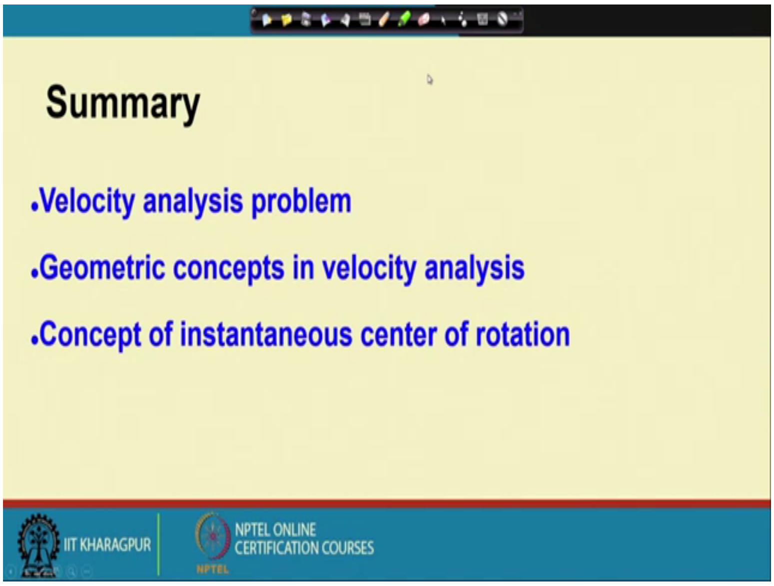

So, to summarize we have looked at the velocity analysis problem, we have looked at some geometrical concepts in velocity analysis and finally, we have come to the concept of instantaneous center of rotation of a rigid body. So, with that I conclude this lecture.