Mechanism and Robot Kinematics
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Lecture – 19
Displacement Analysis: Closed Chain Robot – II

We are going to continue our discussions on close chain parallel manipulators. So, close chain manipulators. So, to give you the overview of what we are going to discuss in this lecture.

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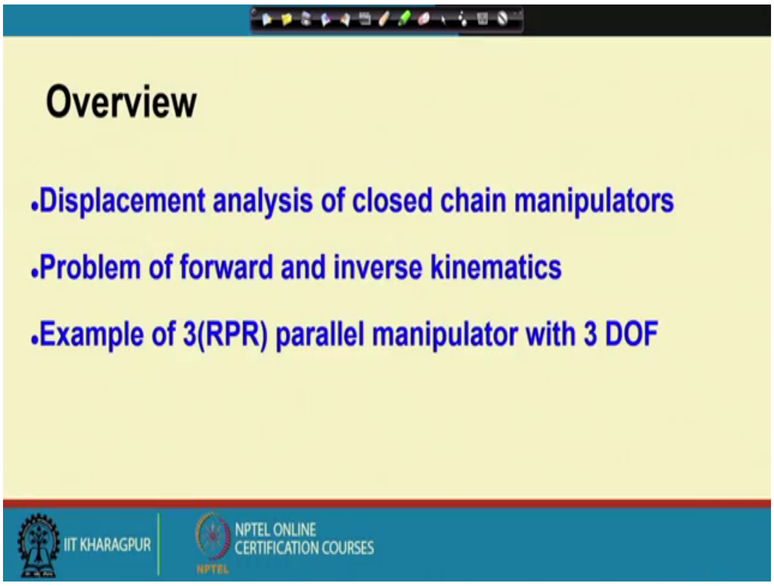

So, the displacement analysis of closed chain manipulators, we are going to look at the example of a 3 RPR parallel manipulator which has 3 degree of freedom we have discussed what a closed chain robot is.

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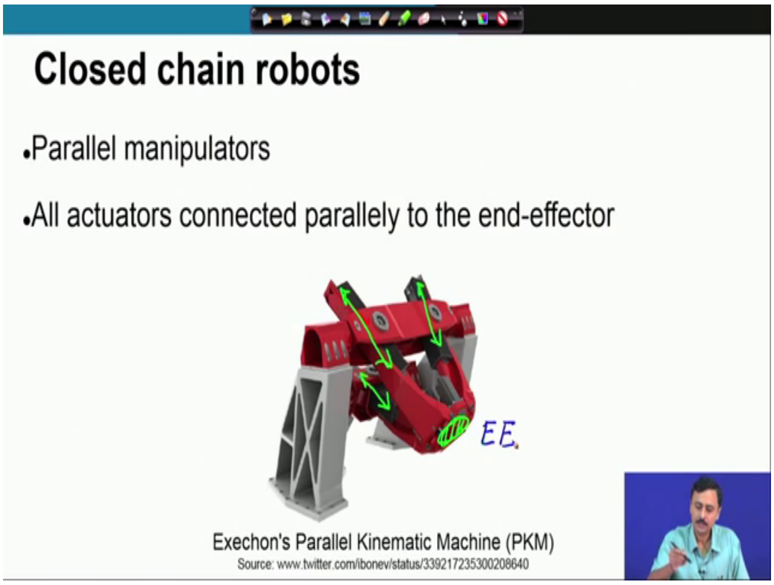

It is also known as a parallel manipulator because all the actuators are connected parallely to the end-effector as this figure shows. So, this is one actuator this is a second actuator the third actuator they are all connected parallely to the end-effector.

So, this is the end-effector and all the actuators are directly connected through links to the end-effector. So, this is a parallel manipulator.

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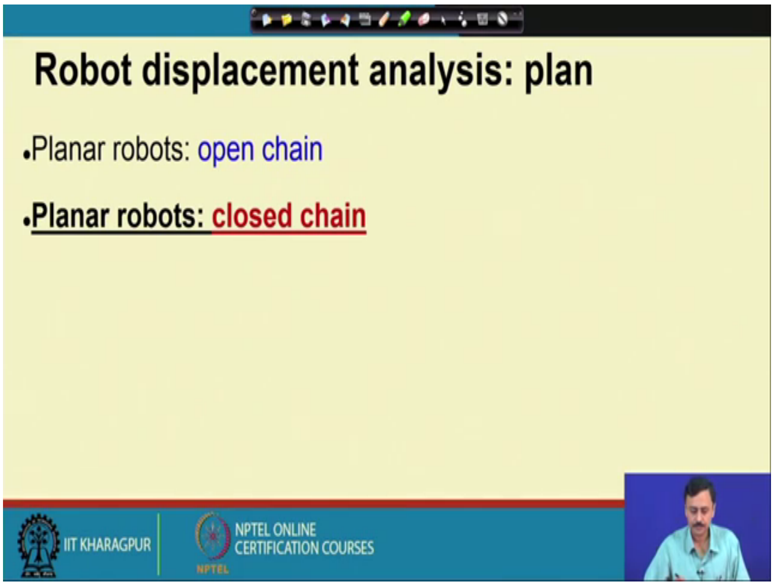

So, we are going to continue our discussions on closed chain planar robots let us look at some of the 3 degree of freedom closed chain planar robots.

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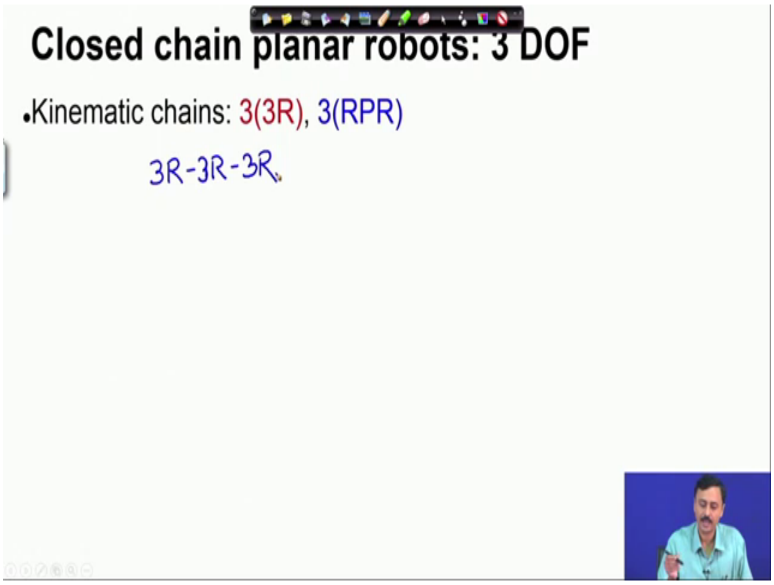

The first chain I have mentioned is the 3; 3 R. So, according to this nomenclature actually it is 3 dash 3 R. So, as per our previous nomenclature, this should be 3 R dash 3 R dash 3 R. So, written a compact form as 3 times 3 R.

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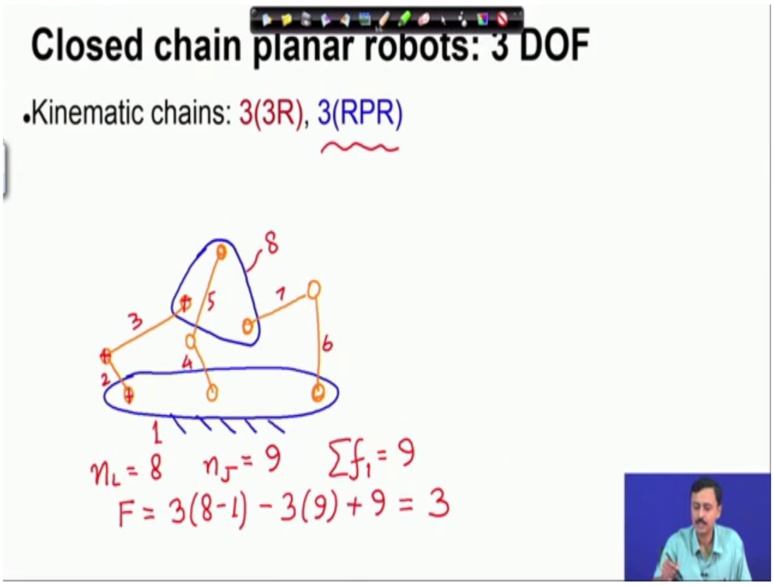

So, let us look at this kinematic chain. We have a ground link and the end-effector link as I have shown now we have 3; 3 R legs. This is one 3 R leg, this is the other 3 R leg, this is the third 3 R leg, let us calculate the degree of freedom of this. This ground is 1, 2, 3, 4, 5, 6, 7 and the end-effector is 8 link; 8.

So, number of links is 8 number of joints 1, 2, 3 plus 3; 6 plus 3; 9. So, we have nine joints nine kinematic pairs each having degree of freedom one since they are all revolute pairs. So, therefore, summation of degree of freedom is nine. So, therefore, degree of freedom is 3 times number of links minus one minus 3 times number of joints plus summation of degree of freedom of each joint. So, this gives us 3. So, this closed chain planar manipulator has 3 degrees of freedom.

Next is the 3 times RPR which we are going to look at for that also the degree of freedom calculation remains the same because some of the revolute pairs have been replaced by prismatic pairs. So, 3 revolutes have been replaced by prismatic 3 prismatic pairs.

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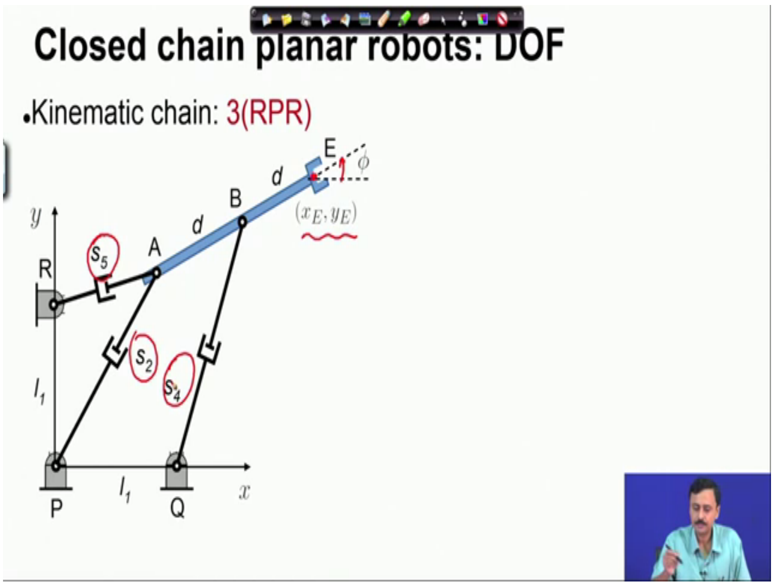

So, let us move forward and look at our 3 RPR chain as shown here we have this end-effector link in blue and there are 3 legs which are which carry a prismatic and actuated prismatic pair.

So, you can see that the coordinates of the end-effector point x E and y E are to be and the coordinates of the end-effector point and the orientation since this has 3 degrees of freedom. So, we can control 3 things. So, position of the end-effector and the orientation of the end-effector link and we have actuations at these 3 prismatic actuators in terms of S 2, S 4 and S 5.

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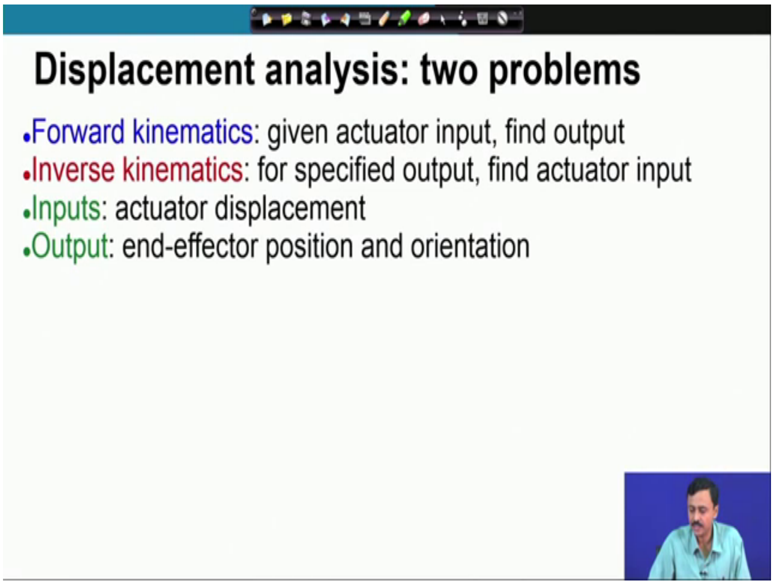

We know that the displacement analysis problem has 2 parts one is the forward kinematics problem in which the actuator inputs are given and we are to find out the output; output means the end-effector position and orientation in the inverse kinematics problem for a given output which means the end-effector position and orientation is specified we have to find out the actuator inputs.

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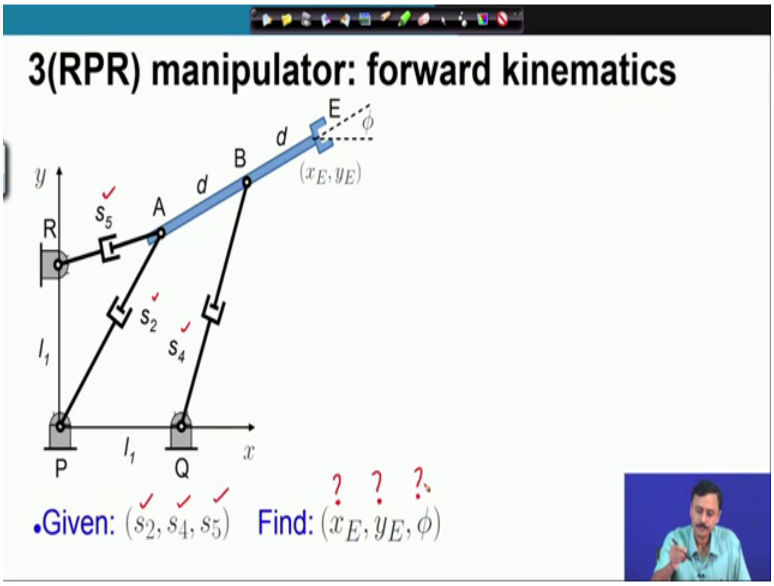

So, let us look at the forward kinematics problem for the 3 RPR parallel manipulator here we are given the actuator throws S 2, S 4, S 5. So, we are given the lengths of these actuators or these legs we have to find out the coordinates of the end-effector x E and y E and the orientation of the end-effector link. So, this is the forward kinematics problem.

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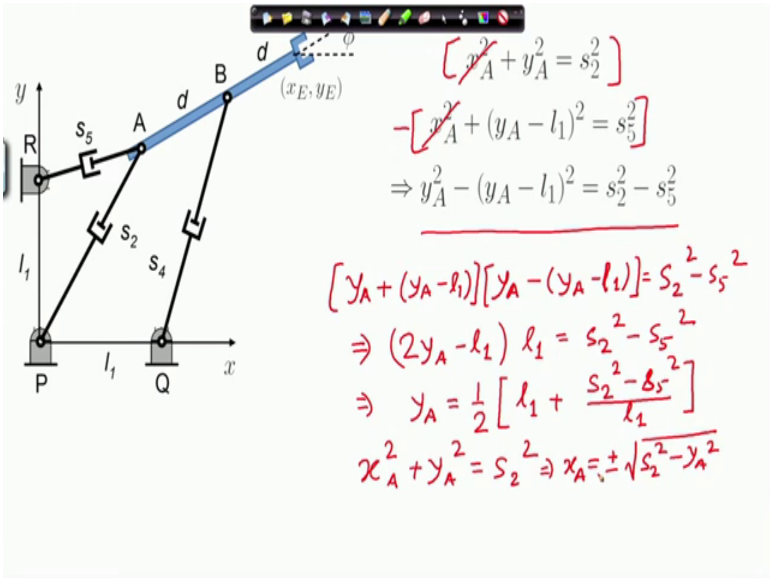

So, let us begin with writing out the coordinates of the points on the end-effector link. So, here I have written out the relations between the coordinates of point a and the actuator throws S 5 and S 2. So, you can very easily derive this expression. So, this is the origin of our coordinate system. So, x A square. So, x coordinate of a square plus y coordinate of a square is the length of actuator 2, S 2 square. So, its length square the length R a square which is S 5 square is given by x S square plus y A minus l 1 whole square.

So, essentially these are equations of circles. So, the first equation is a circle about the origin with radius S 2 the second equation is the equation of a circle with radius S 5 and centre 0 l 1. So, R essentially the point R; so, therefore, if I subtract the second equation from the first if I subtract the second equation from the first, then x A cancels off what I arrive at is this which can be written as y A plus y A minus l 1 into y A minus y A minus l 1 is S 2 square minus S 5 square. So, that gives us 2 times y A minus l 1 times l 1 is equal to S 2 square minus S 5 square. So, from here I can solve for y A. So, one half of; so, this is the solution for y A. So, I have found the y coordinate of point A and then finding out x coordinate of point A is trivial because I have this equation this is the first equation. So, I can solve for x A.

So, we have 2 solutions with plus or minus signs. So, let us look at this formula.

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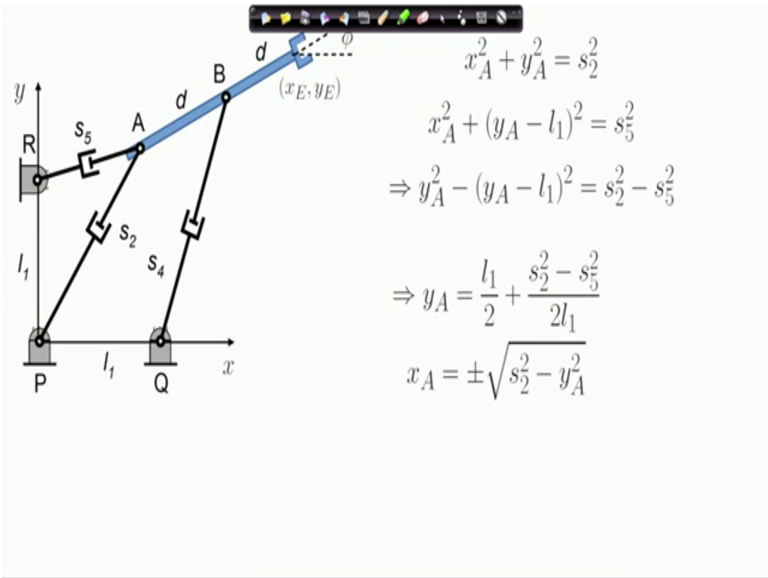

So, this was the solution of y and x A this has 2 solutions.

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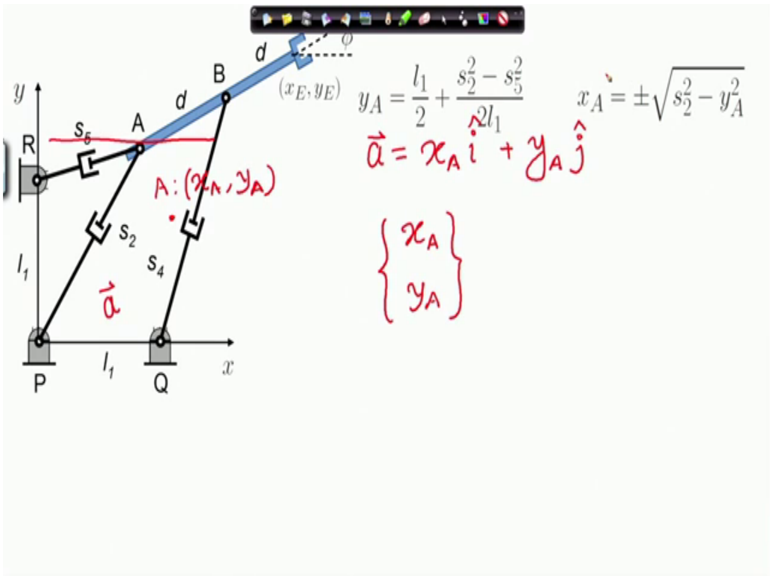

So, I have written out these 2 solutions here these are the coordinates of point A. So, the 2 solutions are specified in term given in terms of these 2 signs.

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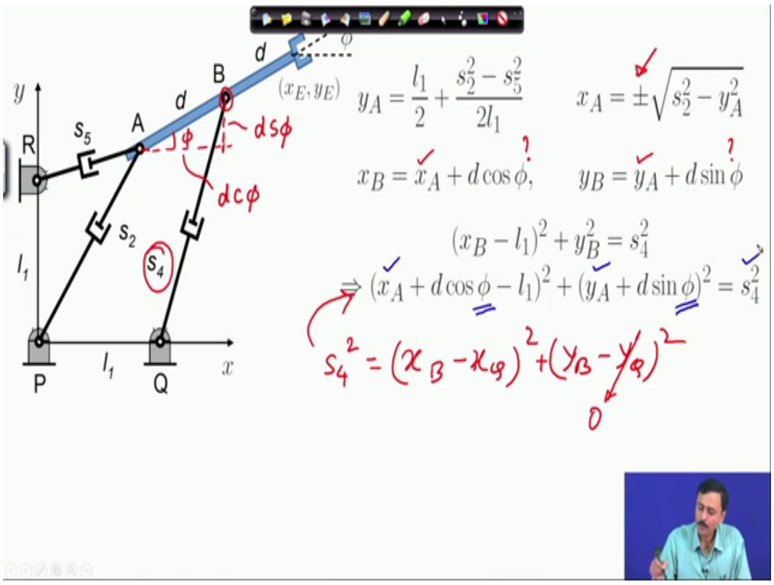

Now, I can express x B in terms of coordinates of x A and I will now introduce this phi. So, x B; so, I am expressing the coordinates of point B in terms of coordinates of point A which is now known to me plus d times cosine phi. So, this distance ay B is d as you can see this is d cosine phi this angle being phi and this is d sine phi.

So, therefore, x B x coordinate of point b is x coordinate of point A plus d cosine phi and y B is equal to y A which is not also known to as plus d sine phi. So, once I have the coordinates of point B of course, in terms of this unknown phi; phi is unknown as yet then I can express the length of S 4; I can express the length S 4 square as x b minus x cube whole square plus y B minus y q whole square. So, that is S 4 square that is what I have written here by substituting the expressions of x B and y B and also x Q y Q is of course, 0.

In this equation, you will find what is not known is phi; phi is as yet unknown x A is known y A is known and S 4 is given. So, phi is the only quantity that is not known.

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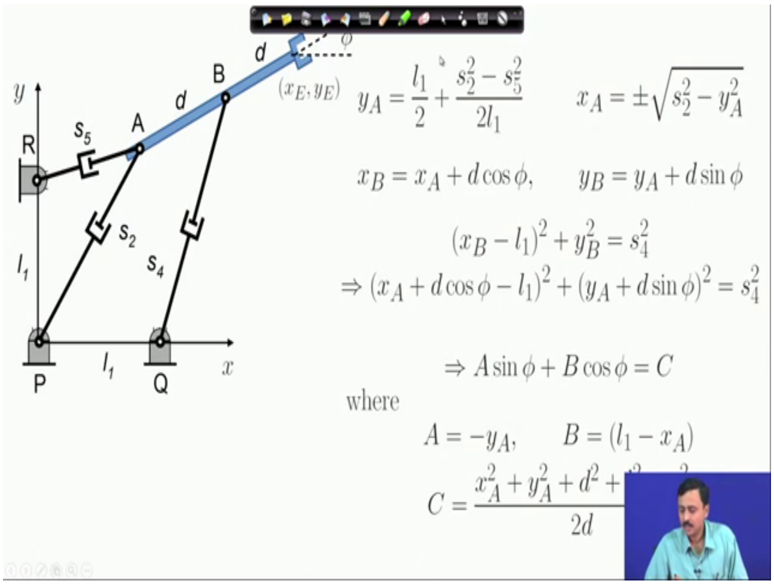

So, when I open up and simplify this equation I can express it in this form which you can very easily do. So, since phi is unknown to us I express it in terms of sine phi and cosine phi where a b and c, they are completely known because y A we have calculated x A we have calculated and S 4 is given to us. So, the only thing that is not known is phi and we know how to solve this equation we make the substitution.

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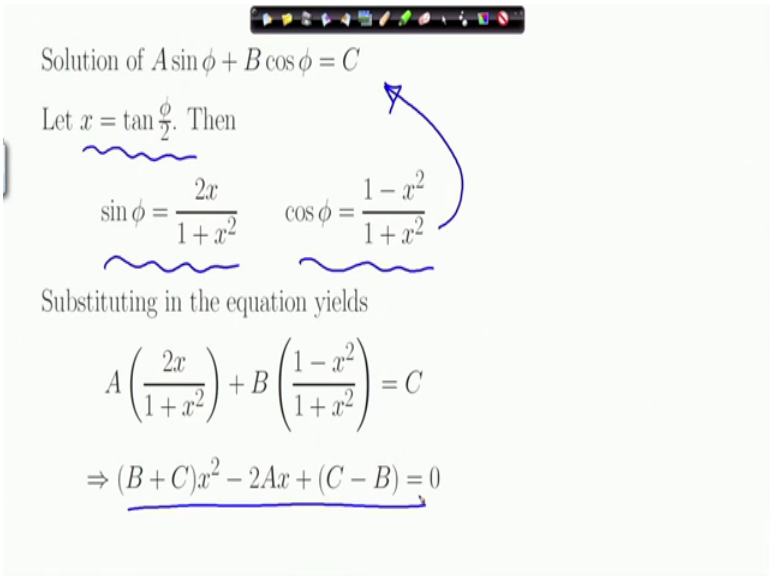

We define x as tan phi by to express sine phi and cosine phi in terms of x substitute back in our equation get this quadratic in x whose solution is known to us.

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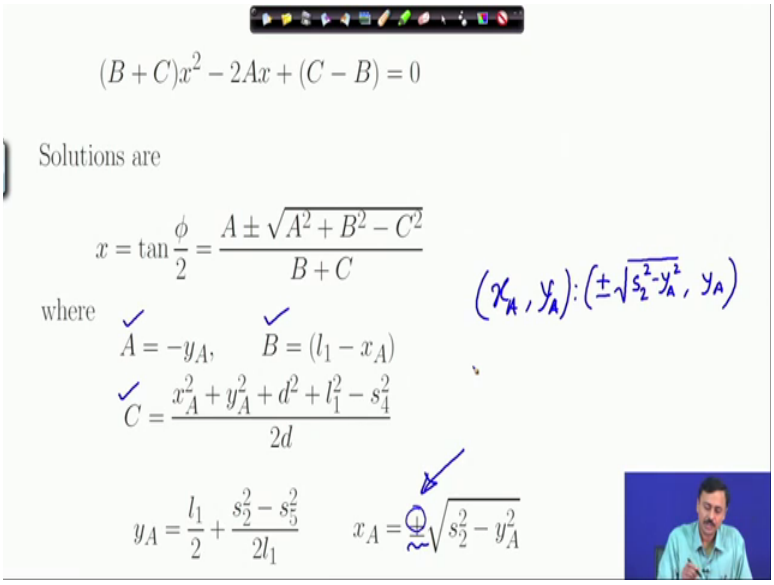

So, we know tan phi by 2 in terms of A, B, C which are known to us once we have phi we have this expression of tan phi by 2.

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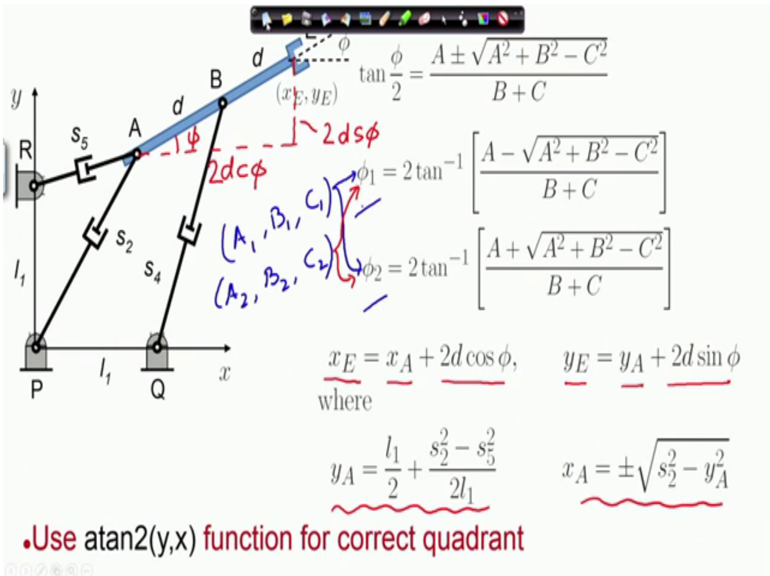

So, I have written out the expression of tan phi by 2 in terms of A, B, C and there are 2 solutions of phi from here phi one and phi 2 which are because of these the sign positive choice of the sign positive or negative here. So, this is with a negative sign and this is with the positive sign. So, these are the 2 solutions of phi.

Here we need to go back once to look at this expression and recall that there were 2 solutions of x A y a. So, x A y A had 2 solutions one with positive sign of x A the other with a negative sign of x a. So, there were 2 solutions of x A and y A corresponding to these 2 solutions of x A and y A you will have 2 values of A, B, C; you may call them A 1, B 1, C 1 and A 2, B 2, C 2. So, therefore, for each value of A, B, C, you will get 2 solutions of phi, this is what we have looked at. So, for each value of A, B, C, you have 2 solutions of phi, phi 1 and phi 2, now since there are 2 sets of A, B, C call them A 1 B 1 C 1 and A 2, B 2, C 2.

We will now have 4 solutions of 5. So, 2 solutions for this and 2 solutions for this set, there will be 4 solutions. Now once we have the solutions of phi we can now express the end-effector coordinates x C in terms of x A plus 2 times d cosine phi. Now this is what is d. So, 2 d cosine phi this being phi and this is 2 d sine phi. So, x E will be x A x coordinate of a plus 2 d cosine phi and y E similarly will be y coordinate of a plus 2 d sine phi and remember that we have these solutions of x A and y A.

So, finally, we have these 4 solutions which we try to understand graphically.

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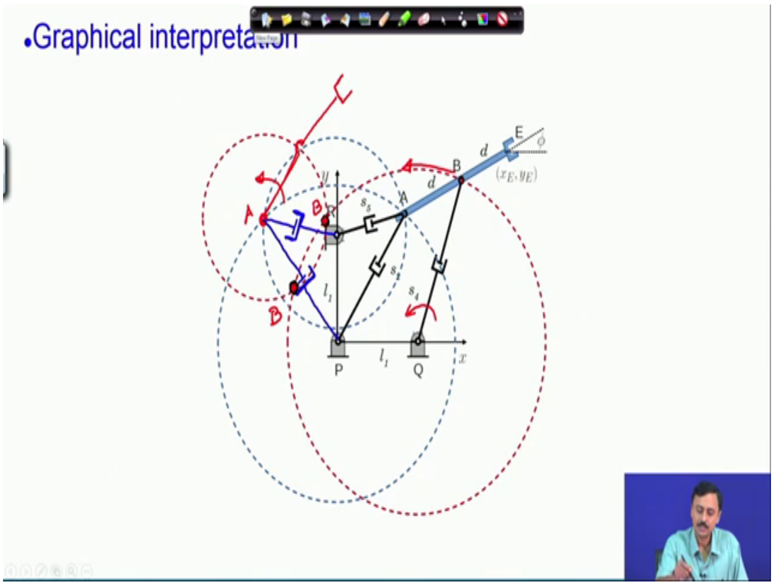

So, we were given S 2, S 5, S 4. So, these were given we were to find out x E y E and phi. Now we think of how we will assemble this mechanism. So, this point a belonging to this link this leg can move on this circle. So, as we rotate this leg a will move on this circle since S 5 is also specified if we rotate this link then a will move on another circle given by this. So, this is the circle with radius R a or S 5 same as S 5.

So, wherever you have intersections of these 2 circles a can lie. So, a can lie on the intersections of these 2 circles, there are 2 possibilities one is this possibility that has been shown the other is this possibility. So, these are the 2 solutions of x A, you can see you can check that y A remains the same the y coordinate of a remains the same. So, the x coordinate of a has 2 solution. So, as an ordered pair a can have 2 coordinates one is here the other is here now corresponding to this solution that is the first solution as shown.

Now, since S 4 is also given. So, the revolute pair b can lie on the intersection of the circle obtained by rotating this leg. So, when you rotate this leg. So, b moves on this circle now because A is fixed here you can rotate the end-effector link. So, therefore, B can move on another circle which is this. So, on the end-effector link B can rotate on this circle.

So, therefore, the intersections of these 2 red circles which are here and here are the possible solutions where b can be assembled and that will fix up the end-effector link. So, therefore, x C y E and phi get fixed. Now A has this solution there is a second solution of a. So, therefore, when you choose this solution of a then this end-effector link when you rotate; so, let me draw out these 2 legs. So, this is the second solution for A and the end-effector link can now be made to rotate about this point A and the S 4 link still rotates. So, B on the S 4 link still rotates on this circle. So, there are 2 intersections of this as you can see here and here.

So, this is one intersection this is the other intersection. So, this is where b can lie. So, there are 2 solutions of B for this second location of A for the first location of A there were 2 solutions for B for the second location of A; there are 2 locations of B. So, there are 4 solutions. So, these are the 4 solutions that we have. So, let us look at this once again. So, we started off with this circle of radius S 2, we want to first locate A. So, we first make a circle with radius S 2 about the origin which is P, then this second circle with center at R and radius S 5.

So, there are 2 intersections of these 2 circles these 2 intersections correspond to the 2 solutions of A. Now once A is fixed then we have to locate B. So, the end-effector link A

E can rotate about A and hence b describe the circle and similarly the leg q b of length S 4 can rotate about q as a center with radius S 4. So, I have drawn this circle with center at q and radius S 4 and with centre A the end-effector link can rotate with radius. So, then rotate about A the radius is D. So, that is the second red circle. So, the intersections are the, correspond to the point B, the intersections correspond to point B. So, there are 2 intersections. So, 2 solutions for B this is for the first choice of A.

For the second choice of A the center at a shifts to the second solution so, b can lie on the smaller red circle on the end-effector link. So, the intersection of the 2 red circles again correspond to the; to solutions of B corresponding to the second solution of A. So, these are the 4 solutions.

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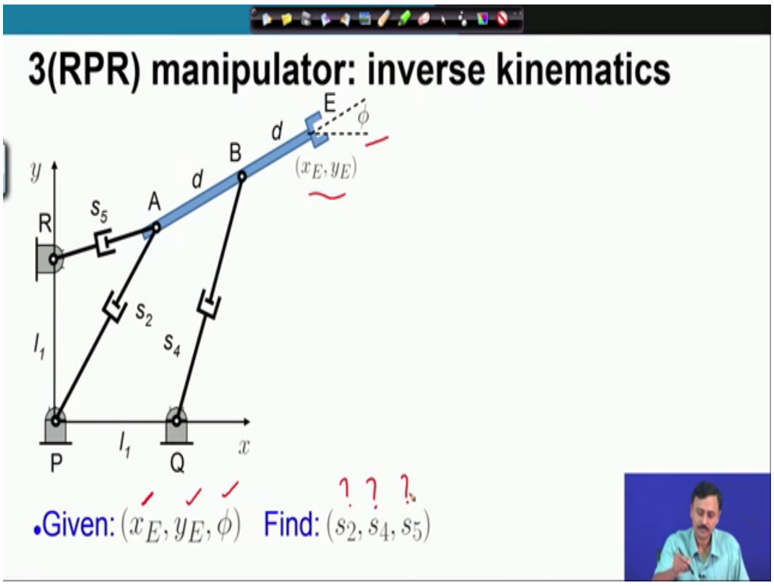

Let us quickly look at the inverse kinematics problem. So, here we are specified x E y E and phi which are the position and orientation of the end-effector; end-effector and we have to find out S 2, S 4, S 5 which are the actuator throws the prismatic actuators.

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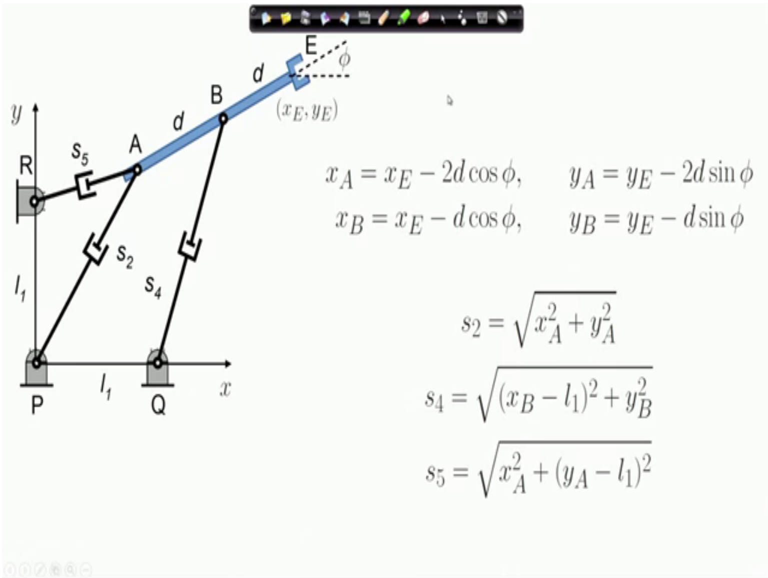

So, here I have written out the coordinates of point A in terms of the known quantities you see x E is known phi e is known y E is known. So, all these are known x E y E and phi these are known. So, therefore, I can calculate x A y A x b y b the coordinates of these 2 points. Now once I know the coordinates of these 2 points it is a matter of simple geometry to express S 2 and see S 2 is this S 2 is nothing, but square root of x A square plus y A square then S 4 is nothing, but square root of x b minus l 1 whole square plus y b square from simple geometry.

And similarly S 5 is square root of x A square plus y A minus l 1 whole square. So, given x E y E phi so, these are given to us we have been able to calculate the lengths of the actuators S 2 S 4 and S 5. So, this is absolutely straight forward.

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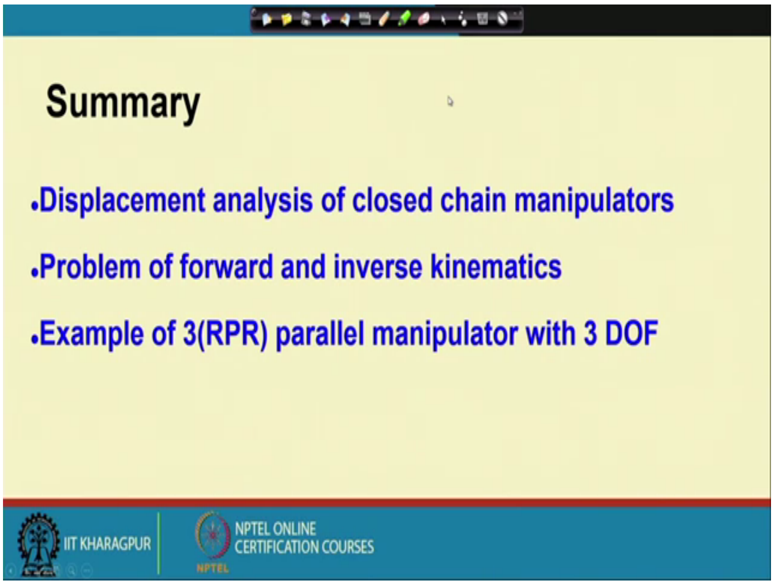

So, let me summarize; we have looked at the displacement analysis of a closed kinematic chain manipulator and we discussed the example of the 3 RPR kinematic chain we looked at the forward and the inverse kinematics problem for this chain with that I will close this lecture.