Mechanism and Robot Kinematics
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Lecture – 17
Displacement Analysis: Open Chain Robot – IV

In this lecture we are going to discuss the kinematics of planar open chain manipulator. So, we are discussing the displacement analysis problem. We will today look at the RPR open chain manipulator.

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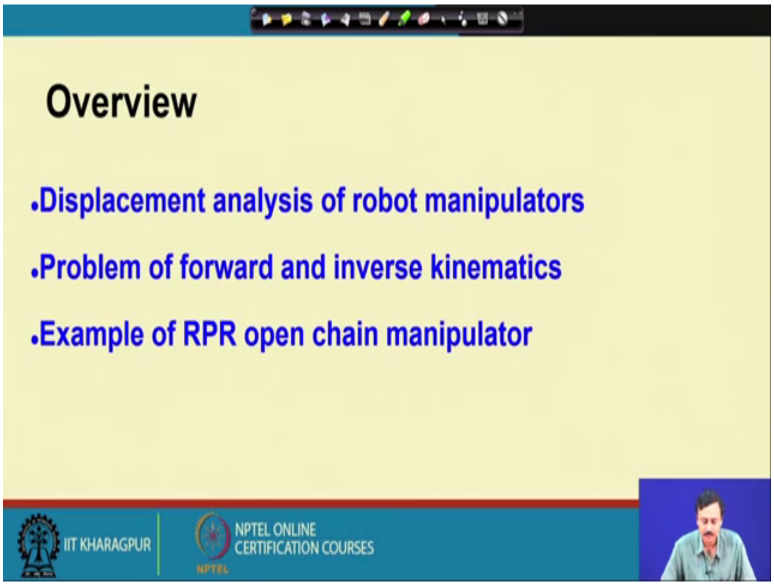

So, here is the overview of today's lecture. So, the forward and inverse kinematics of the RPR open chain manipulator. So, we are continuing with the analysis of open chain planar robots. Later on we will also look at planar robots with closed chain configuration.

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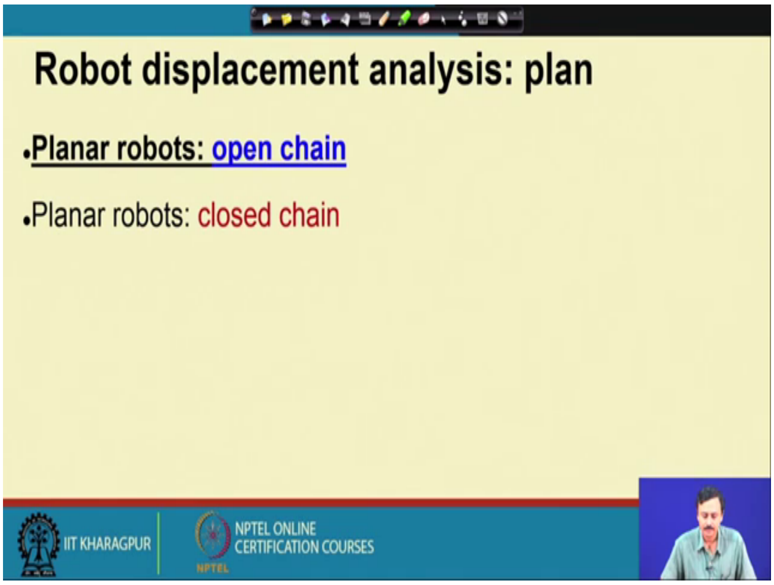

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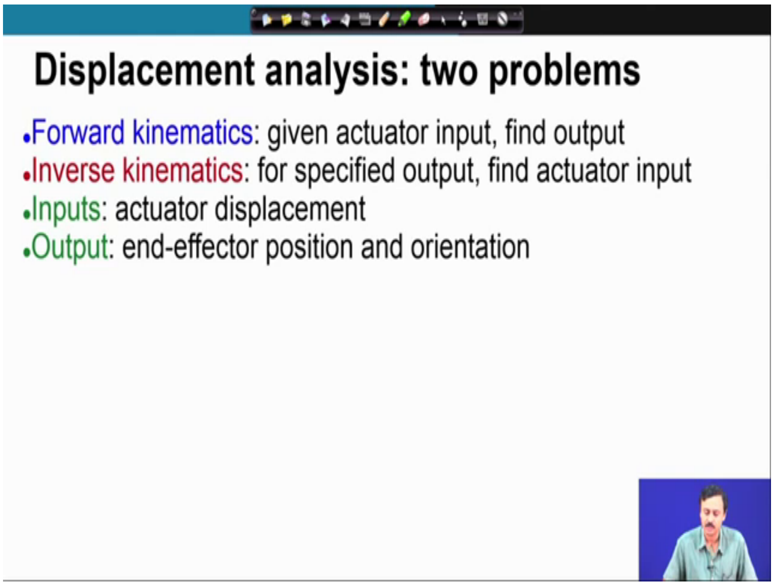

So, these are the two problems of displacement analysis that we have, we will discuss today the forward kinematics problem in which the actuator inputs are given, and the end effector position and orientation are to be found. The reverse is the inverse kinematics problem. So, the output is specified; that means, the end effector position and orientation are given to us. We have to find out the actuator inputs which means the throw of the actuator or the displacement or the angular displacement of the actuators.

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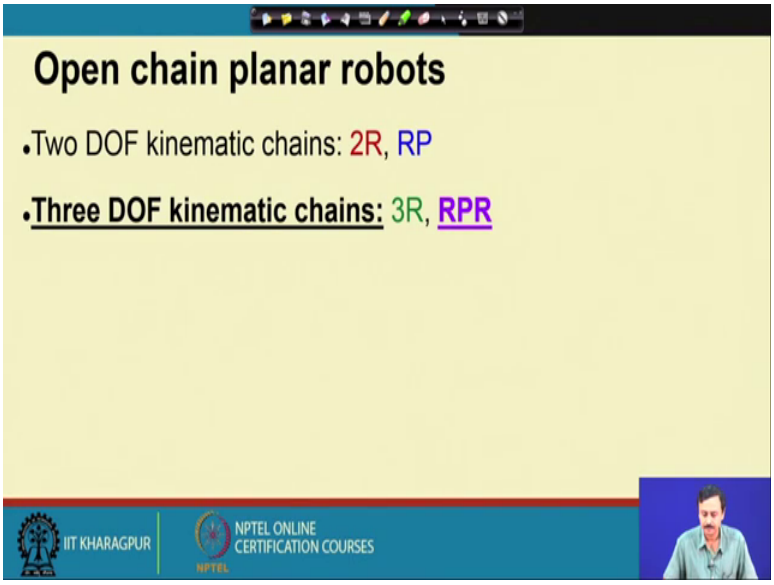

So, today we are going to look at the three degree of freedom planar chain RPR. So, revolute prismatic revolute manipulator.

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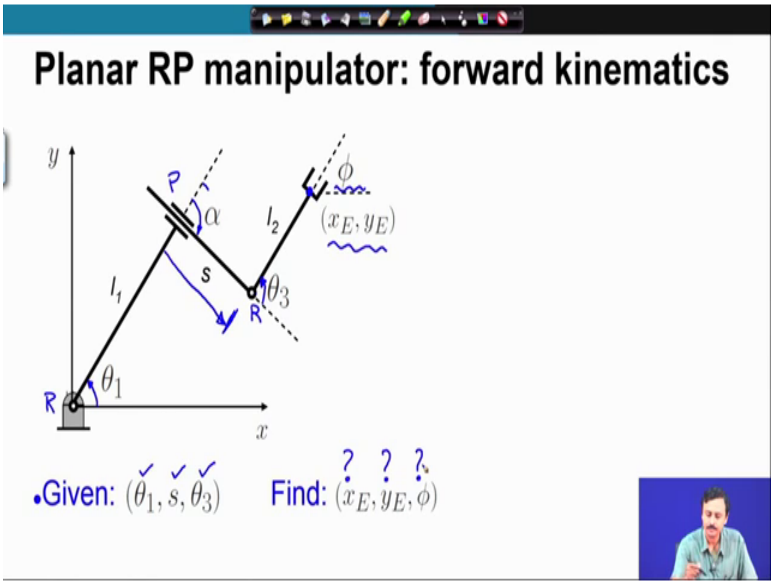

So, here is a schematic of an RPR planar manipulator and the statement of the forward kinematics problem. So, we have these inputs at the three joints. So, one this is the revolute joint. So, here we have this theta 1 angle measured from the x axis. The second joint is a P pair, and what is specified is the throw of the prismatic pair. Here this angle alpha is fixed, this angle alpha is fixed. So, S is the pair variable corresponding to the P pair. So, that is the throw of the prismatic actuator. Third one is this angle theta 3, which is the pair variable for the revolute pair here. finally, we have the end effector position positional coordinates xE and yE and the orientation coordinate phi. So, we have three inputs and three outputs. So, the forward kinematics problem is given theta 1 S and theta 3, we have to find out xE yE and phi. So, that is the forward kinematics problem.

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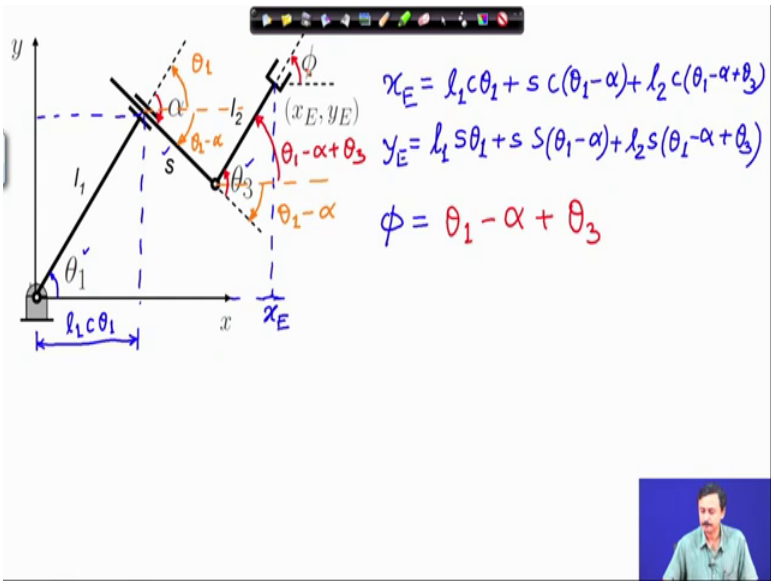

So, this is the schematic once again. Now we have been specified theta 1 S and theta 3, we have to find out first the locational coordinates of the end effector. So, this coordinate is xE, if I have to express S E in terms of this theta 1 S and theta 3. So, what I need to do is find out the projections of this link l 1, this distance S and this link l 2 onto the x axis.

So, let me start expressing this xE. The first component is the projection of l 1 along the x axis, which is l 1 cosine theta 1. So, c theta 1 stands for cosine theta 1; so l 1 cosine theta 1 plus the projection of this S on the x axis. Now this angle alpha is measured in the negative sense. So, therefore, if I look at if I draw this horizontal, this angle is theta 1, and therefore, theta 1 minus alpha would give me this angle.

So, this angle is theta 1 minus alpha; so theta 1 in the counterclockwise sense and alpha in the clockwise sense of theta 1 minus alpha. So, that gives me this angle. So, therefore, I have this term plus S times cosine theta 1 minus alpha. Then there is this next term, because of the projection of this thread link. Now if I draw a horizontal line, as you can see this angle is theta 1 minus alpha, this angle is theta 1 minus alpha, to that I must add theta 3.

So, from this reference dashed line which is the reference line theta 3 is measured counterclockwise; so theta 1 minus alpha plus theta 3 that is the angle that the third link makes with the x axis. So, therefore, the projection will be plus here it the length I have considered as l 2 cosine theta 1 minus alpha plus theta 3. So, that is the x coordinate of the end effector. Now it is not very difficult to write the y coordinate. So, its the projection on the y axis. So, l 1 sin theta 1 is the contribution from the first link, then we have the prismatic throw, so plus S times sin theta 1 minus alpha. And finally, the third link theta 1. So, sin of theta 1 minus alpha plus theta 3.

So, these are the expressions of the positional coordinates of the end effector. Now the orientation coordinate phi. So, the orientation coordinate you can check. So, this is the orientation coordinate. So, it is nothing, but this angle. Now that angle we have already expressed as theta 1 minus alpha plus theta 3. So, this angle is theta 1 minus alpha plus theta 3. So, that becomes the orientation coordinate; that is phi. So, this completes the forward kinematics relations for the RPR open loop planar manipulator. So, this is what I have written out for you

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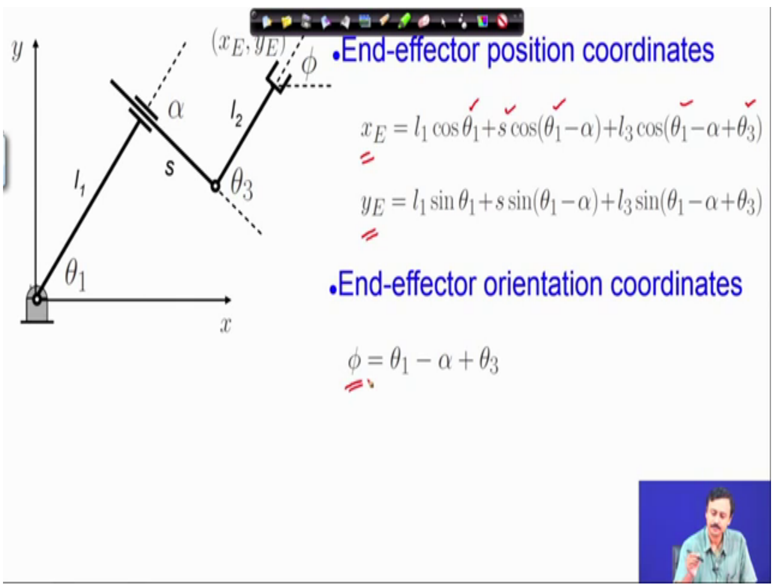

So, xE yE expressions and finally, the end effector orientational coordinate. So, which is phi. So, this is the forward kinematics problem. So, given theta 1 theta 1 S and theta 3, I will be able to find out xE yE and phi.

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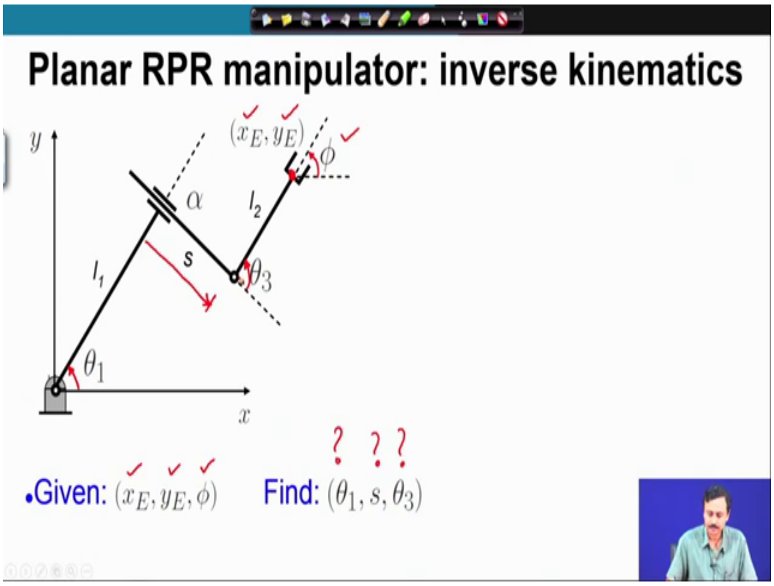

Now the inverse kinematics problem. So, here we have the angles theta 1 is the S and theta 3, and the end effector position and orientation. So, in the inverse kinematics problem we are specified xE yE phi. So, here is xE yE phi, so the position of the end effector and the orientation of the end effector. So, these are the three things that are specified, we have to find out theta 1 S and theta 3. So, theta 1 is the first revolute pair angle, S is the throw of the prismatic pair and theta 3 is the second revolute pair angle.

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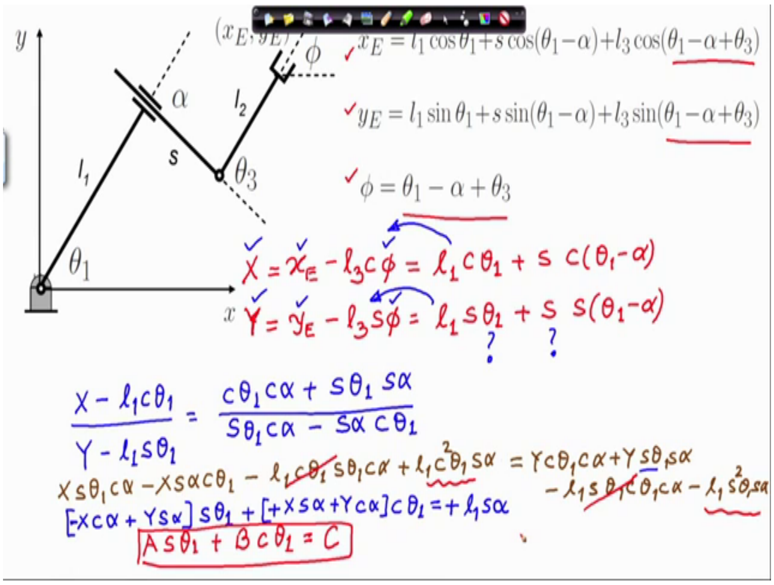

So, here is the schematic of the RPR chain once again. We will start with the forward kinematics relations. So, these are the forward kinematics relations we have derived. Here we have to remember that we are specified xE yE and phi, and you have to solve for theta 1 S and theta 3. Now, if you notice here phi appears in the expressions of xE and yE. So, therefore, if I replace this theta 1 minus alpha plus theta 3 by phi, I can rewrite the first equation like this and the second equation in this form.

So, therefore, if I redefine this as capital X and this as capital Y, then I find that since xE and phi these are specified, yE and phi these are specified. So, capital X and capital Y are completely known to me. On the right hand side theta 1 and S, these are the unknowns. So, theta 1 and S, these are the unknowns which I need to solve. So, the solution procedure follows the standard route.

What I am going to do is? I am going to eliminate S between these two equations in this form. So, capital X minus l 1 cosine theta 1. So, I bring this on the left hand side and take this ratio by bringing l 1 sin theta 1 on the left hand side further in the second equation. So, y minus l 1 sin theta 1 and that is equal to. So, on the right hand side S is eliminated in the ratio, what we are left with, I can write it as cosine of theta 1 cosine of alpha plus sin of theta 1 cosine of alpha this in the numerator, and in the denominator sin of theta 1 cosine alpha minus sin of alpha cosine of theta 1.

Now, I simplify this equation, I multiply. So, what I have is x sin theta 1 cosine alpha minus x sin alpha cosine theta 1 minus l 1 cos theta 1 sin theta 1 cos alpha plus l 1 cos theta 1 sin alpha cos theta 1. So, essentially this becomes cosine square. Let me write this as cosine square. And on the right hand side I have y cos theta 1 cos alpha plus y sin theta 1 sin alpha minus l 1 sin theta 1 cos theta 1 cos alpha minus l 1 sin square theta 1 sin alpha.

Now, you can check that this term gets cancelled with this term, and we can combine this term and this term. So, if I bring it to one side you will have l 1 sin alpha cos square theta 1 plus l 1 sin alpha sin square theta 1. So, that gets combined. So, therefore, I can rewrite my equation as x cos alpha. So, I am collecting the coefficients of sin theta 1. So, x cos alpha and I have. So, here is another term.

So, minus y sin alpha into sin theta 1, then I collect the terms of cosine theta 1. So, I have minus x sin alpha minus y cosine alpha cosine theta 1 and that is equal to. So, I have this these terms, on the right hand side I have minus l 1 sin alpha. Other terms have been taken care of. Now here I can make the sin inversion throughout. So, I make it plus plus plus plus and here minus, and rewrite this as A sin theta 1 plus B cosine theta 1 equal to C. So, this is the simplified form and with you can read out A B and C from the previous equation.

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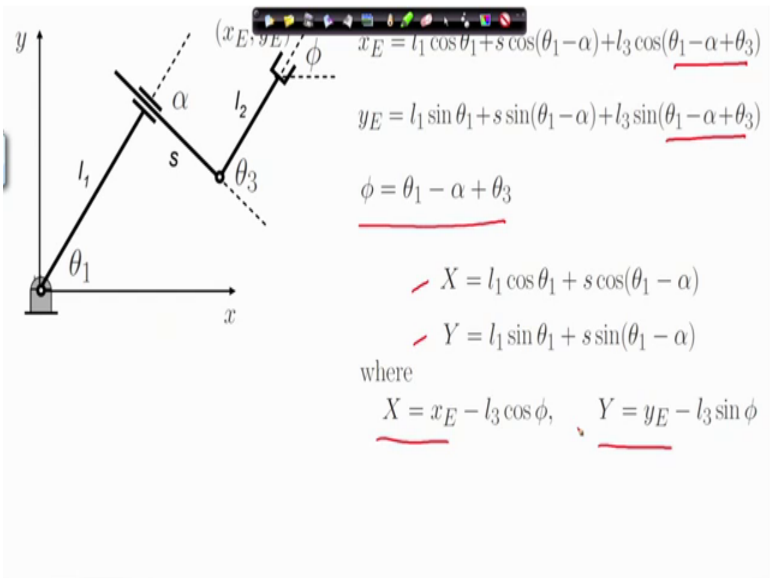

So, this is what I will show you in steps. So, as I have mentioned that I have defined these functions x and y in this form, because phi is known, this is phi. So, I have substituted phi and I have redefined these terms X capital X and capital Y, then taken the ratio I have eliminated S, I find this equation after eliminating S, which I simplified to obtain an equation which has only theta 1 as unknown. Here A B and C are completely known, because this capital X and capital Y are known. So, therefore, I can solve for theta 1 from this equation.

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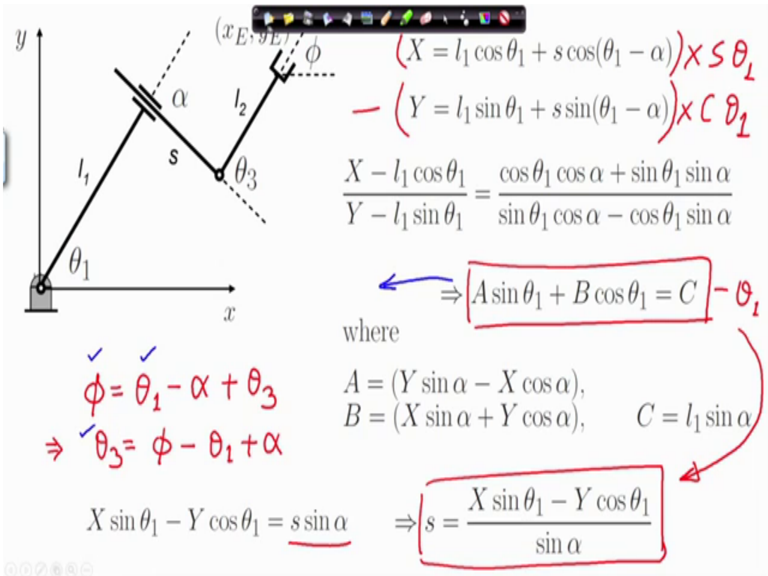

Now, in order to solve for S, I have multiplied the first equation by sin theta 1, the second equation by cosine theta 1 and I have taken the difference. When I do that on the right hand side I am left with only S sin alpha therefore, to solve for S I just divide throughout by sin alpha. So, this is my expression for S. Now, therefore, our solution procedure, is first find, first solve for theta 1.

So, solve for theta 1, substitute that in here, solve for S, and then we are left with one more angle which is theta 3, but we also know that phi is theta 1 minus alpha plus theta 3. So, therefore, theta 3 is equal to phi minus theta 1 plus alpha. Remember that here phi is known, theta 1 we have solved from here. So, therefore, I can calculate theta 3. So, this will complete the inverse kinematics problem.

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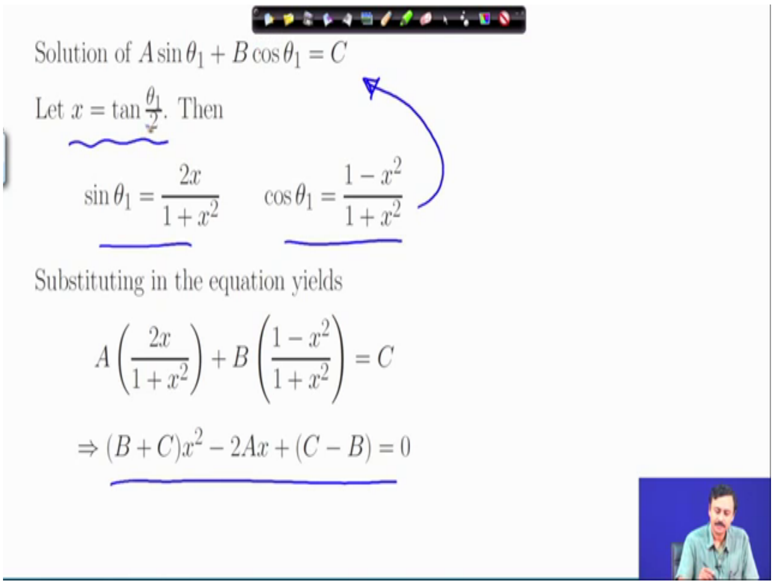

So, let me show this solution of A sin theta 1 plus B cosine theta 1 equal to C. So, we have followed this procedure, many times we make this substitution, we redefine redefine this variable x and x plus sin theta 1 plus cosine theta 1 in terms of x, which when substituted in the master equation gives us this quadratic equation, whose roots give us tan theta 1 by 2.

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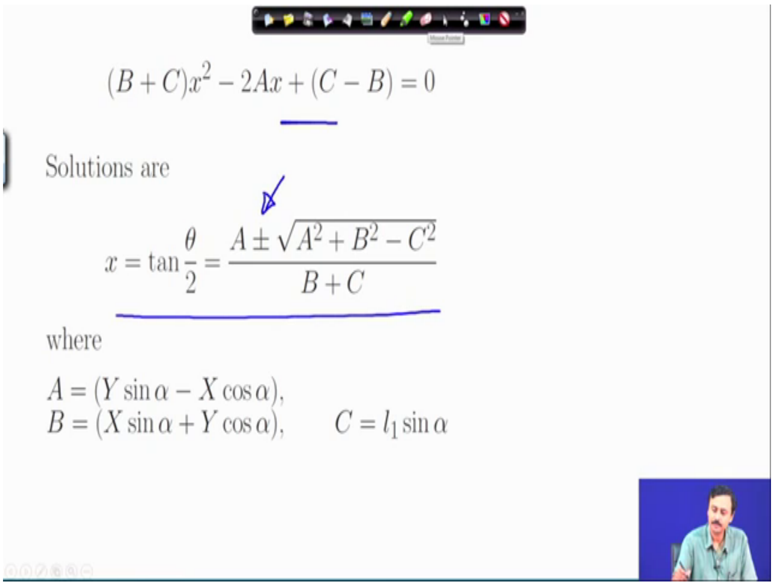

So, these are the. So, this is our quadratic equation. The roots are obtained like this. So, there are two roots, corresponding to this positive and the negative signs. Here A B and C we have derived earlier. So, from here we can solve for theta 1.

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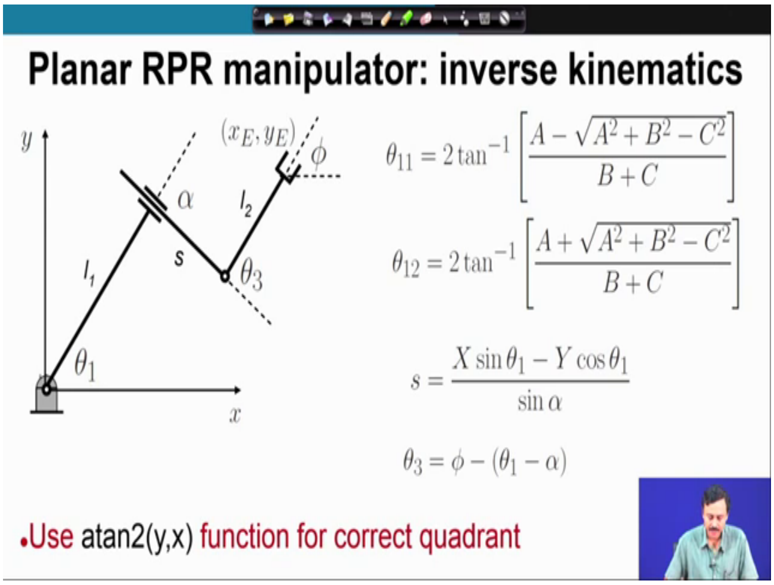

So, theta 1 has two solutions. Once again this tan inverse you have to use atan 2 function to get the correct quadrant. So, we have two solutions of theta 1, theta 1 and theta 1 2. S is obtained in this form which we have derived. Finally, we have theta 3 which can be found in terms of phi and theta 1 and alpha is a fixed angle. So, this completes the solution for the inverse kinematics problem.

So, we have determined theta 1, S and theta 3. So, we have determined all these three joint variables. So, as I have mentioned we have to use this atan 2 function. Now let us understand the solution graphically.

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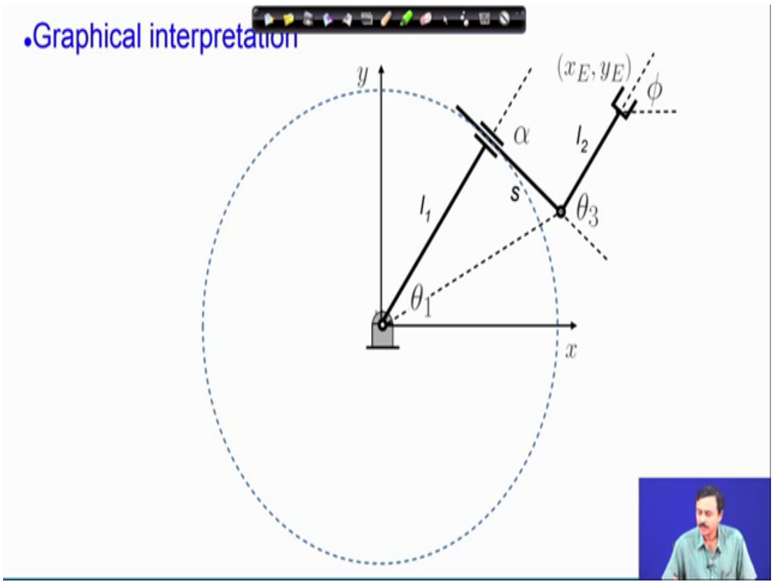

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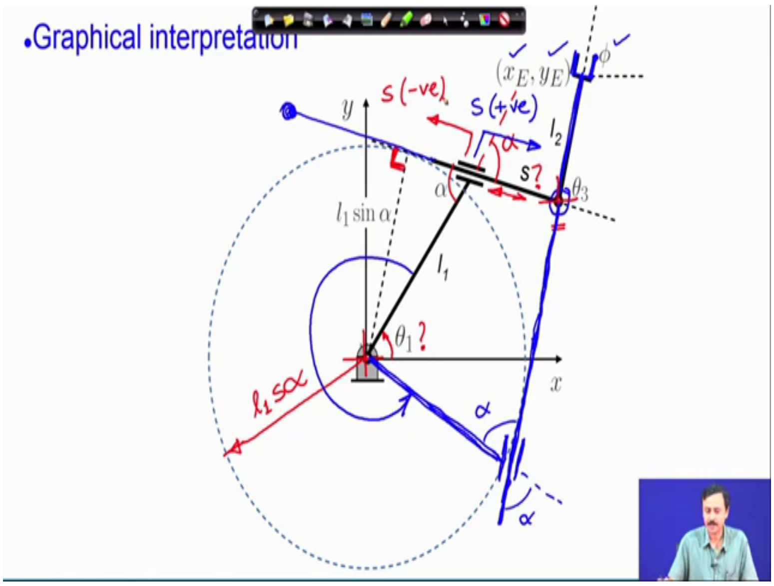

We will understand this for an arbitrary angle alpha as I have shown here. Now what is specified to us is xE yE and phi these are specified. So, once these three things are specified, this length gets fixed, this link gets fixed. So, which means this kinematic pair is now fixed, this is the ground joint, ground hinge. So, that is definitely fix. So, these two are fixed. Now what remains to be found, is this angle theta 1 and just throw of the prismatic actuator which is S.

Now, because this angle has to be maintained this angle was alpha, and so therefore, this angle is alpha also alpha. Now this kinematic pair, from here, because this is fixed, from here I drop tangents to this circle whose radius is l 1 sin of alpha. Why do I drop tangents, because I have to maintain this angle alpha, and this angle is 90 degree, since this is the radial line. This link can slide in this direction of the prismatic pair, which makes an angle alpha.

So, therefore, to a circle this sliding link is tangent. So, this is one configuration, the black configuration. I can have another configuration, another tangent which I will show here, I can drop another tangent to this circle, this is the other tangent. So, therefore, this will look like this in the other configuration. So, here this angle is alpha.

So, this is the other solution, but this solution is slightly unphysical, because this means that this side has to be extended and a hinge has to come here, only then when you rotate this and bring it to here, then this hinge comes on this side, but that is a little unphysical which means, I mean this revolute pair has to cross this prismatic pair.

Now, this will require very special construction, but nevertheless mathematically this is a solution, and if you can construct appropriately you can achieve this solution as well in the same manipulator. So, that is the negative solution. So, here this is the positive solution. So, S is positive in this direction, S is negative in this direction. So, mathematically you will get a solution with S negative. So, this is the graphical interpretation of the inverse kinematics solution.

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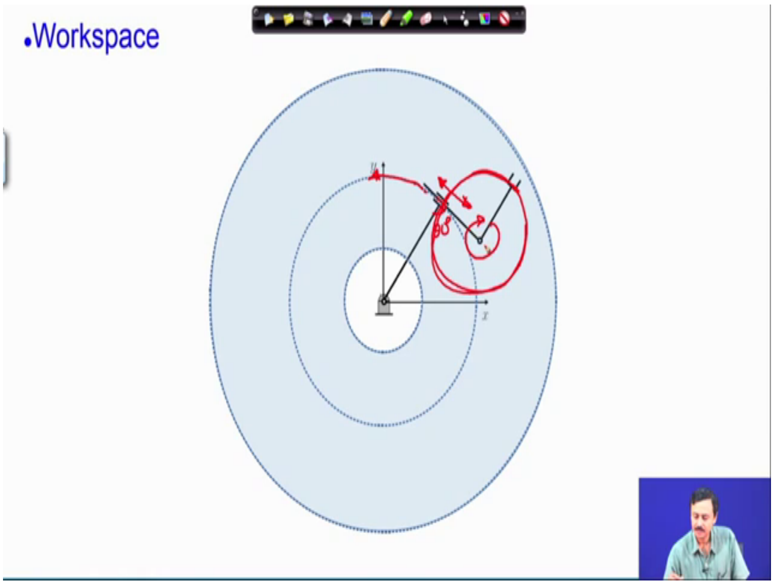

Now, the workspace, here you can see that this prismatic pair will rotate on this circle. So, therefore, this sliding ring sliding element will always be moving in this direction, in this particular case with alpha is 90 degree this will be tangent to the circle. If this angle is 90 degree then this link moves tangent to this circle. Now this revolute pair can rotate; therefore, this describes the circle.

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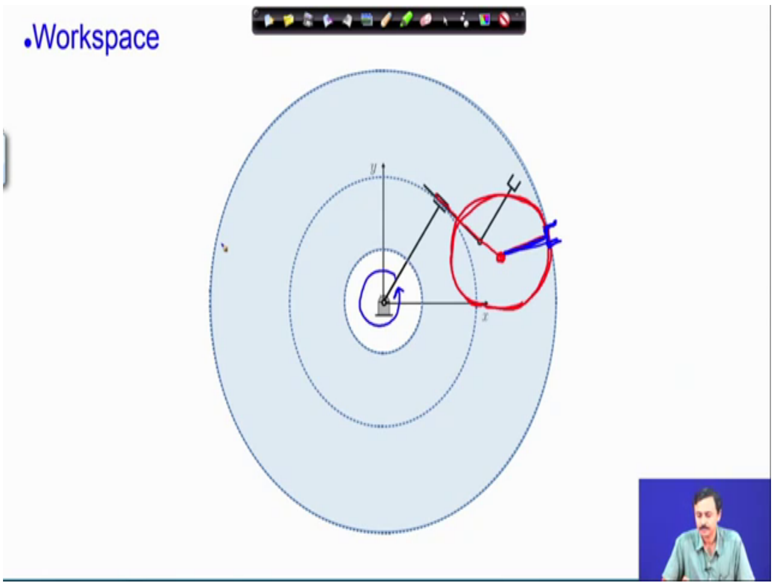

And the farthest reach is when you have, this comes here this revolute pair and you can generate a full circle. So, this is the third link of the manipulator. So, this is the farthest reach, and now if you rotate this angle then you sweep on the outer circle.

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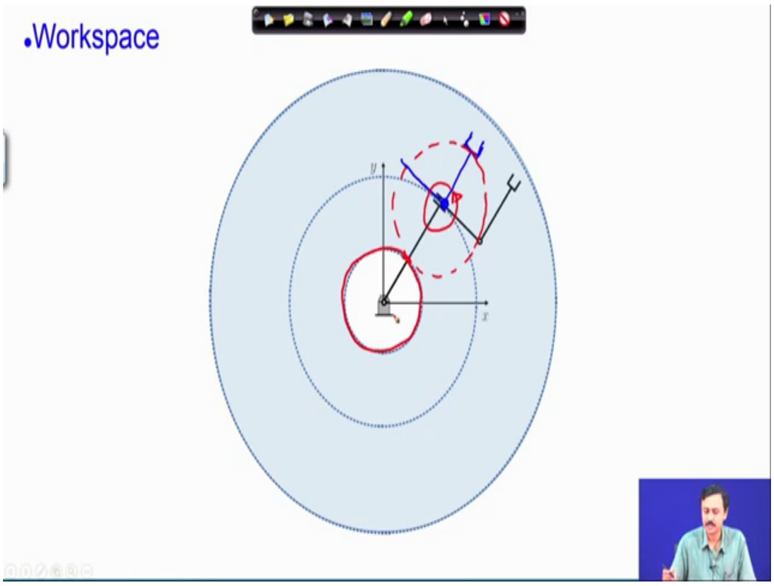

Now, on the inside the closest approach would be when this comes like this, and so the revolute pair, this revolute pair goes here and I described a circle. So, the second revolute pair goes in a circle. So, this is the limit of the approach on the inside. So, this is the limiting circle, inside this circle the end effector cannot reach, for the given, for the shown length specifications, for the shown lengths it cannot enter this region, but if you can have l 2 long enough you can reach inside this as well. There can be restrictions on this workspace if there are joint angle limits of course. So, you can correspondingly find the workspace of this manipulator.

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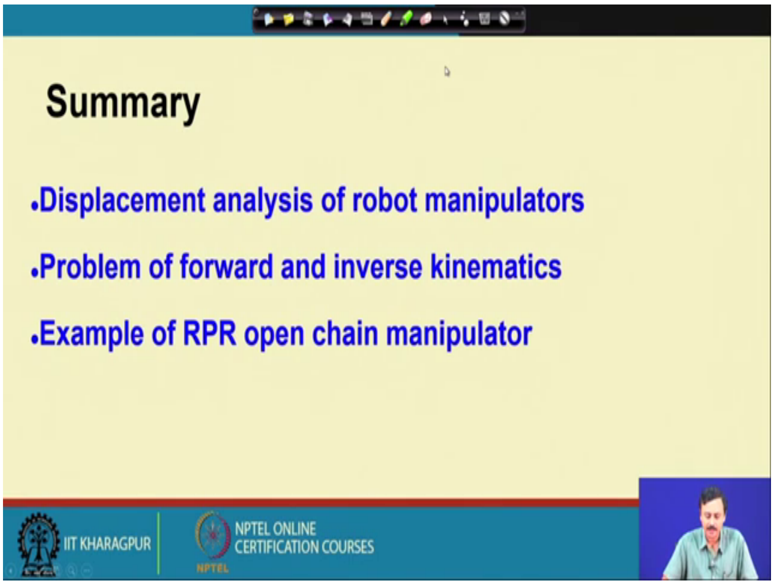

So, here I have the summary for you. We have looked at the displacement analysis or the forward and inverse kinematics problem of the RPR kinematic chain. We have looked at the solutions; we have looked at the geometric interpretations of the solutions. And finally, we have discussed the workspace of this manipulator.

So, with that I will complete this lecture.