Mechanism and Robot Kinematics
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Lecture – 16
Displacement Analysis:
Open Chain Robot – III

We will continue our discussions on the displacement analysis of robots. So, we have looked at 2 degree of freedom manipulators. Today we are going to look at a 3 degree of freedom parallel robot manipulator, open chain manipulator. So, this is the overview of today's lecture. So, the displacement analysis problem, forward and inverse kinematics of a 3R opens chain planar manipulator.

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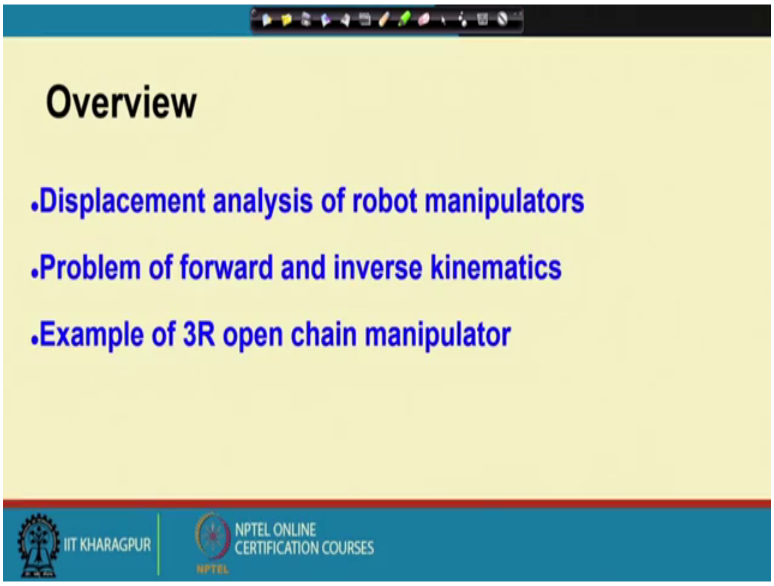

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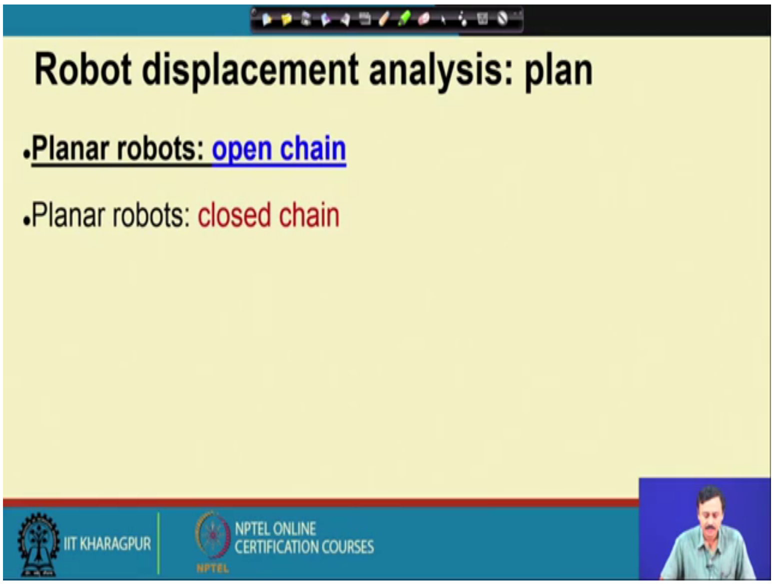

So, as we have discussed our plan, we are discussing planar robots with open chain, subsequently we will discuss planar robots with closed chain, configurations.

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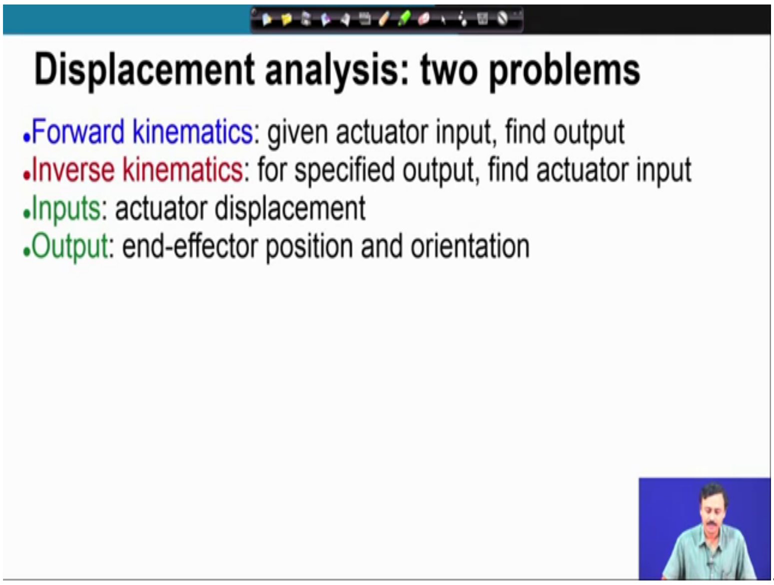

So, we have already defined this forward and inverse kinematics problem. So, there are two problems in displacement analysis. So, given the actuator input to find out the output motion of the end effector, and the inverse kinematics problem is for a specified output of the end effector; that means, specified position and possibly orientation of the end effector, we have to find out the actuator inputs. So, inputs of the actuator displacements and the output is the end effector position and orientation.

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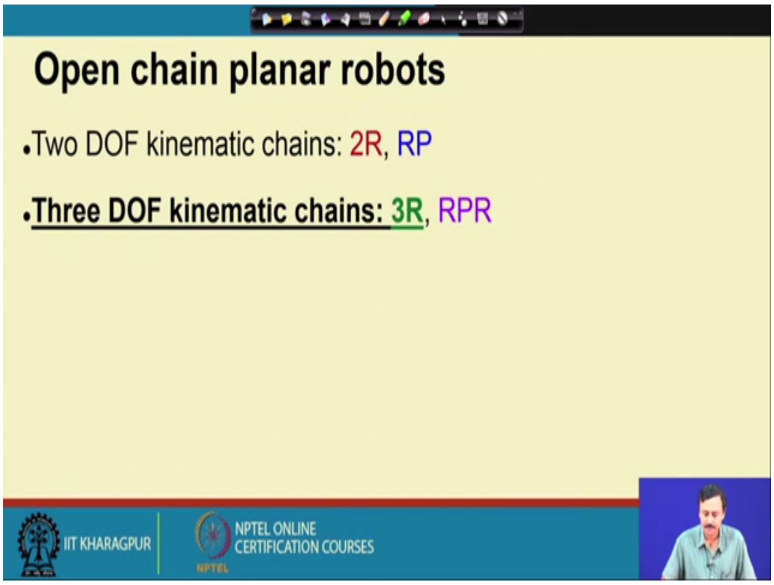

So, now we are going to look at this 3 degree of freedom kinetic chain a 3R planar open chain manipulator. So, this is a schematic of a 3R planar open chain manipulator. And here I have written out for you the forward kinematics problem.

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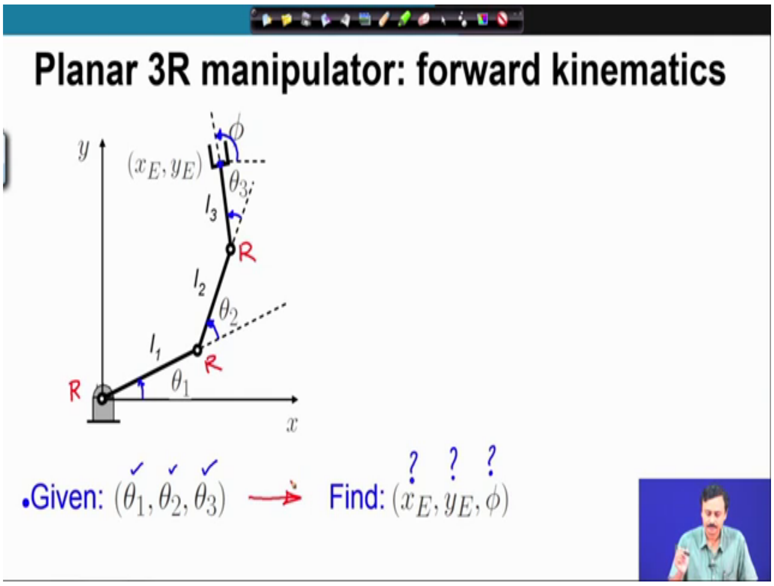

So, given these angles theta 1, theta 2 and theta 3, we have to find out the location of the end effector. So, xE and yE. Since this is a 3 degree of freedom manipulator, so we can have 3 variables at our control. So, this xE and yE and this orientation angle phi. So, given theta 1, theta 2, theta 3, the forward kinematics problem is to find out xE yE plus phi. So, xE yE these are the location, location of coordinates of the end effector and phi is the orientation of the orientation angle of the end effector.

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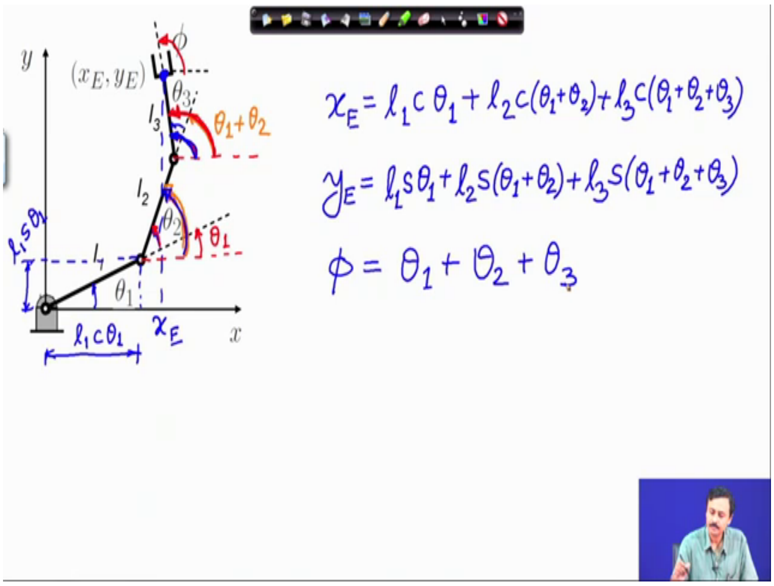

So, here we have three Revolute pairs and we have to now find this relation. So, here is the schematic once again. So, what do you have to do? we need to, first we need to locate this coordinates of the end effector in terms of theta 1, theta 2 and theta 3. So, coordinates of the end effector. So, this is the xE, I can write that this xE is the summation of the projections of the 3 linked link lengths on the x axis that will give me xE. So, therefore, xE, I can write as projections of the link lengths, summation of the projections of the link lengths on the x axis. So, the projection of link length l 1 is l 1 cosine theta 1, it is very easy to see; that is the projection of l 1. So, c theta 1 stands for cosine theta 1. Similarly the projection of l 2 and to add to this.

Now what is the projection? So, if I draw a line parallel to the x axis. So, cosine of this full angle now this full angle is, this is theta 1 remember. So, theta 1 and this is theta 2; so theta 1 plus theta 2. So, l times cosine theta 1 plus theta 2. So, that gives me the projection of link length two, link length l 2 on the x axis; similarly the projection of link l 3 once again.

So, this is line parallel to the x axis; so, projection of l 3 on the x axis will be l 3 cosine of this whole angle. Now this whole angle is composed of the following parts, this angle is same as this angle as you can see. So, this is theta 1 plus theta 2. So, therefore, the whole angle that l 3 makes with the x axis is theta 1 plus theta 2 plus theta 3. So, therefore, I must add this projection as l 3 cosine of theta 1 plus theta 2 plus theta 3. So, that completes the x coordinate of the end effector.

Similarly, I can write the y coordinate as the projection of these link lengths on the y axis. So, to start with this is the projection of, this distance is the projection of l 1 on the y axis and this can be written as l 1 sin theta 1. So, therefore, the first term here is l 1 sin theta 1 plus.

Now the projection of l 2 on the y axis; so is nothing, but l 2 sin of this whole angle, and this whole angle is nothing, but theta 1 plus theta 2. So, l 2 sin of theta 1 plus theta 2. Similarly the projection of l 3 on the y axis is l 3 sin of this whole angle. now that whole angle is theta 1 plus theta 2 plus theta 3 as we have noted. So, this becomes l 3 sin of theta 1 plus theta 2 plus theta 3. So, that completes the yE locational coordinate of the end effector, what remains is the orientation angle phi.

Now, you can very easily see that this, this full angle same as this angle phi. So, therefore, I can write phi as theta 1 plus theta 2 plus theta 3. So, that completes the forward kinematics problem of this 3R planar open chain manipulator. So, I will show this to you formally.

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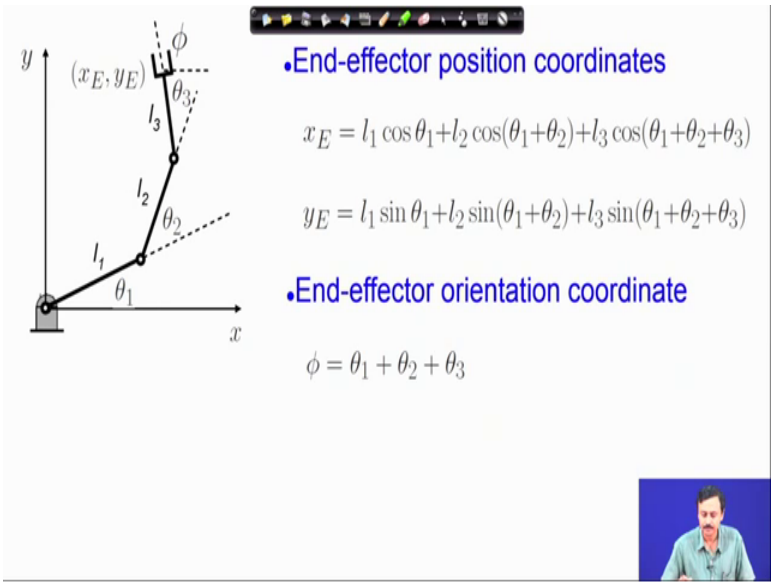

So, this is the end effector positional coordinates xE and yE, and we have the orientation coordinate phi as theta 1 plus theta 2 plus theta 3.

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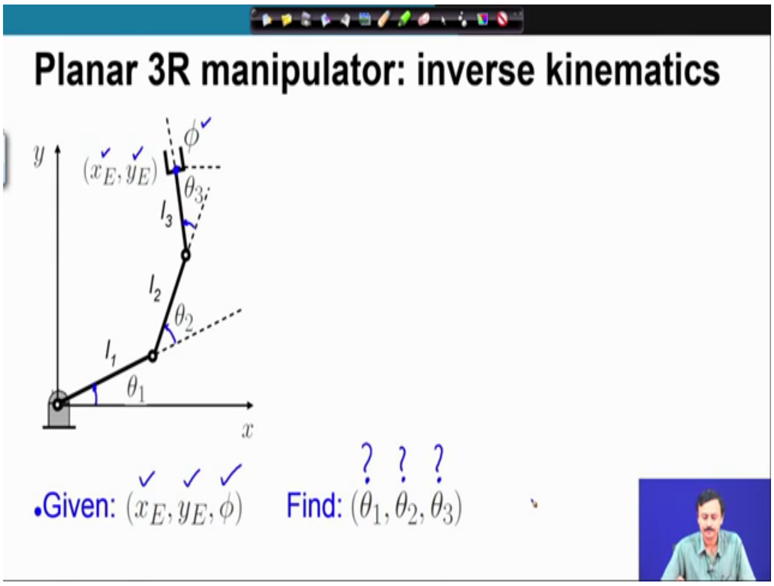

Now, the inverse kinematics problem; once again we have these angles theta 1, theta 2 theta 3, which are now unknown, but what is known, is the location of the end effector; that means, xE and yE and the orientation of the end effector phi. So, these are the three quantities which are specified, and what we have to find out are these joint angles theta 1, theta 2 and theta 3. So, we will start with the forward kinematics expressions.

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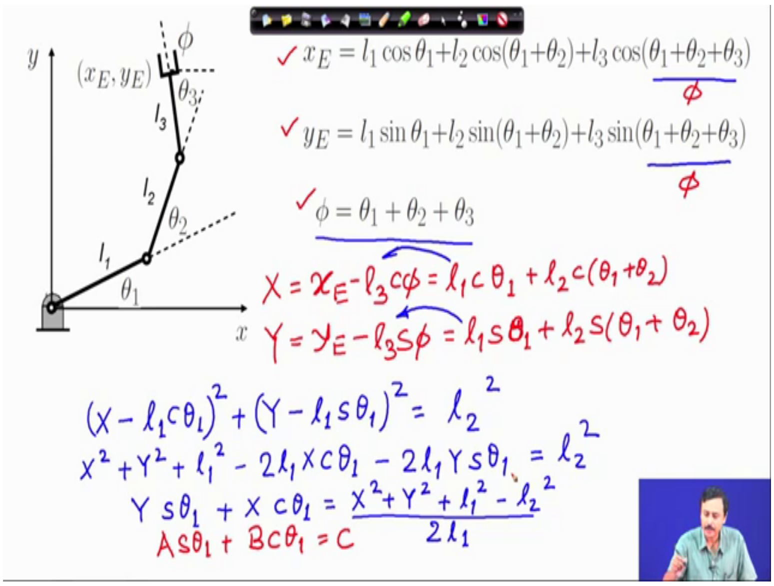

So, the relation between xE yE phi and theta 1, theta 2 theta 3. Then we note that you see this phi, this appears here and here in the locational coordinates of the end effector. So, therefore, I can replace these by phi which is specified. So, remember these quantities are specified. So, phi is known to me. Now once I substitute this theta 1 plus theta 2 plus theta 3 by phi, this term the third term is completely known to me. So, I can bring it to the left hand side and write. So, from the first equation and similarly from the second equation.

Now once I have these two equations, the left hand side remember is completely known to me, I call them X capital X and capital Y. So, capital X and capital Y are completely known, because xE and phi are known to me. So, the problem now boils down to finding out theta 1 and theta 2 given capital X and capital Y. So, the simplest approach would be, to look at this x minus l 1 cosine theta 1 whole square plus y minus l 1 sin theta 1 whole square. So, I have taken this term to the left, this term to the left the two equations squared them up and added them.

So, I get on the right hand side l 2 square. So, when I open this up, I get x square plus y square plus l 1 square minus 2 l 1 x cosine theta 1 minus 2 l 1 y sin theta 1 is equal to l 2 square. So, I can now manipulate this and rewrite in this form by sin theta 1 plus x cos theta 1 is equal to x square plus y square plus l 1 squared minus l 2 square divided by 2 l 1. So, I have re expressed this equation in this form. So, this form again is familiar A sin theta 1 plus B cosine theta 1 equal to C. So, you can find the correspondence A is y, B is x and C is this, an expression on the right hand side.

So, let us look at these steps formally. So, I had the scab definitions of capital X and capital Y.

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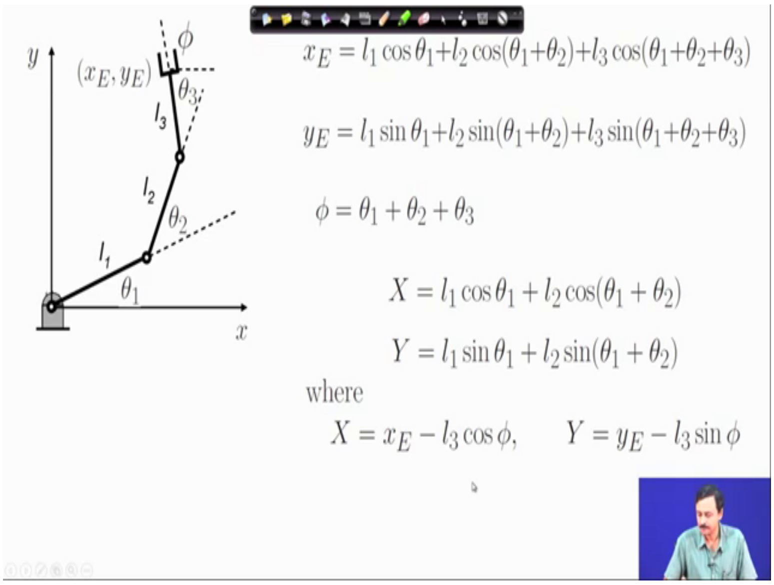

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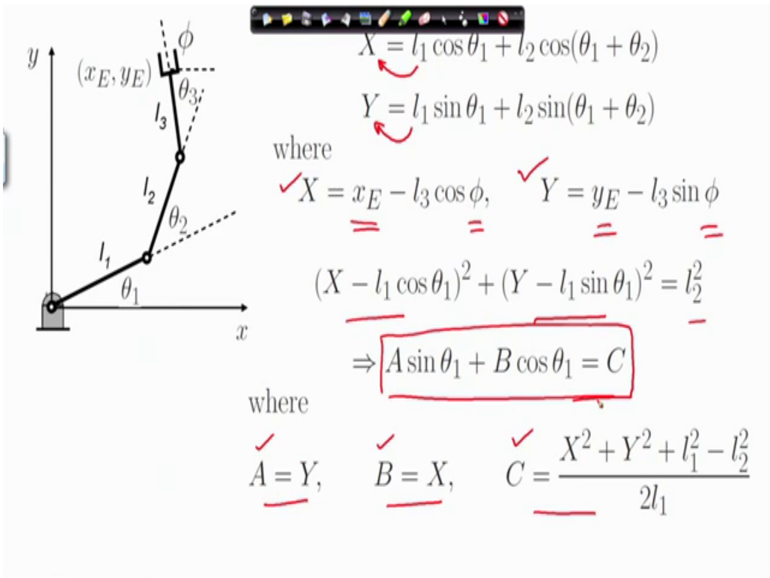

Now these capital X capital Y in terms of theta 1 and theta 2. So, I took these terms on the left hand side squared them added and express this, which can be reframed in this form A sin theta 1 plus B cosine theta 1 is equal to C, where this A B and C we have seen how to derive. So, we know A B and C, because this X and Y are known to us; xE phi y e phi. So, these are all known to us; so capital X capital Y unknown. Therefore, A and B unknown. So, we are left with the only unknown theta 1 in this equation which we can solve.

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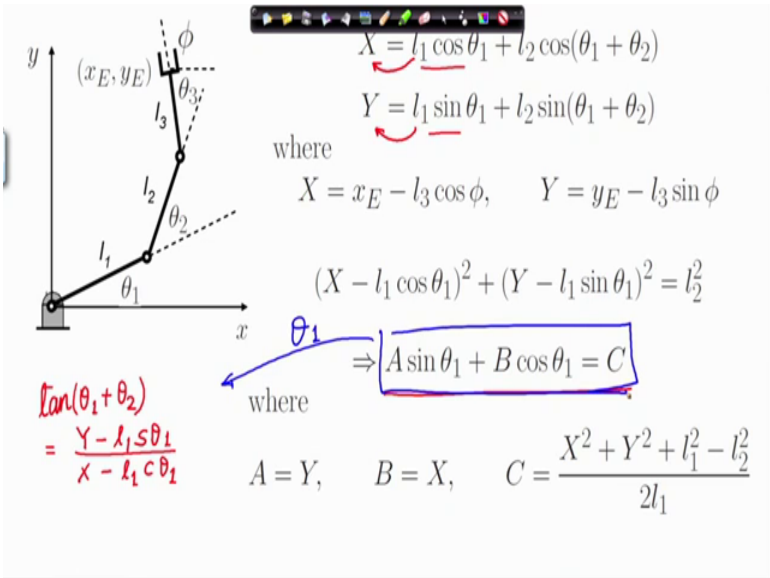

So, that will give us the solution of theta 1. What about theta 2? So, theta 2, you can see that if I take these terms to the left hand side, and take the ratio of the second equation with the first, then I have tangent of theta 1 plus theta 2 is equal to y minus l 1 sin theta 1 by x minus l 1 cosine theta 1.

So, I have taken these terms to the left hand side and taken the ratio the second equation with the first and I have this. Now from here I will solve first solve for theta 1, then substitute thisns theta 1, solution of theta 1 into this and solve for theta 2. So, we are going to look at this step by step. So, first we are going to look at this equation. So, solution of A sin theta 1 plus B cosine theta 1 equal to C.

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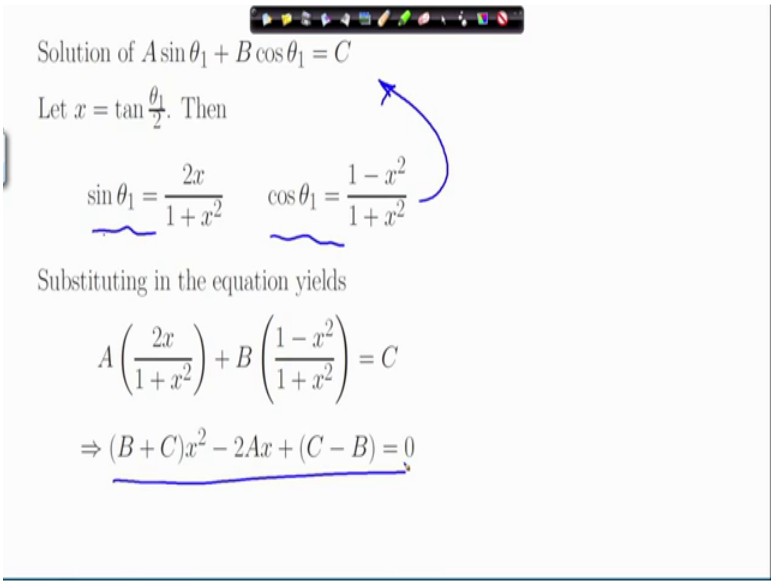

Once again we make this definition, so that we express sin theta 1 and cosine theta 1 in this form in terms of x. These expressions when substituted into the master equation results in this quadratic equation in x, which we know how to solve.

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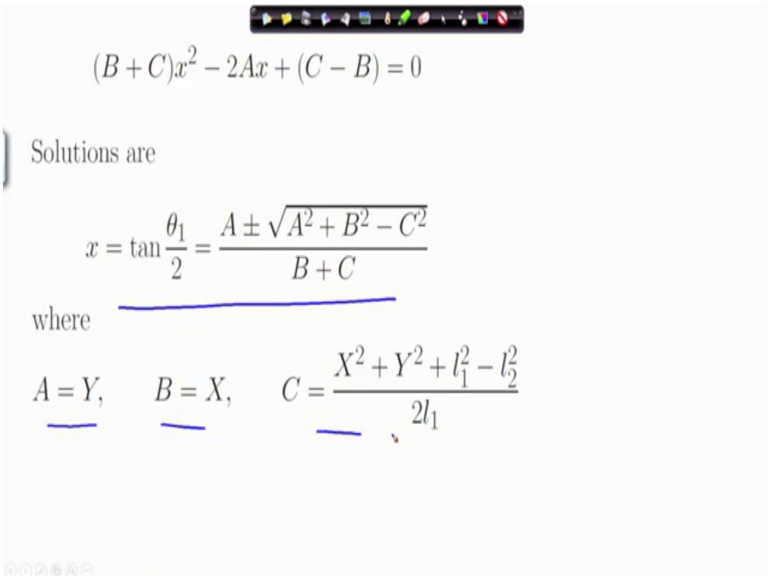

So, the solutions are given by this expression and x is tan theta 1 by 2. So, I know how to find out theta 1. Here A B and C are completely known. So, we will know x and hence tan theta 1, and finally, we have the solution of theta 1.

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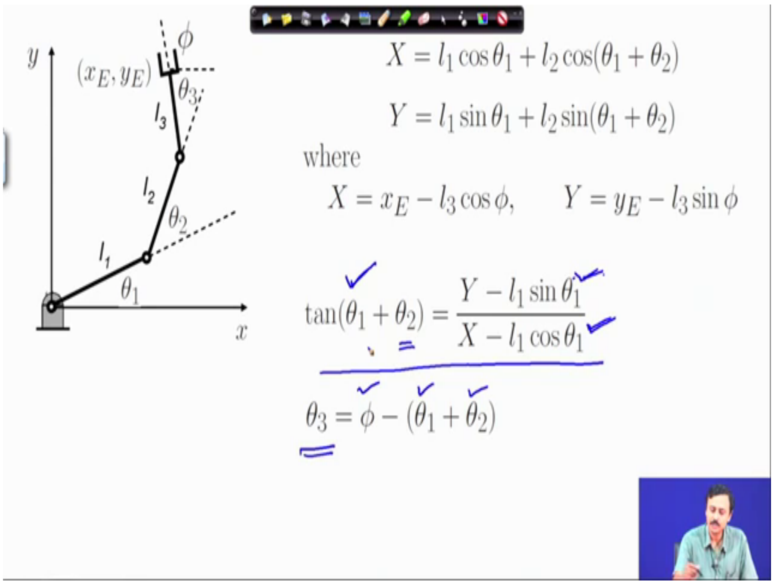

Once we have the solution of theta 1 as we have discussed we form this expression. So, theta 1 is completely solved there are two solutions. So, therefore, I know theta 1. So, there are three places we need to substitute, and take the tangent inverse and get theta 2. Once I solve for theta 2, so I have theta 1 and theta 2 then I can solve for theta 3, because phi is known. So, that will complete the solution. and once again when you take tangent inverse here or in the previous step, we must use the atan 2 function.

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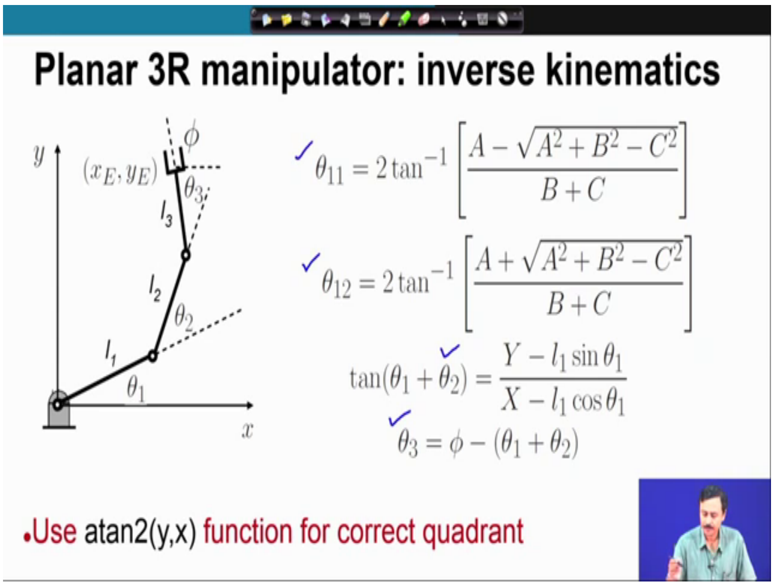

So, here is the complete solution in one go. So, we have these two solutions of theta 1, correspondingly we will find the solution of theta 2 from here and finally, theta 3. So, once again we use this atan two function, when we solve these things on the computer.

So, let us now understand the solution graphically.

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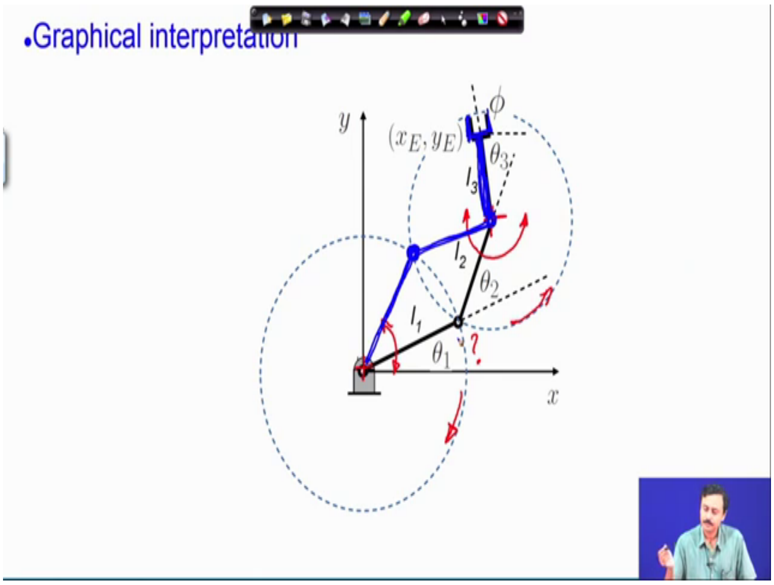

Here remember that xE yE and phi these are specified. Once these three things are specified this link gets completely fixed, this gets completely fixed, because one point is fixed and the angle is fixed. So, this link is fixed completely. Therefore, this point is completely fixed. This revolute joint is completely fixed. What is not known is theta 1 and theta 2.

So, therefore, this is fixed, I can then rotate l 2 the link l 2 about this fixed point, and similarly l 1 can be rotated about this fixed point to find out where this hinge lies. So, this is the circle on which this hinge on l 2 moves, and this is this circle, this is the circle on which the hinge on link one moves. Therefore, the intersection points are the possible solutions. So, one solution is given in this black configuration.

Let me draw the other configuration, other possible solutions in blue, so this. So, here is the revolute pair the hinge, and in the blue configuration also this link remains fixed, this is fixed. So, one is the black configuration the other is the blue configuration. These are the two solutions of the inverse kinematics problem.

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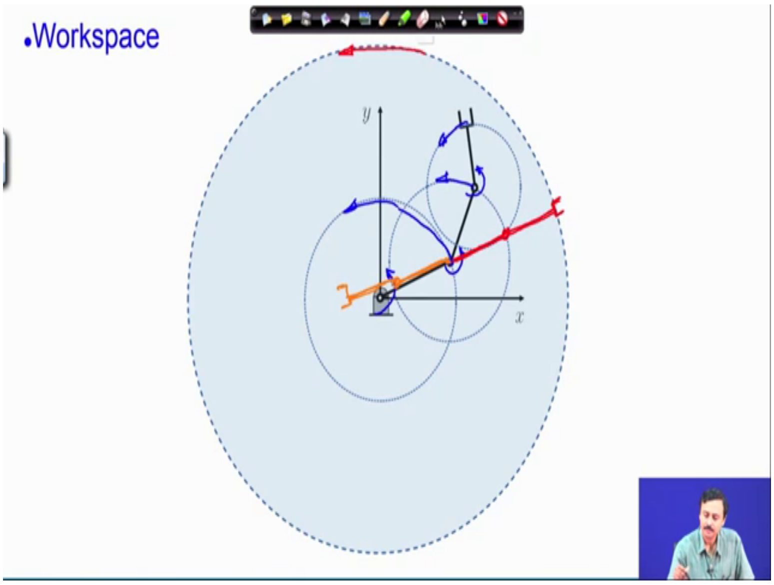

Next, we look at the workspace. If all the revolute pairs can rotate completely then we find that this revolute pair can go on this circle, this revolute pair can move on this circle and the end effector can rotate. So, these are the three Revolute pairs. So, the farthest reach is when, the manipulator is completely extended like this; that is the farthest reach. And by rotating this completely I can generate the boundary of the reach of the manipulator. On the inward side this can be folded, I can reach any point like this, I can reach points like this.

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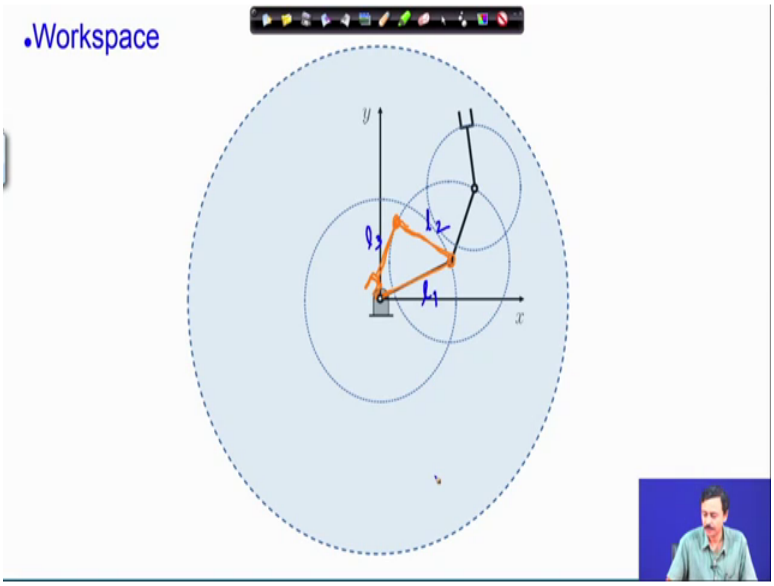

So, I can cover the complete space within the outer circle. So, I can reach any point in this region; so, for a 3R manipulator all points can be reached. So, that of course, will depend on these link lengths. So, these link lengths l 1, l 2, l 3, the way I have drawn you can reach, but if l 2 and l 3 are short, then there might be some regions that are left out. So, that is the workspace for the 3R manipulator. There is another restriction that is on the joint angles, if they have limits then also the workspace gets restricted or reduced.

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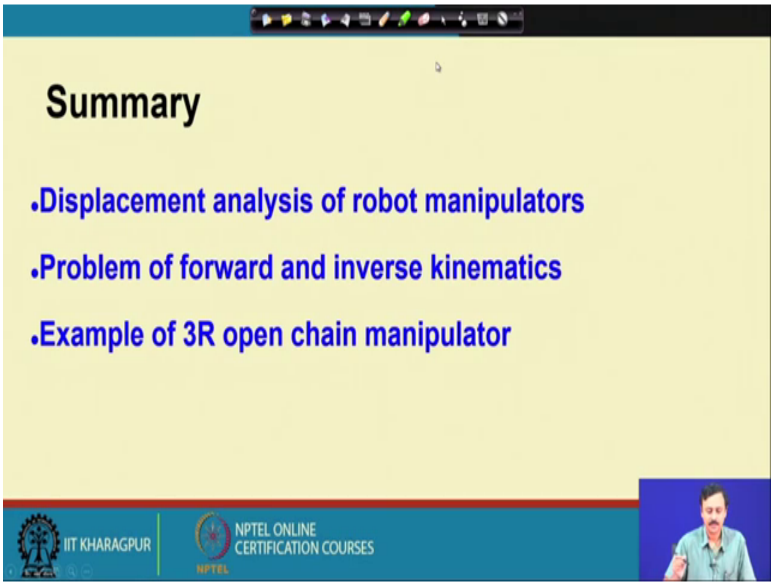

So, here is a summary. So, we have looked at the 3R open chain manipulator, looked at its forward and inverse kinematics, and we have discussed the geometric interpretation of the solutions of the inverse kinematics problem.

So, with that I will close this lecture.