Mechanism and Robot Kinematics

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Lecture – 10
Displacement Analysis: Constrained Mechanism – I

So, when we use a mechanism there is an input or a set of inputs which are actuated and an output. Now, we need to relate these inputs to the output. There are various requirements for example, if I ask, if I need this output what should be my inputs this is one kind of relation I see; or if I specify the inputs what should be the or what will be the output. So, in today's lecture we are going to look at these issues.

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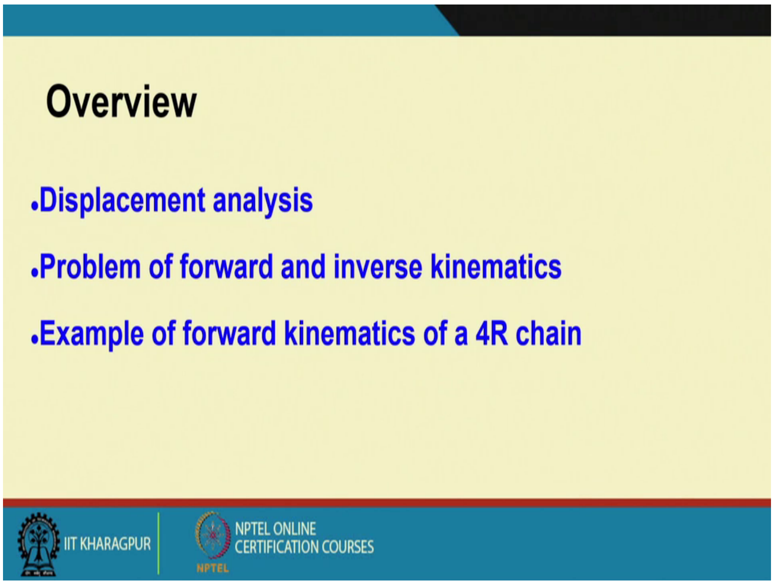

This problem is known as the displacement analysis problem. We are going to define and look at problem of forward and inverse kinematics. So, displacement analysis has these 2 problems forward kinematics and inverse kinematics and we are going to restrict ourselves today to this 4R chains forward kinematics of a 4R chain.

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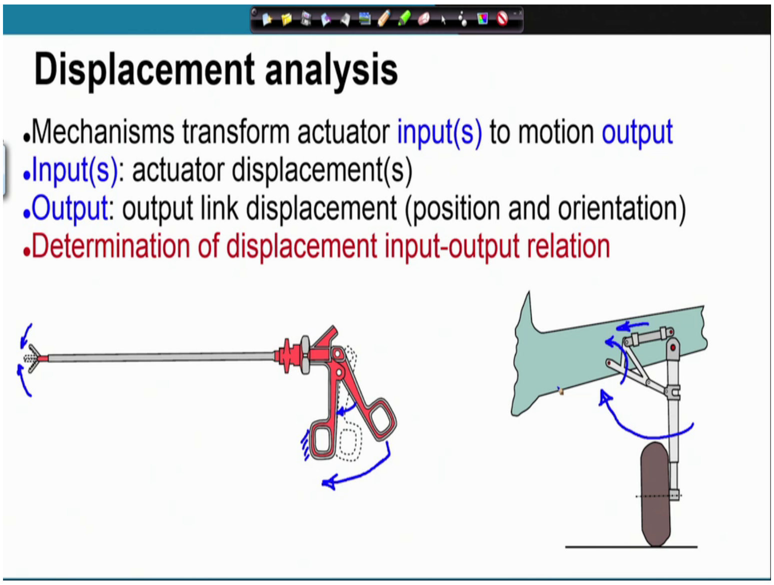

So, displacement analysis as I just now mentioned, that we like to know the input output relation we know that a mechanism transforms the actuator inputs to certain motion outputs there can be multiple inputs, but usually there is one output. So, this input output relations is what is the subject matter of displacement analysis.

So, inputs are the actuator displacements for example, in case of a rotary motor what is the angle by which the motor rotates. In case of a hydraulic actuator it will be what will be the throw or extension of the hydraulic actuator. So, these are the inputs. The output would be in terms of the link displacement. So, output link displacement this could be an angle, it could be position and orientation like in a robot, combination therefore, of linear and angular displacements. So, these are the outputs or output.

Determination of this displacement input output relation is what we are going to study under displacement analysis. So, let us look at this example. So, here I have an example of scissor that is used or a clamp maybe it is used for laparoscopic surgery so we have the surgeons finger operating, this handle, this handle can be considered to be fixed. So, when I change this angle when the input angle input link moves by this much angle from this configuration to this dashed configuration how much is the output motion of the clamps. This is one kind. Here I have shown you the landing gear mechanism of an aircraft, if I expand this actuator then as I have shown you this rotates this link rotates the turnery and that folds the wheel or the wheel ink. So, the question is how much should this actuator expand. So, that the wheel gets completely folded.

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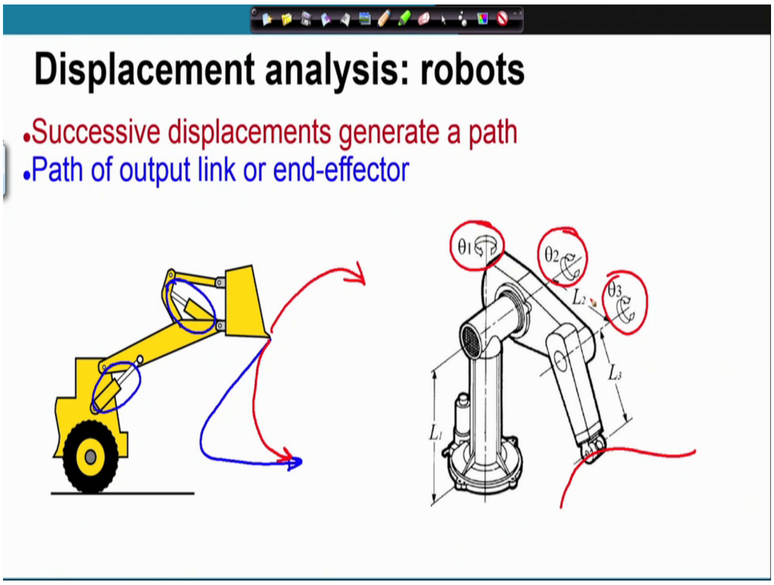

Now, in case of robots remember robots are mechanisms with degrees of freedom 2 or more. Now, as soon as you have 2 or more degrees of freedom you have that many actuators, and you can control the displacement of the output of the robot. So, for example, you can control the motion in x direction and the motion in y direction let us say. So, 2 degrees of freedom, I can control these two.

Now, successive because these two are independently controllable, I can if I want to go from this point to this point, I can choose to go like this or I can choose to go like this or I can choose to go in this manner, so successive displacements of the output generates a path. Now, in case of robots therefore, this displacement analysis is about finding the actuator inputs so that it can generate a certain path.

So, the path is of the output link or the end effector. Let us look at this example of an excavator we know that this has got 2 degrees of freedom we can ask the question, if I want to take this been in this path how should I actuate how should I actuate how should I actuate the hydraulic actuator. So, that the bin travels in this path.

So, the displacement analysis in the case of robots it is a little more complicated because it affords this freedom I can choose my path I can choose a different path or when I am dumping I might choose another path. So, the question is what should be the input actuation, same case for this robot puma robot. If I want to have this end-effector move in a certain path how I should move the angles of these motors so that the end-effector moves along this path.

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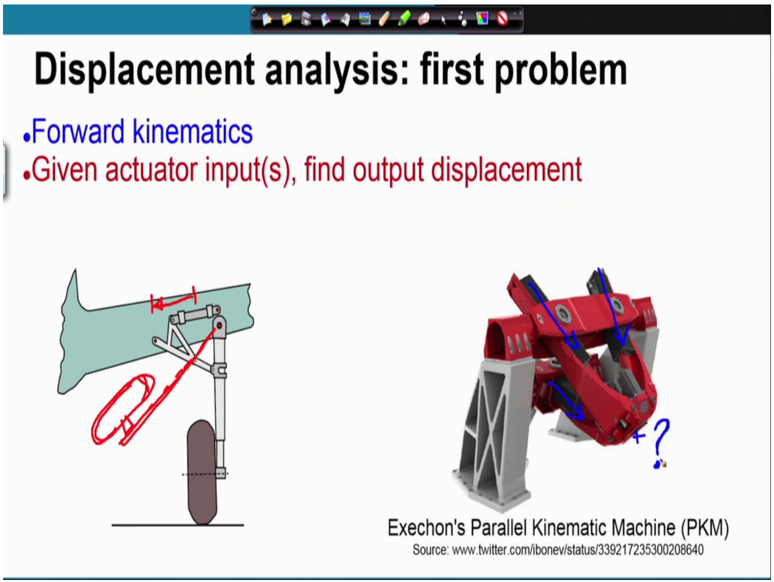

Now, there are two problems in displacement analysis as I was mentioning. The first problem is given the input; that means, given the actuator motion it could be angle, it could be throw off the hydraulic actuator, given this input what will be the output. So, for example, if I say that this angle should be say 0 degree and this should be 90 degree where is my hand this is called the forward kinematics problem or the direct kinematics problem.

So, let us see this. So, forward kinematics problem, given the actuator inputs find the output displacement. So, where is the output? As I mentioned this like before if I give this displacement where is this link, if I give displacement of this much where is this wheel link this is the forward kinematics problem.

Similarly, in this parallel kinematic machine if I specify these are the actuators if I specify this actuator through this actuator through and there is another actuator at the bottom if I specify these 3 actuators where is the end point the, end-effector point. So, that is the forward kinematics problem.

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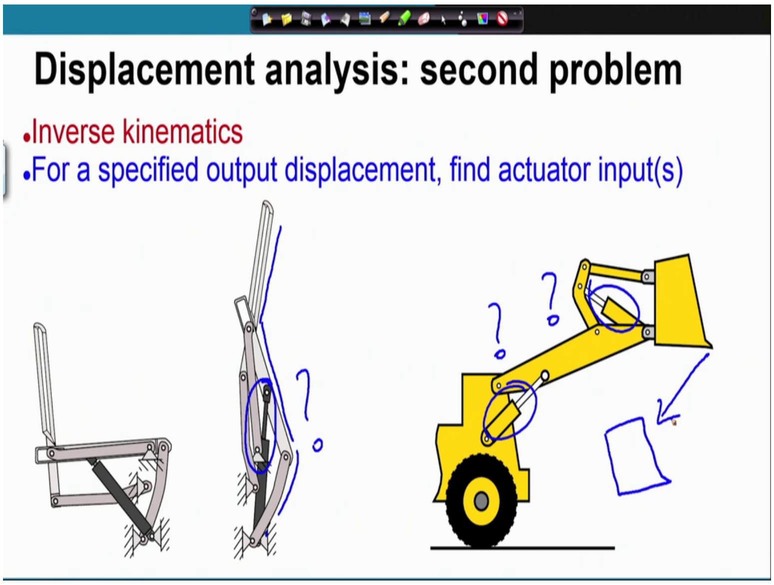

The second problem is that of the inverse problem, so inverse kinematics problem. For a specified output displacement find the actuator inputs this is very relevant, if I want to position for example, this chair to the standing position I am specifying this position what should be the throw of this actuator that is the inverse kinematics problem. Or if I want to specify the location of the bin at a certain position and orientation maybe a certain position let us say what should be the actuator inputs. So, this is the inverse kinematics problem.

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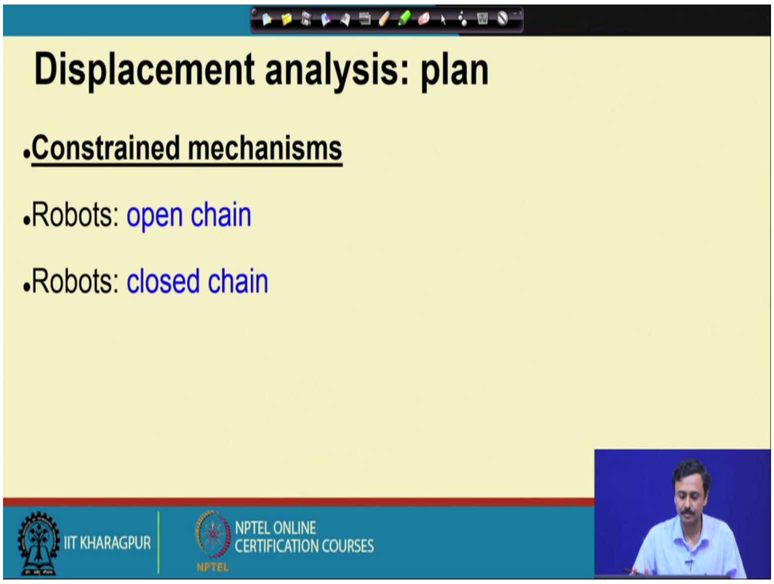

Now, let us plan our study of displacement analysis. So, as I have planned, we will first discuss constraint mechanisms you know that constraint mechanisms are mechanisms with 1 degree of freedom. So, we are going to study displacement analysis of constraint mechanisms we will start with that subsequently will go to robots. So, we will look at open chain robots and closed chain robots.

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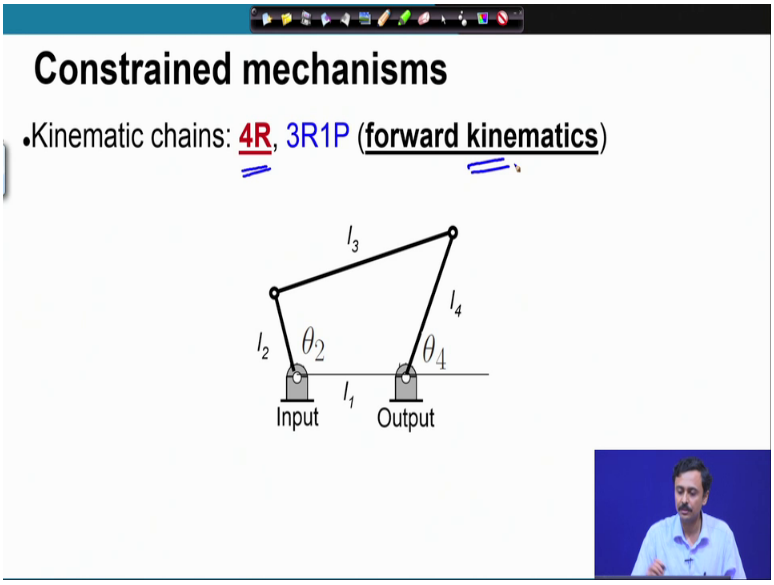

So, today we will start with the analysis displacement analysis the forward kinematics analysis of a 4R chain. So, we are going to start with constraint mechanisms and under constraint mechanisms we are going to discuss 2 kinematic chains, a 4R and a 3R1P in this lecture we are going to discuss 4R chain the forward kinematics of it. So, here I have shown a 4R chain. So, what is the problem let us see.

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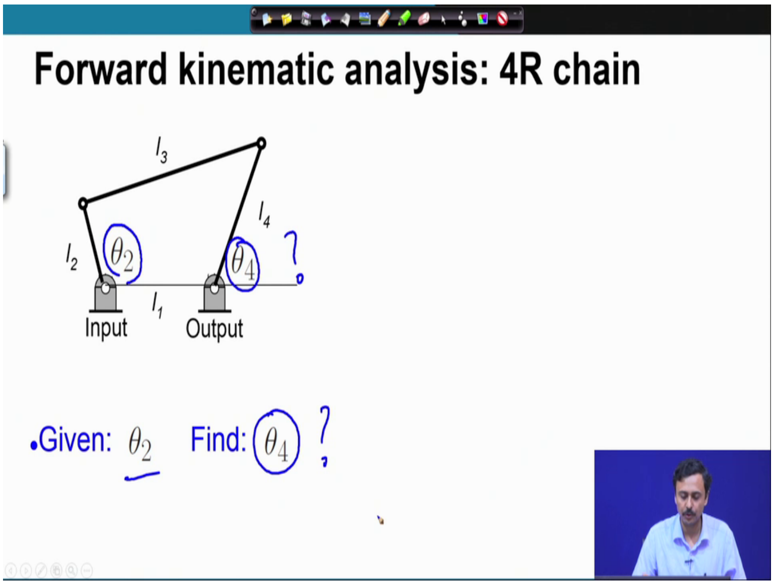

So, the forward kinematic problem kinematics problem for a 4R chain is given theta 2 which is the input which I call the input. So, given this find theta 4 the output, this is the forward kinematics problem for the 4R chain. So, this is what we are going to study.

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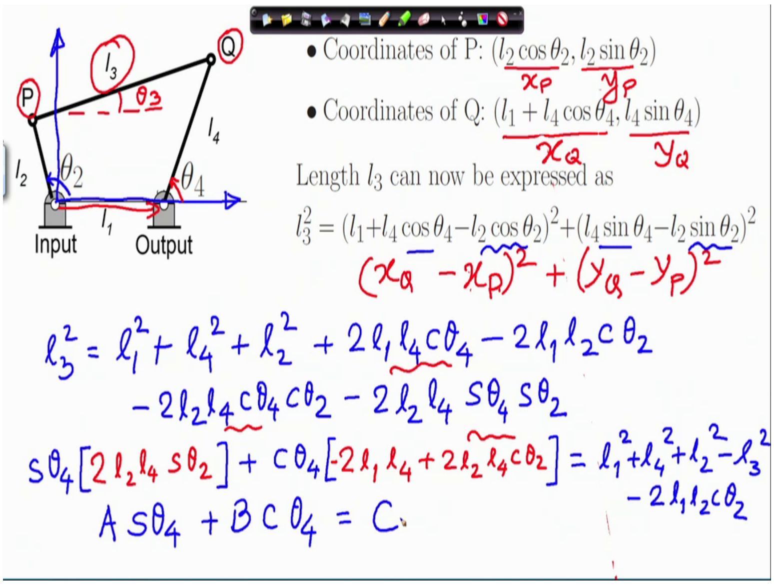

Let me draw the coordinate system that I choose. So, theta 2 is as a sphere I think theta 2 is this angle, theta 4 is this angle and we can also have theta 3 as a byproduct of a calculation. But we are primarily interested in finding a relation between theta 2 and theta 4; that means, given theta 2 what is theta 4.

So, here I have marked 2 points P and Q these are the two hinges. So, coordinates of point P in this coordinate system is l 2 cos theta 2 and l 2 sine theta 2. So, this is the x coordinate and this is the y coordinate of point P. Similarly I can write coordinates of point Q.

So, in order to write that I must add this l 1 along x, so l 1 plus l 4 cos theta 4 which is, so l 1 plus l 4 cos theta 4 is the x coordinate of point Q and l 4 sine theta 4 you can see from the figure is the y coordinate of point Q. So, therefore, this link length l 3, link length l 3 can be expressed as l 3 square is equals to as you can see here x Q minus y Q whole square x cube sorry this is x Q minus x P whole square plus y Q minus y P whole square and that is l 3 square, that I have written out.

Now, if you open this up if you open this up then what do you have. So, l 3 square is equals to l 1 square plus, now l 4 square cos square theta 4 and from here I will have l 4 square sine square theta 4, so it is l 4 square. Similarly l 2 square cos square theta 2 l 2 square sine square theta 2 will come in addition there therefore, this will become l 2 square. Now, the cross terms 2 l 1 l 4 cos theta 4 which I will write as C theta 4 minus 2 l 1, l 2 cos theta 2, C theta 2 and minus 2 l 2 l 4 cos theta 4 cos theta 2 and one cross term from the second square, that will be minus 2 l 2 l 4 sine theta 4 S theta 4 is sine theta 4 and S theta 2 is sine theta 2. So, this is what you can get.

Now, I can rearrange, I can rearrange it like this sine theta 4 times. So, let me see what are the sine theta 4 terms. So, here I have the sine theta 4 terms, this becomes 2 l 2 l 4 sine theta 2. Then I will look at the cos theta 4 terms, so I have cos theta 4 this is one term, this is one term, that is it. So, I can write, this comes with, this term I have taken to the left hand side so that this I have written positive.

So, I must take all terms to the left hand side - cos theta 4 2 l 1, this will come with a negative sign to l 1 l 4 and plus 2 l 2 l 4 cos theta 2 and that is equal to l 1 square plus l 4 square plus l 2 square minus l 3 square, I am bringing l 3 square to the right hand side minus 2 l 1 l 2 cos theta 2. So, this is what I have. Now if I do a little bit of simplification then I can write it as A sine theta 4 plus B cos theta 4 is equal to C where ABC are functions of theta 2 as you can see.

So, let me formally show you this, step.

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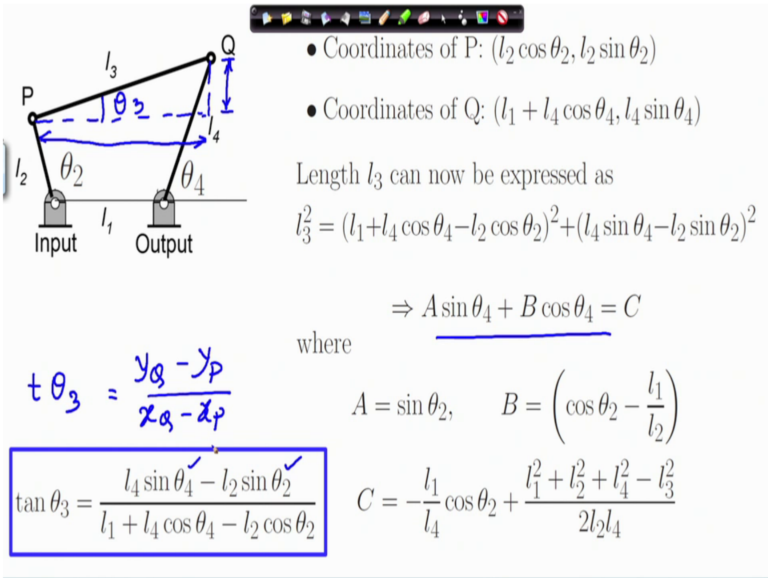

So, A sine theta 4 plus B cos theta 4 equal to C I have done some simplifications. So, A is sine theta 2 and B has these expressions and C has this expression. So, what is our goal now? Goal is to solve this. Why? Because I am given theta 2 if I am given theta 2 I know a I know B I know C they involve theta 2 I know them. What I have to find? I have to find out theta 4. So, I have to essentially solve this equation. So, I have to solve this equation to solve theta 4.

Now, we are going to do it in a manner which can be computerized. So, in a computer as you know that these trigonometric functions will have two solutions, double solutions. So, we are going to revise an approach which will take care of this thing and we are going to get both the solutions so how to do that. By the way you can relate theta 3 as tangent theta 3. So, you can see this, this is theta 3. So, tangent of theta 3 is this distance divided by this distance and this distance is y Q minus, y Q minus y P by x cube minus x P that is tangent theta 3. So, t theta 3 stands for tan theta 3 and you can just substitute these expressions and find that this is what it is.

So, we can once I know theta 4, once I have solved for theta 4 because theta 2 is already known once I have solved for theta 4 from here I can substitute here and get theta 3. So, that is our plan. So, right now this is our plan solving this equation in a manner which can be computerized we want to get all the possible solutions.

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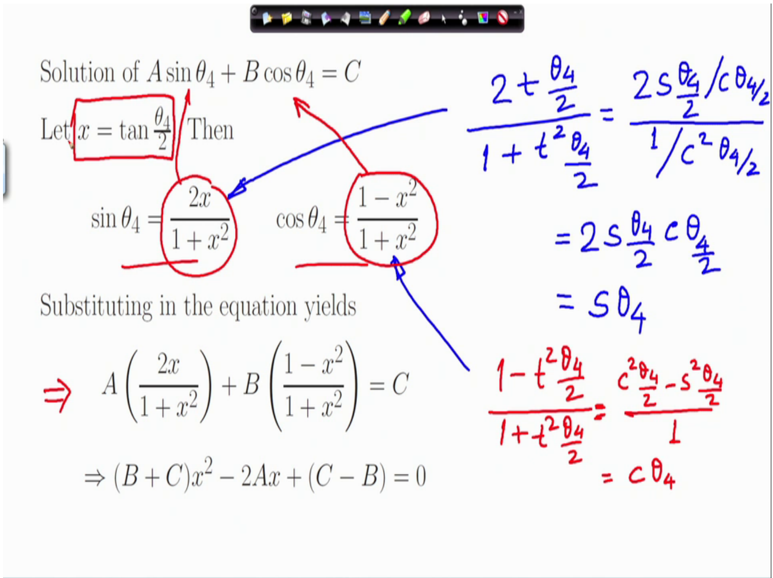

So, solution of A sine theta 4 plus B cos theta 4 equal to C, here we make a substitution x is equal to tan theta 4 by 2, now how does that help. Now, tan theta 4 by 2 can be used to write sine theta 4 by 2 like this which is a standard because 2 times tangent of theta 4 by 2 by 1 plus tangent square theta 4 by 2 that is this expression on the right hand side is equal to.

Now, if I write tan tangent theta 4 by 2 as sine theta 4 by 2 and cosine theta 4 by divided by cosine theta 4 by 2 then you can very easily see that, this is sine theta 4 by 2 divided by cosine theta 4 by 2 divided by now here it becomes sine square theta 4 by 2 plus cos square theta 4 by 2 which is 1, divided by cos square theta 4 by 2. And that turns out to be 2 sine theta 4 by 2 and this goes to the numerator and cancels off with 1 cos theta 4 by 2. So, this only leaves 1 cos theta 4 by 2. So, 2 sine theta 4 by 2 into cos theta 4 by 2 is sine theta 4.

In a similar manner this cos theta 4 by 2, so if I look at the right hand side then 1 minus tan squared theta 4 by 2 by 1 plus tan squared theta 4 by 2. Now, again I write tan theta 4 by 2 as sine theta 4 by 2 divided by cos theta 4 by 2. So, in the numerator I have cos square theta 4 by 2, minus sine square theta 4 by 2 divided by cos square theta 4 by 2 and on the denominator I will also get 1 by cos square theta 4 by 2, so this becomes 1, now cos square theta 4 by 2 minus sine square theta 4 by 2 is cos theta 4. So, we have looked at these two expressions.

Now, if I substitute this in case of sine theta 4 by 2 this expression and in case of cosine theta 4 by 2 this expression then I get this equation which I can simplify this is very easy step I leave it to you to show that you get this quadratic equation in x. Remember x is tangent theta 4 by 2 tan theta 4 by 2 is x. Now you know that the quadratic, I mean quadratic equation has this, these are the roots of a quadratic equation there are two roots of the quadratic equation which can express explicitly.

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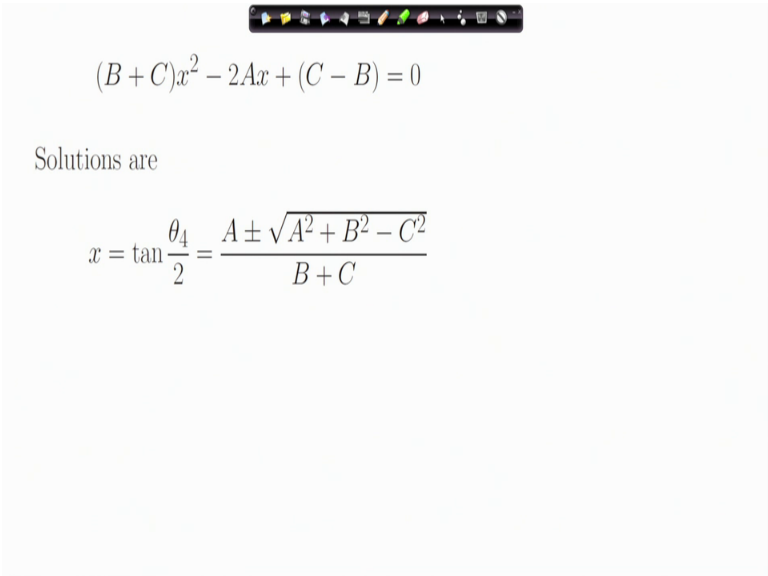

So, this is what we are going to look at next. So, this was the quadratic equation that we obtain the solutions which is essentially tan t theta 4 by 2, so x is tan theta 4 by 2. So, you know this solution of quadratic equation roots of a quadratic equation, so you have it in this form. And you also remember that we have these ABC expressed previously. So, these are the definitions of ABC in terms of theta 2 and the other link lengths. So, I can find out x and hence I can find out tan theta 4 by 2 in terms of theta 2. So, let us do that.

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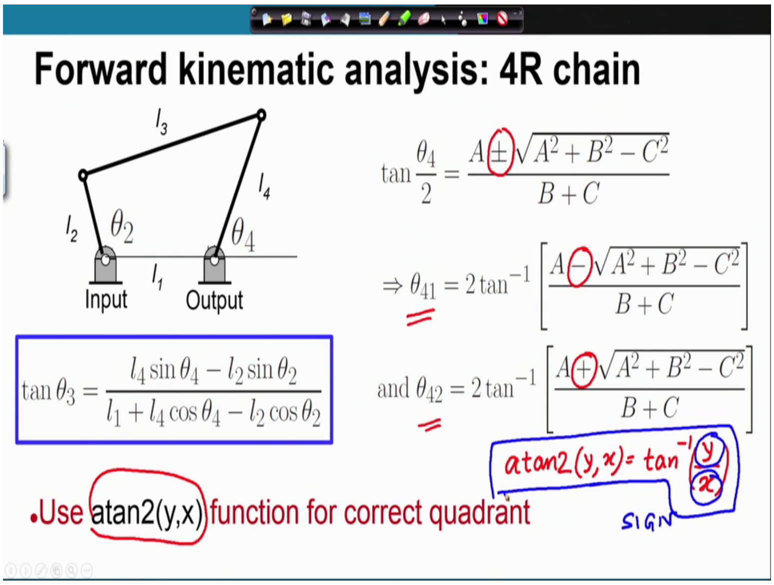

So, tan theta 4 by 2 is this expression in terms of ABC. So, there are these two possible solutions theta 41 and theta 42, theta 41 and theta 42 with here you have this plus minus so I have taken one by one. So, here I have taken the minus and here I have taken the root with the positive sign and I do tan inverse.

Now, this tan inverse is quite powerful. In most softwares in programming languages this tan inverse is given in terms of a function called atan 2. So, once I have theta 4 and theta 41 and theta 42 I can solve for theta 3. So, for these two solutions theta 41 and theta 42 I have 2 solutions of tan theta 3 or theta 3.

Now, this tan inverse is to be or the inversion of tan for inversion of tan to get the correct quadrant of the angle you need to use this function atan 2 this is available in most programming languages and in softwares numerical softwares atan 2 or sometimes also known as arc tan 2. And it gives it has 2 arguments y and x, so it calculates, atan 2 y comma x is actually nothing but tangent inverse, tan inverse y divided by x. What it does extra? It looks at the sign of the numerator and the denominator and there by decides, so sine of numerator and denominator. So, it looks at the sign of the numerator and denominator and thereby decides the quadrant. So, this is very powerful. So, it will tell you the correct quadrant of the angle.

As you know that in the first quadrant everything is positive, in the second quadrant sine is positive which means that the numerator if it is positive and the denominator if it is negative, then it must be in the second quadrant. In the third quadrant both sine and cosine, are negative and therefore, tangent is positive. So, if you have y also negative and x also negative then it will give in the third quadrant.

And if you have in the 4th quadrant cosine is positive and sine is negative. So, if it sees that the denominator is positive while the numerator is negative, it will give the angle in the 4th quadrant. So, this function does all that automatically for you. So, you did not worry about the quadrant. So, you should explore this function atan 2 y comma x.

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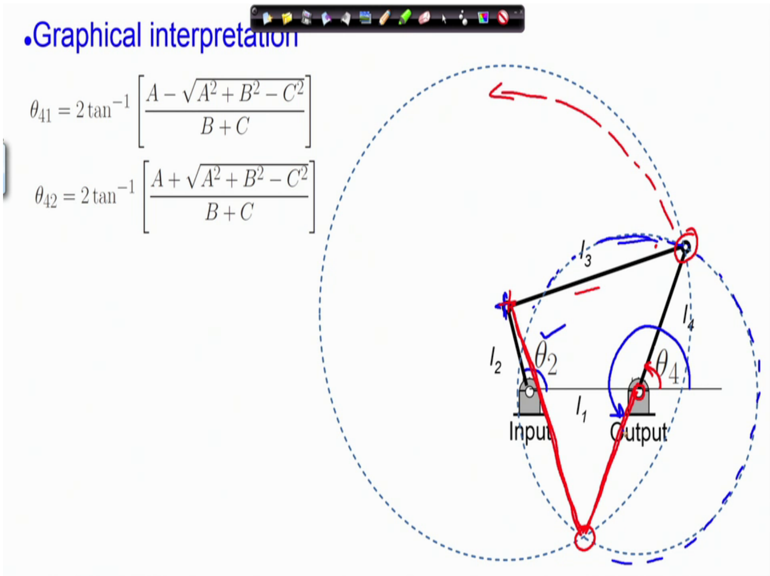

Let us look at the graphical picture, the geometric interpretation of the two solutions. Now, theta 2 is specified, as you know theta 2 is specified we want to find out theta 4 now this hinge therefore, can rotate because theta l 4 is specified l 4 is fixed, so this theta 4 can be any angle such that this hinge lies on this circle. Since this hinge is fixed because theta 2 is fixed, this hinge is fixed; therefore, on the link l 3 can rotate with this as the center on this circle. Now, these two circles have two intersections, this is one intersection this is the other intersection. So, you can assemble the mechanism in this configuration the black one or this red one.

Now, whether these two are distinct assembly more that will be decided by whether this mechanism is a Grashof chain or not, but you can assemble the mechanism in two ways like this. So, these are the two solutions that we are obtaining. So, 4 has two solutions. So, this is one solution and this is the other solution. So, you can see that one is in the first quadrant the other is in the third quadrant. So, this will be done automatically by the atan 2 function. So, this is the graphical picture, this is how we can understand the solution.

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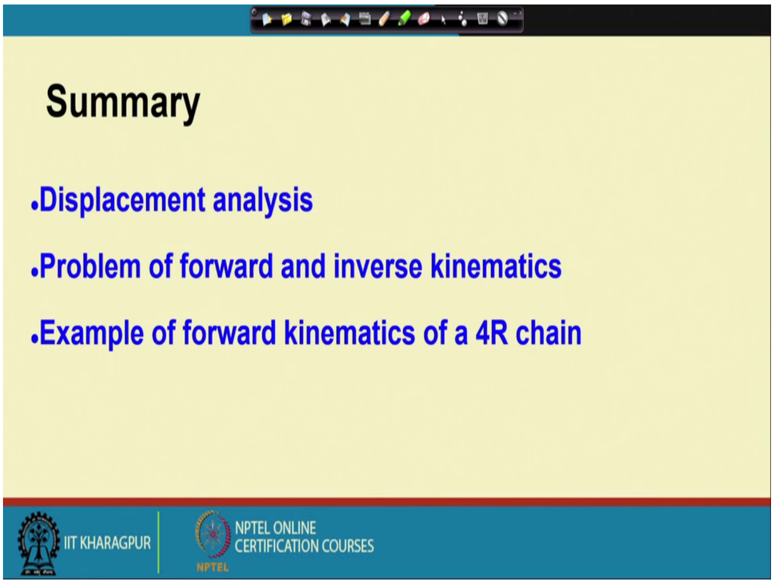

To summarize we have looked at the displacement analysis of mechanisms in general and define the forward and inverse kinematics problems. And here today we have looked at the forward kinematics of a 4R kinematic chain, the constraint kinematic chain. So, with that I will close this lecture.