

**Traditional and Non-Traditional Optimization Tools**  
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**Lecture – 42**  
**Genetic Algorithm as Evolution Tool (Contd.)**

Let us try to solve. So, this numerical example and let us see how to find out the output for a set of inputs, and how to a design and develop with the help of the genetic algorithm.

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**Solution:**

GA-string

10110 01101 11011 1000101010111001

$b_1$   $b_2$   $b_3$  RB

To determine the real value of  $b_1$ ;  
D.V. = 22  
Using linear Mapping rule, we get  
 $b_1 = 3.419355$   
Similarly, we get  $b_2 = 9.193548$ ,  $b_3 = 1.370968$

Mamdani App.

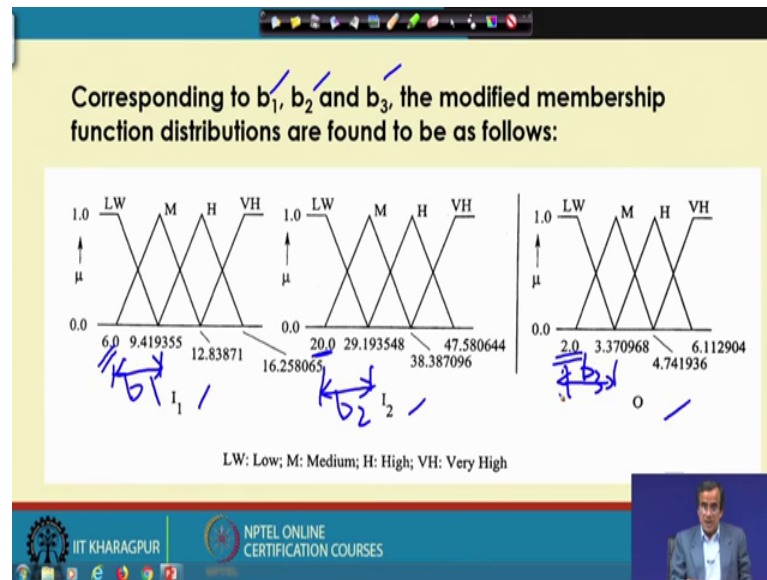
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Now, let me concentrate on the first GA-string which is nothing but this 10110 to represent  $b_1$ , 01101 to represent  $b_2$  11011 to represent  $b_3$  and 16 bits are used to represent the rule base.

So, this particular GA-string carries the information of the whole fuzzy reasoning tool, and we are going to consider the mamdani approach of fuzzy reasoning tool. We are going to consider the mamdani approach. Mamdani approach of fuzzy reasoning tool. Now, to determine the real values for this particular  $b_1$  all of us we know how to find out the decoded value of this particular pipe this bits is nothing but 22. Now, using the linear mapping rule I can find out the real value for this particular the  $b_1$ . Similarly, I can also find out the real value corresponding to this that is this, and the real value corresponding to that is  $b_3$  equals to this.

So, I can find out the real values of  $b_1$   $b_2$   $b_3$ . And now, corresponding to these 16 bits I will have to find out the rule base the modified rule base. Now, let us see how to find out the modified the rule base.

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But, before that corresponding to these modified values or the real values for this particular the  $b_1$   $b_2$  and  $b_3$ , so I can modify the membership function distribution.

Now, to modify the membership function distribution the starting value of  $I_1$  has been kept the constant. Now, this particular the half base width of the triangle, so that is varied and we are getting the modified membership function distribution for  $I_1$ . Similarly, this is nothing but the modified membership function distribution for  $I_2$ , the modified membership function distribution for the output. And we can see that the starting value for each of these variables like your  $I_1$   $I_2$  and  $O$ .

So, those are kept constant and this particular like your the  $b$  value say if this is  $b_1$ . So, this is  $b_2$  and this is your the  $b_3$  and those values are going to control what should be the length of this. So, so this is actually the modified the membership function distribution of the fuzzy reasoning tool.

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Corresponding to the sub-string:  
1000101010111001 the RB is found to be as follows:

		I <sub>2</sub>			
		LW	M	H	VH
I <sub>1</sub>	LW	LW	-X	-X	-X
	M	LW	-	H	
	H	M	-	H	VH
	VH	M	-	-	VH

Handwritten notes: 1000, 16 bits

Now, corresponding to the 16 bits which have already a discussed; now, let me just see that particular rules 1000. So, 1 10001000 similarly, we have got a few other bits also I am not going to discuss. Now, what it do is we have got 16 such bits here. So, we are going to start from here. So, this is the starting I can also start from here. So, let me start from here, now we will start from here then we will move in this particular direction.

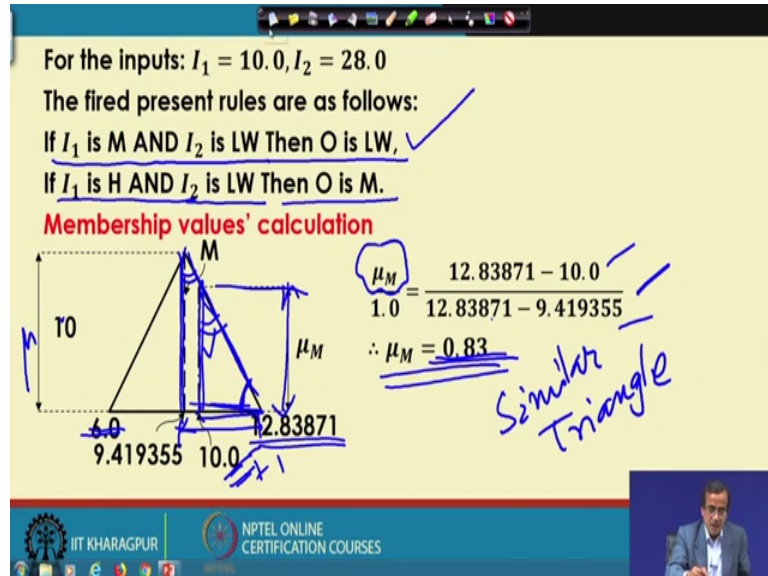
So, here we have got 1; that means, I have got one means this particular output is present; that means, this particular rule is present. Next is 0 0 0 so, this particular rule is absent this is absent this is absent and once it is over, Now, we will concentrate here and we will move in this particular direction. Next, we will concentrate here and try to move in this particular direction and will concentrate here.

So, this is the way actually we can assign this particular number and wherever you find one that particular rule is present. And if it is 0 so, that particular rule is absent. So, corresponding to that 16 bits which are there in first GA-string. So, I can find out that this particular rule is present this is present this is present this is also present, present, present, present, and present. So out of this 16 1 2 3 4 5 6 7 8; so 8 rules are present; that means, out of these 16 bits I have got 8 1s and the remaining 8 0s.

So, this is actually the modified rule base corresponding to the first GA-string the last 16 bits of the first GA-string. So, this is the way actually we can modify actually the database that is the membership function distribution and this particular the rule base.

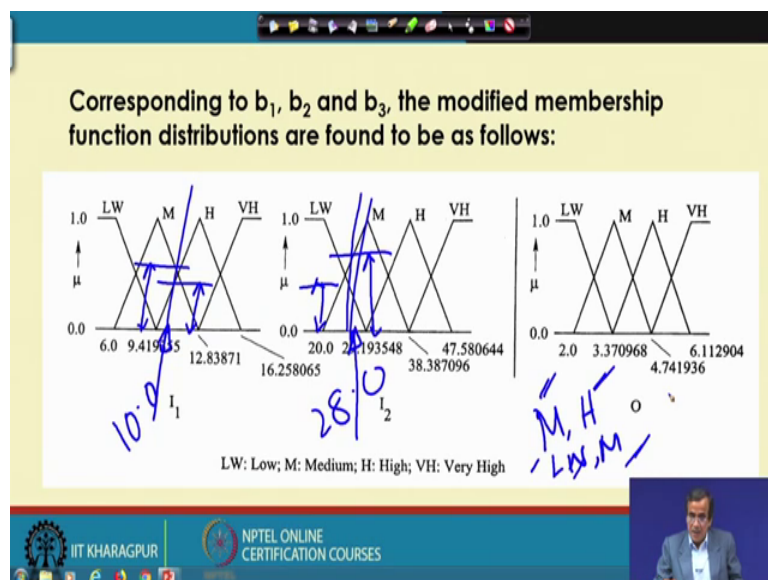
And once we have got this particular the modified database and the rule base now, we are in a position a to find out like what should be the output for a set of input.

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Now, let me consider the inputs are correspondingly the first training scenario I1 is 10 and I2 is 28.0. Now, let us let us try to see like what happens like if I1 is 10 and I2 is 28.

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Now, if I1 is 10; that means, I am here I am more or less I am here. So, I1 is 10; that means, I am here, so this is 10.0 and I2 is 28. So, most probably I am here so, this is 28.0. Now, corresponding to this I1 equals to 10 and here I2 equals to 28.0. So, I2 equals

28.0 so, corresponding to this I1 equals to 10. So, it can be called high with some  $\mu_m$  value, and it can also be called a the medium with some other  $\mu_m$  value.

Similarly, this 28 value of I2 can be called low with some  $\mu_m$  value, and it can also be called medium with some other the  $\mu_m$  value. So, this I1 could be medium and high. So, I1 could be your medium and high and this particular I2. So, it could be like your the low and medium. And considering these 2 and 2 so, I can find out 4 possible combination. The combinations are as follows, like if I1 is medium and I2 is low then output is something. The next is if I1 is M and I2 is M then the output is something, the third one if I1 is high and I2 is low then the output is something. Next is the 4th one if I1 is H and I2 is M then the output is something.

So, there are 4 such possible combinations of the inputs here. So, a maximum of 4 rules will be fired. Now, out of these 4 rules so, all 4 rules may not be present in this particular the GA this particular the rule base; so out of these 4 so, I will have to check how many are present. Now, fortunately or unfortunately out of those 4 only 2 areas lying here. So, 2 ares found to be present here. So, we will have to find out the output of this particular fuzzy reasoning tool using only those 2 rules which are present here, and the remaining 2 rules actually we will have to forget.

Now, So, out of those 4 only these 2 rules are found to be present here, in this rule based the rules are as follows if I1 is medium and I2 is low then the output is low. Next is if I1 is H that is high and I2 is low then the output is your the medium. Now, using these particular rules let us see how to find out the output. Now, let us concentrate on the first rule. So, if I1 is M now what I do is corresponding to this I1 equals to 10. So, we try to find out the membership function distribution whenever we are going to consider it is the medium, so for this medium the starting value of this particular triangle. So, here we have got 6, here it is 12.83871 and the mid value is a 9.419355 and corresponding to I1 equals to I1 equals to 10. So, this is the scenario. So, this will be my  $\mu_m$  value and using the principle of this similar triangle. So, very easily I can find out what should be this particular the  $\mu_m$  value.

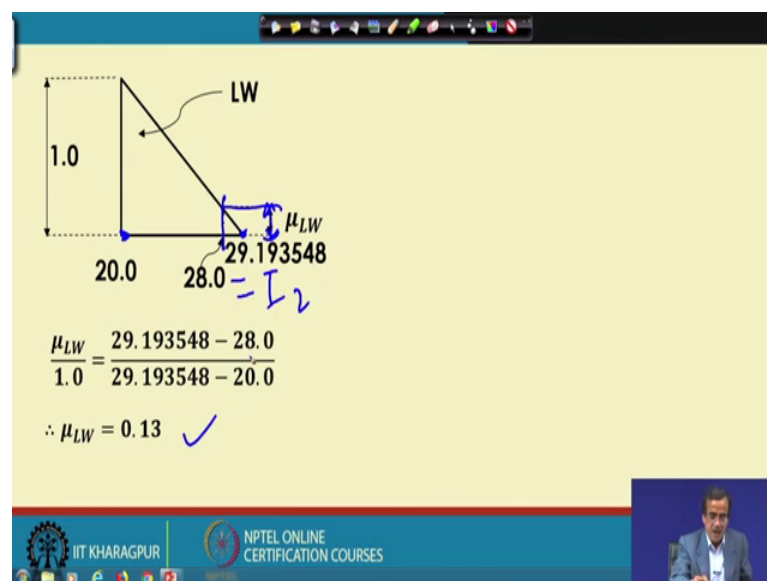
Now, there are several ways to determine out of those the principle of similar triangle will be easier. Like, if I consider so, what is  $\mu_m$ ?  $\mu_m$  is nothing but this and one. So, this is actually this is 1.0 not 10 this is  $\mu_m$  that is 1.0. Now. So,  $\mu_m$  divided by 1.0;

that means, corresponding to this particular triangle. It is opposite side is this and it is opposite side is this the moment we consider this triangle and this particular triangle. So, these 2 triangles are similar. So, these 2 triangles are similar. So,  $\mu_m$  is this divided by 1.0 is nothing but so, this particular distance that is 12.83871 minus 10.0 divided by 12 point divided by this particular distance 12.83871 that is minus 9.419355.

So, I can find out what is this particular the  $\mu_m$ . So, these 2 triangles are similar so, the see the common angle. And this particular angle is equal to this so, I can use the principle of the similar triangle. So, using the principle of the similar triangle actually, I can find out what should be the value for this particular the  $\mu_m$ . And if I calculate  $\mu_m$  so,  $\mu_m$  will become equal to 0.83.

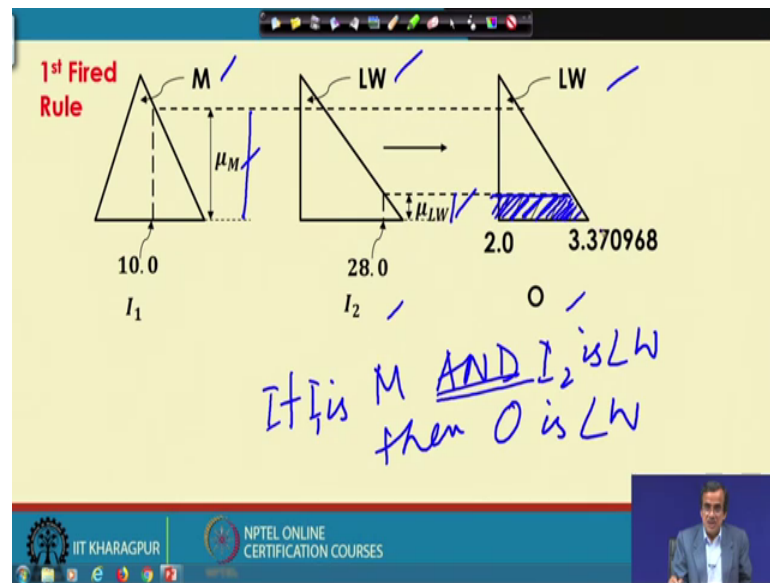
Now, once I have got this particular the  $\mu_m$ . Now, we are in a position to find out corresponding to low that is I2.

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So, what should be the  $\mu_m$  value by following the same principle of similar triangle. So, for this low I2 a I2 is the second input. So, this is the right-angle triangle. So, this is 20.0 this is 20.0 and this is your 29.193548 and 28.0 is the value of I2 the input value for this particular I2 and corresponding to this I2. So, this is nothing but the  $\mu_m$  value and this  $\mu_m$  low using the principle of similar triangle, I can find out 0.13 and once I have got this particular the value. Now, let us try to concentrate on the first fire rule.

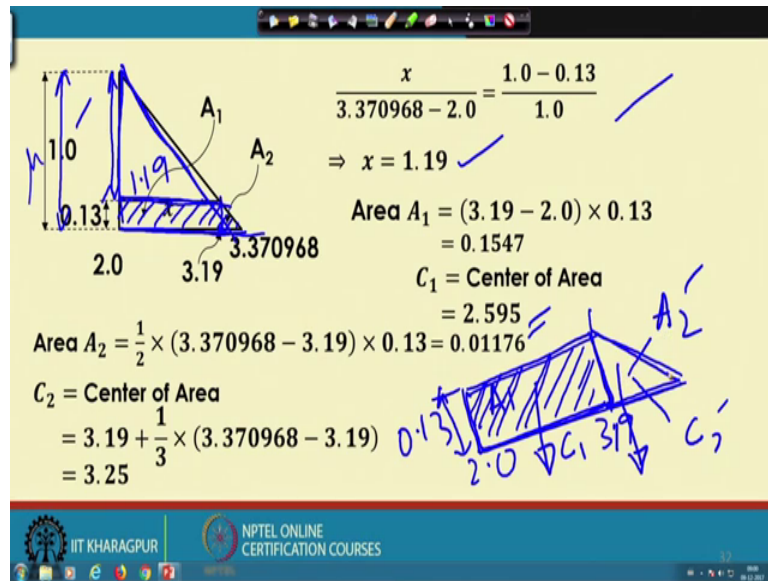
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So, the first fire rule was if  $I_1$  is equal to if  $I_1$  is M and  $I_2$  is low then the output is low. Now, corresponding to these particular  $I_1$  equals to 10, we have already seen that  $\mu_m$  we have already calculated and corresponding to this  $I_2$  we have already calculated  $\mu_{m\ low}$ . Now, if I see this particular rule so, there is a and operator here; that means, if  $I_1$  is if  $I_1$  is medium and your this  $I_2$  is low then, the output O is low so, this is the rule. So, here we have got the AND operator and this is nothing but the mean operator. So, we will have to compare. So, this particular  $\mu_m$  value this particular  $\mu_{m\ low}$  value and we will have to consider the minimum.

Now, if I consider  $\mu_m$  and  $\mu_{m\ low}$ . So, we will be getting. So, this particular shaded area as the output that is called the fuzzified output corresponding to the first fired rule. So, this type of the truncated area will be getting and this particular shaded portion is going to indicate what should be the output of this particular the fired rule. Now, by following the similar procedure So, I can also find out what should be a actually the output of the second fire rule, that I am going to discuss after some time let me once again concentrate on these.

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Now here, so this is nothing but actually the fuzzified output corresponding to the first fired rule. Now, if this is the fuzzified output so, this is not a crisp output. So, directly it cannot be used so, we will have to defuzzify. So, to defuzzify will have to find out the area of this particular shaded portion and the center of area. Now, to find out the area at the center of area like what you do is so, first we try to find out. So, we know that this is the  $\mu_m$  corresponding to that 0.13 and this is 1.0 is the total  $\mu_m$  value. So, this is the  $\mu_m$  and this is 2.0 and 3.370968 and I will have to find out. So, what is this? What is the this particular point to find out actually? What you do is so, we use this particular principal  $x$  divided by so, 3.370968 your minus 2; that means once again I am going to use the principle of the similar triangle.

Now, if you use the principle of similar triangle; that means, one triangle you consider like this and another triangle you consider like this and these 2 triangles are similar and if I use the principle of similar triangle. So,  $x$  is this divided by this particular the base is nothing but your 1.0 minus 0.13; that means, this particular thing divided by 1.0; that means, this particular thing. So, use the using the principle of the similar triangle I can find out the value of  $x$ . So,  $x$  is nothing but 1.19 and once I got this particular  $x$ . Now, very easily I can find out the area and center of area because, this is almost similar to a something like this.



So, what we will have to do is first we will have to find out the area and center of area of this particular portion. So, I can find out the area very easily because, this is 2.0 this is 3.19 and this side is 0.13. So, very easily I can find out the area of this shaded portion, that is your like 3.19 sorry this is 3.19. So, 3.19 minus 2.0 that is this side multiplied by 0.13. So, I will be getting this particular the area of the shaded portion, and the center of the area is nothing but here. So, that is the mid and that is coming as 2.595.

So, I can find out the area that is  $A_1$  and center of area that is  $C_1$ . Similarly, I can find out for this particular area. So, I can also find out what should be the area and center of area. Now, this is nothing but a triangle. So, very easily I can find out the area half base into height. So, I can find out the area and the center of area is nothing but. So, here it is one third 2 third. So, from here this will be one third distance apart. So, I can find out what is  $A_2$  and what is  $C_2$  that is area and center of area.

So, for this small area rectangle I can find out area and center of area, for this triangle I can find out area and center of area. And if I combine now, I can find out what should be the area and center of area.

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The slide displays the following calculations:

$$\text{Total Area } A' = A_1 + A_2 = 0.16646$$

$$\text{Center of combined area } C' = \frac{A_1 C_1 + A_2 C_2}{A_1 + A_2} = \frac{0.1547 \times 2.595 + 0.01176 \times 3.25}{0.16646} = 2.64$$

Handwritten blue annotations on the slide include a diagram of a rectangle with area  $A_1, C_1$  and a triangle with area  $A_2, C_2$ . The combined area is labeled  $A'$  and its center is  $C'$ .

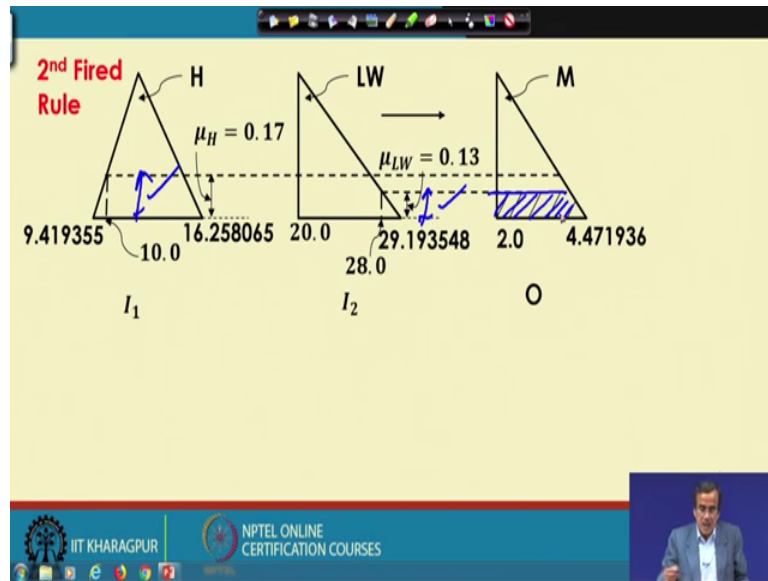
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The total area of this particular portion like if I see so, this particular portion the total area of this the total area and the center of area I can find out. So, total areas this and center of area. So, this was  $A_1, C_1$  and here we have  $A_2, C_2$ . So, using this particular

formula I can find out what should be the center of area considering the whole, and that is nothing but C prime and the total area is denoted by A prime.

So, for the corresponding to the first fire rule; so I can find out the area and center of area.

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Following the same principle for the second fire rule I the second rule is if I1 is high and I2 is low then, the output is medium. So, once again corresponding to 10 you find out the mu m value and that is nothing but. So, mu m value is this mu mch and corresponding to this I2. So, we find out the mu m value you compare this mu m value and these. So, this is the minimum. So, I can find out this is nothing but the area that curve that is nothing but the fuzzifier output corresponding to the second rule.

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$$\text{Total area } A'' = \frac{1}{2} \times (1.9785 + 2.741936) \times 0.13$$

$$= 0.3068$$

Center of the fired area  

$$C'' = 3.370968$$

By using Center of Sums method of defuzzification:

Crisp output  $O = \frac{A' C' + A'' C''}{A' + A''}$

$$= \frac{0.16646 \times 2.64 + 0.3068 \times 3.370968}{0.16646 + 0.3068}$$

$$= 3.1138$$

Now, using the same principle actually what I can do is, I can find out like what should be the area of this shaded portion. So, I can find out the area of this particular shaded portion very easily, and the area is nothing but  $A$  double prime and that is nothing but half then this side plus this side  $\mu$  multiplied by the height. So, this is a trapezium sort of thing. So, I can find out the area as the center of area will be midpoint  $C$  double prime is nothing but these. So, this is the center of area.

Now, I am going to use the center of sums method for defuzzification because, what we need is actually the crisp value of the output and using the center of sums method of defuzzification. So, I can find out the crisp output by combining say both this particular your the shaded portion or the both the  $A$  the area. So, for the first rule I have got one shaded area. Now the shaded area was something like this. So, this was the shaded area corresponding to the first fire rule and corresponding to the second rule. So, the shaded area was something like this and they have got some own unit they have got some own unit. So, these 2 areas will have to superimpose and if you superimpose and if you try to find out the combined  $A$  the crisp output that will be nothing but  $A$  prime  $C$  prime corresponding to this first fire rule then  $A$  double prime  $C$  double prime corresponding to this particular fire rule and if you combine then I will be getting this particular the crisp output, and this crisp output is nothing but the value of the deviation.

So, corresponding to the first input that is say  $I_1$  and the second input that is  $I_2$ . So, this is nothing but the deviation  $O$ . So, I can find out the deviation corresponding to the set of inputs.

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Deviation  $d_1 = |3.5 - 3.1168|$   
 $= 0.3861$   
corresponding to the first training case.  
Pass all the training cases and determine the deviation values as follows:

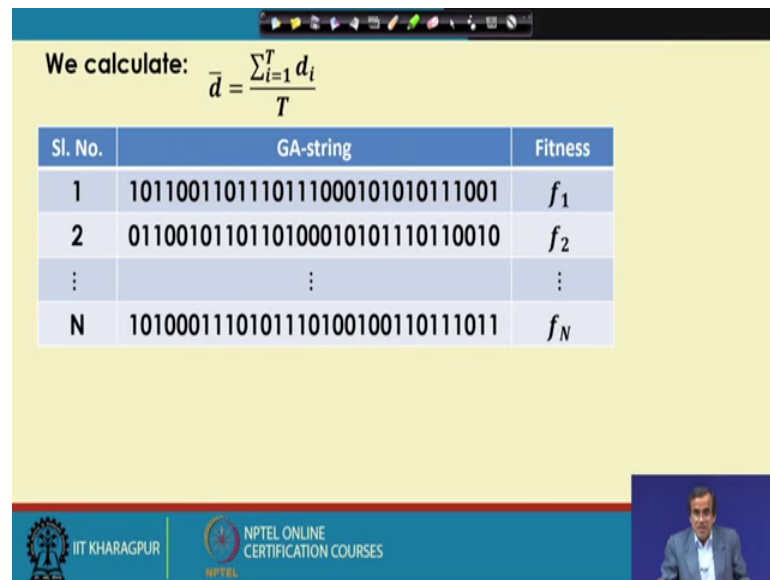
Sl. No.	$I_1$	$I_2$	O	Deviation in prediction
1	10.0	28.0	3.5	$d_1$
2	14.0	15.0	4.0	$d_2$
⋮	⋮	⋮	⋮	⋮
T	17.0	31.0	4.6	$d_T$

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So, I hope it is understandable like how to find out a the crisp output corresponding to the set of input. And once I have got this particular the calculated output the crisp output supposing that this is 3.1168 and the target value is 3.5. So, I can find out the deviation the mod value of that is this, and once I you have got the mod value of the deviation that is  $d_1$  the same thing corresponding to the same GA-string the first GA-string.

So, we just pass the second training scenario that is 14.0 15.0 targeted 4.0 find out the deviation. Similarly, for the  $t$ th training scenario you find out the deviation. And let me repeat once again I am concentrating till now, only on the first training for the first GA-string and corresponding to the first GA-string. So, we have sent all the training scenario one after another and then what we do is we try to find out the average deviation, that is nothing but the sum of all deviation values divided by the number of training scenarios and that is nothing but this.

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We calculate:  $\bar{d} = \frac{\sum_{i=1}^T d_i}{T}$

Sl. No.	GA-string	Fitness
1	1011001101110111000101010111001	$f_1$
2	0110010110110100010101110110010	$f_2$
⋮	⋮	⋮
N	1010001110101110100100110111011	$f_N$

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And once we have got that this particular  $\bar{d}$  prime is the average deviation and that is nothing but the fitness for the first GA-string that is  $f_1$ .

Follow the same principle for the second GA-string once again you pass the whole capital T number of trainings cases one after another. Follow the same principle so, you will be getting the fitness that is  $f_2$ . Similarly, for the last GA-string that is the Nth one I will be getting the fitness; so for the whole population of the GA. So, I have got the fitness information and once you have got the fitness information as usual you go for the reproduction scheme, who are getting the mating pool you go for the crossover operator for exchange of properties go for the mutation that completes one generation of the GA. And this particular process will go on and go on and the GA through a large number of iteration I will try to find out what should be a the optimized fuzzy reasoning tool.

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**Automatic design using the GA**

Sl. No.	GA-string
1	101100110111011100010101011100100011010010101001001011011101011
2	011001011011010001010111011001010000110101011100010101000010101
⋮	⋮
N	101000111010111010010011011101100111010100011100101010001100101

LW, M, H, VA  
 00 01 10 11  
 32 bits

$5+5+5+16+32 = 63$

So, by following this particular principle the GA is actually able to or evolve the optimized fuzzy reasoning tool, which will be able to make the prediction more accurately.

Now, here actually I am just going to tell one problem that to develop this type of fuzzy reasoning tool it might be required that we need some information problem information. But, we may not get the problem information always and that is why actually what we do is, we try to give the whole task to the GA to find out what should be the optimized a knowledge base for this particular the fuzzy reasoning tool. And we do not design this particular the rule base manually and that is called the automatic design of this particular fuzzy reasoning tool using a genetic algorithm. The GA will take the whole responsibility to evolve a suitable fuzzy reasoning tool. So, that it can make the prediction as accurately as possible.

Now, here let us see how to design this particular the GA-string. We have already seen that to represent b1 we use 5 bits. So, let me use 5 bits here to represent b1, the next 5 bits to represent your the b2, the next 5 bits to represent like b3. Next is the next 16 bits like 5 6 7 8 9 10 11 12 13 14 15 16. So, the next 16 bits to represent your the rule base. So, this is the rule base this is b1 this is b2 and this is b3, and what you can do is. So, there are 16 rules and these 1s and 0s are going to represent whether a particular combination is present or not.

Now, if it is present what should be the output that will also be represented with the help of this type of the GA-string. How to do it? Now, there are 16 rules 16 combination of the input parameters for each combination what should be the output? User is not going to design manually and the whole task will be given to the GA, GA will try to find out what should be the rule base. Now, here actually what we do? If we see the output is having 4 linguistic term like low medium high and say very high very high. Now, what to do is to represent these 4-linguistic term we use 2 bits for example, we use a 00 to represent low, 01 to represent medium, 10 to represent high and 11 to represent very high.

So, how many bits we need here; so for 16 rolls 16 multiplied by 2. So, there will be 32 bits here. So, there will be 32 bits just to represent what should be the output of those 16 combination of the input parameters. So, the GA-string will be 5 plus 5 plus 5 plus 16 for the rule base and 32 for your the outputs of these particular rules, and total it is how much 31 and 32. So, this is 63 bits.

So, GA will be 63 bits long. And here, the whole responsibility we are giving it to the GA to design and evolve a sweetable fuzzy reasoning tool for this particular your to solve this particular problem in a very efficient way. But, as I told that this task is more difficult for the GA to evolve both the your the data base as well as the sweetable rule base for the fuzzy reasoning tool.

Now, here this method is very efficient, but it has got one drawback in the sense like there is a possibility that the GA actually can select a few redundant rules, let me take a very simple example.

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To identify the redundant rules:  
Important factor = probability of occurrence  $\times$  worth

Important

No-firing situation

16  $\rightarrow$  8  $\rightarrow$  6

\$ 0/0/0

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Supposing that say I am going to start with the 16 rules now, if I do the GA base training in the first stage there is a possibility that I will be getting say 8 good rules. So, 8 remaining good 8 the remaining rules will be removed. So, out of 16 only 8 will be selected as the good rule.

Now, if I go for the second stage tuning of the fuzzy reasoning tool using the same GA, there is a possibility that out of 8 only 6 will be your the selected as a good rule. So, 2 will be removed. So, if you just go for this type of stage wise GA base tuning there is a possibility that I will be getting a very less number of rules, but those rules may not be sufficient just to a a trigger all the possible combination of this particular the inputs. And there could be a possibility that I am sending one set of input and there may not be any fire rules, that particular situation is known as the no firing situation.

Now, there should not be any no firing situation for this particular the fuzzy reasoning tool. Now if there is no firing situation. So, there will be no output corresponding to that particular set of input which is dangerous, and at the same time if there are some redundant rules. The algorithm will be computationally expensive and unnecessarily will have to store those rules, and if I just go for the hardware implementation. So, each rule is represented with the help of one electronic circuit. So, the more the number of rules the more complex will be that particular the electronic circuit and that is why the redundant rules is also not welcome and we will have to remove that particular the



redundant rule. To remove the redundant rules what we do is? We take the help of one factor that is called the importance factor.

Now, this is actually the importance factor. Now, to calculate the importance factor what we do is we consider the probability of occurrence and worth; that means, during the training. So, each of these rule how many times it has been fired. So, using that particular the information we try to find out the probability of occurrence, and we try to find out the worth of a particular rule.

Now, the probability of occurrence of a rule will lie between 0 to 1, and the worth will also lie between 0 to 1, and if you multiply then the importance factor will also lie between 0 to 1. Now, for each of these particular rules we calculate this importance factor and if this particular importance factor is found to be less than some predefined value; that means, your that particular rule is not a very good rule. And we declare that this is the redundant rule and we we try to we remove that particular rule from this particular the rule base.

So, using this particular principle; so we can optimize both the database and the rule base for the fuzzy reasoning tool using a genetic algorithm, and there is a possibility once tuned once evolved. So, this particular fuzzy reasoning tool is going to a determine the input output relationship of a process very accurately.

Thank you.