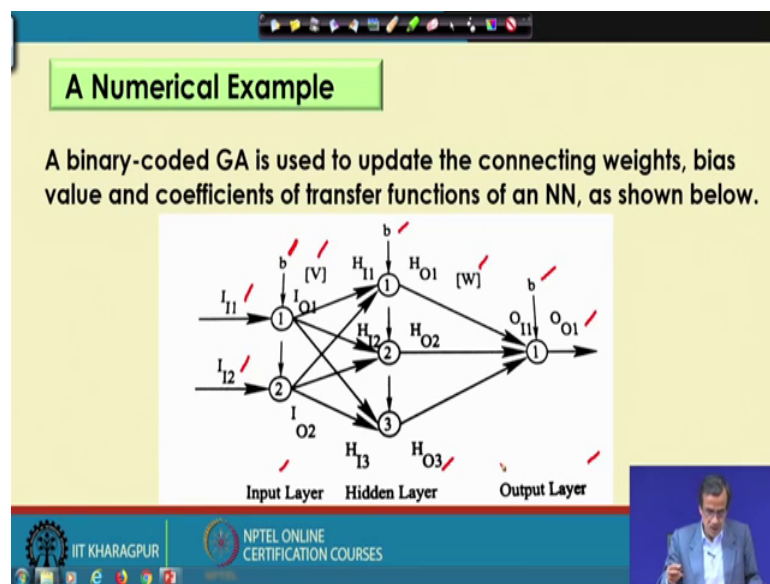


**Traditional and Non-Traditional Optimization Tools**  
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**Lecture – 40**  
**Genetic Algorithm as Evolution Tool (Contd.)**

Now, we are going to solve one numerical example just to explain like how to evolve one neural network using the principle of a binary coded GA.

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Now, let me let me state the problem. A binary-coded GA is used to update the connecting weights, bias value and coefficient of transfer function of a network, as shown below. So, this is actually the small network which we have considered.

Now, for this particular network, once again I am considering 3 layers input layer, hidden layer and output layer. On the input layer we are considering the linear transfer function, hidden layer we are considering the log transfer function and on the output layer we have got the tan sigmoid transfer function.

Now, here we are going to add some bias value like the small bias value denoted by  $b$ . So, this bias value is actually a small value is a small fixed value and we can also keep it as a one of the design variables. Now, for simplicity here, we have consider only two inputs that is  $I_{11}$  and  $I_{12}$ , and there is only one output that is this  $O$  and we have got the

connecting weights  $V$  between input and the hidden layer, the connecting weights  $W$  between the hidden layer and this particular output layer.

Now, let us see how to represent. So, this particular network inside the GA string and how to find out, so how to find out this output and how to find out the deviation or the error in prediction. So, let us let us try to explain this numerical example.

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The neurons of the input, hidden and output layers are assumed to have transfer functions of the forms:

$$y = x; \quad y = \frac{1}{1 + e^{-a_1 x}}; \quad y = \frac{e^{a_2 x} - e^{-a_2 x}}{e^{a_2 x} + e^{-a_2 x}}, \text{ respectively.}$$

The weights:  $[ V ]$  ,  $[ W ]$  vary in the range of 0.0 to 1.0. The bias value  $b$  varies in the range of 0.001 to 0.01.

Both the coefficients of transfer functions  $a_1$  and  $a_2$  are assumed to vary in the range of (0.5,2.0).

Now, as I told that input layer we have got the linear transfer function and on this hidden layer we have got the log sigmoid and on the output layer we have got the tan sigmoid transfer function. The connecting weights  $V$  and  $W$  those are lying in the range of 0.0 to 1.0 and the bias value we are varying in the range of 0.001 to 0.01. So, this is the range for the bias value  $b$  these are the range for the connecting weights 0 and 1, and these are the transfer function.

And the coefficient of transfer function that is this particular  $a_1$  and  $a_2$ , they are lying in the range of 0.5 to 2.0. So, let us see like how to code inside the GA string and how to find out how to get this particular the output for the set of inputs.

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One of the training cases is shown in the Table

Sl.No	$I_1$	$I_2$	O
1	0.6	0.7	0.9
.	.	.	.
.	.	.	.
L	---	---	----

One of the GA-strings is given below, in which five bits are used to represent each real variable.

10110 01101 10001 01011 11011 00011 11001 11110 11101 10101 01110 11000

$V_{11}$   $V_{12}$   $V_{13}$   $V_{21}$   $V_{22}$   $V_{23}$   $W_{11}$   $W_{21}$   $W_{31}$   $a_1$   $a_2$   $b$



Now, this table shows actually the training scenarios. Now, for simplicity here we are going to show only a one training scenario that is input  $I_1$  is 0.6, input  $I_2$  is 0.7 and the output is 0.9.

Now, here we will have to note that this particular output  $I_1$  and  $I_2$  are given in the normalized scale, but if it is given in the real scale so we will have to convert to the normalized scale that is a must. Now, here, these are in the normalized scale and this is actually the target output. Now, the GA string will look as follows supposing that I am going to assign 5 bits to represent each of the variable like  $V_{11}$  that is 10110, then  $V_{12}$  01101, similarly all the  $V$  values, all the  $W$  values then the coefficient of the transfer function that is  $a_1$  and  $a_2$  and the bias value so we have coded in this particular the GA string. And once we have coded in this particular GA string, so we know how to find out their real values.

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Determine the deviation in prediction for the training case shown in the above table, corresponding to this GA-string.

**Solution:**

Sub-string: 10110  
 $v_{11}$

D.V. = 22

$$v_{11} = v_{11}^{\min} + \frac{v_{11}^{\max} - v_{11}^{\min}}{2^5 - 1} \times D.V.$$

$$= 0.0 + \frac{1.0 - 0.0}{2^5 - 1} \times 22 = 0.709677$$

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Now, let me let me discuss this is just like reap recapitulation. So, the substring 10110 is used to represent the V 1, the decoded value we can find out we know how to find out. The real value V 11 is nothing, but V 11 minimum, V 11 maximum minus V 11 minimum divided by 2 raised to the power 5 minus 1 into decoded value. So, you will be getting the real values. Now, using this linear mapping rule, so we can find out what should be the real value. And what is the aim of this numerical example? Aim is to find out what is the deviation in prediction.

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Similarly, real values of the other variables are determined as follows:

Sl. No.	Variable	Binary String	Decoded value	Range	Real value
1	$v_{11}$	10110	22	0.0, 1.0	0.709677
2	$v_{12}$	01101	13	0.0, 1.0	0.419355
3	$v_{13}$	10001	17	0.0, 1.0	0.548387
4	$v_{21}$	01011	11	0.0, 1.0	0.354839
5	$v_{22}$	11011	27	0.0, 1.0	0.870968
6	$v_{23}$	00011	03	0.0, 1.0	0.096774
7	$w_{11}$	11001	25	0.0, 1.0	0.806452
8	$w_{21}$	11110	30	0.0, 1.0	0.967742
9	$w_{31}$	11101	29	0.0, 1.0	0.935484
10	$a_1$	10101	21	0.5, 2.0	1.516129
11	$a_2$	01110	14	0.5, 2.0	1.177419
12	$b$	11000	24	0.001, 0.01	0.007968

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Now, if we see this particular table. Now, corresponding to this serial number one that is V 11 this is the binary string the decoded value, the range and this is the real value. So, similarly we can find out similarly we can find out for this V 12, V 13 and so on. And for all such this design variable so I can find out the binary string the decoded value their respective ranges and their real values we can find out.

Now, here in the table there is a mistake it should be serial number 5 6 7 8 9 10 11 12. So, here for each of these particular your design variables, so I can find out the real values and these real values. Now, we will have to use to find out what should be the output for the set of inputs.

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**Outputs of Input layer:**

$$I_{01} = I_{11} + b = 0.607968$$

$$I_{02} = I_{12} + b = 0.707968$$

**Inputs of the hidden layer:**

$$H_{11} + b = I_{01} \times v_{11} + I_{02} \times v_{21} + b = 0.690644$$

$$H_{12} + b = I_{01} \times v_{12} + I_{02} \times v_{22} + b = 0.879539$$

$$H_{13} + b = I_{01} \times v_{13} + I_{02} \times v_{23} + b = 0.409883$$

**Outputs of hidden neurons:**

$$H_{01} = 0.740219$$

$$H_{02} = 0.791418$$

$$H_{03} = 0.650545$$

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Now, the output of the input layer is nothing but your I O1, is nothing, but I I1 plus b. Now, let me let me try to see what is there, let me try to see this particular network. Now, if you see this particular network you can find that I am trying to find out what is I O1.

So, here we have got the linear transfer function. So, I O1 is nothing, but. So, this I I1 plus this particular the bias value that is nothing, but I O1. Similarly I can find out what is I O2 and once you have got these two outputs very easily I can find out what is I I1, that is the input of the first neuron lying in the hidden layer. And to find out this particular input what I will have to do is, so this output multiplied by the connecting weight, this output multiplied by connecting weight and then we will have to add this particular the bias value.

Now, if you see this the calculation, so it is it is something like this. So, I can find out the output of the input layer then input of the hidden layer the way I discuss  $H_{11}$  plus  $b$  is nothing, but  $I_{O1}$ . So,  $I_{O1}$  multiplied by  $w_{11}$  plus  $I_{O2}$  multiplied by  $w_{21}$  plus  $b$ . So, I will be getting this particular the numerical value. Similarly I can find out  $H_{12}$  plus  $b$  and if you just substitute the numerical values here, so ultimately you will be getting this particular the numerical value. Then  $H_{13}$  plus  $b$  is nothing, but this. So, these are nothing, but the inputs of the hidden layer.

Now, on the hidden layer we have got the transfer function that is we have got the log sigmoid transfer function and using this log sigmoid transfer function. So, I can find out the output of the hidden neuron. Now, this output of the hidden neuron. So, very easily you can find out this particular the expression. Now, if you see this particular the mathematical expression, the mathematical expression for this, so if you see that if we see this particular expression this particular expression if you use. So, I can find out what should be this particular the output of the hidden neuron.

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Input of the output neuron  
 $O_{11} + b = H_{01} \times w_{11} + H_{02} \times w_{21} + H_{03} \times w_{31} + b$   
 $= 1.971413$

Output of the network  
 $O_{01} = \frac{e^{a_2(O_{11}+b)} - e^{-a_2(O_{11}+b)}}{e^{a_2(O_{11}+b)} + e^{-a_2(O_{11}+b)}}$   
 $= 0.981265$

Deviation in prediction  
 $= 0.9 - 0.981265$   
 $= -0.081265$

*(Handwritten red notes: (T-O)² or |T-O| fitness)*

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So, using this we can find out the output of this hidden neuron like  $H_{O1}$ ,  $H_{O2}$ ,  $H_{O3}$  and once we have got the output of the hidden neuron. Now, I can find out the input of the output neuron. So, the input of the output neuron that is  $O_{11}$  plus  $b$  is nothing, but  $H_{O1}$  multiplied by  $w_{11}$  plus  $H_{O2}$  multiplied by  $w_{21}$  plus  $H_{O3}$  multiplied by  $w_{31}$  plus  $b$ . And if you substitute the numerical values, so you will be getting, this is actually the

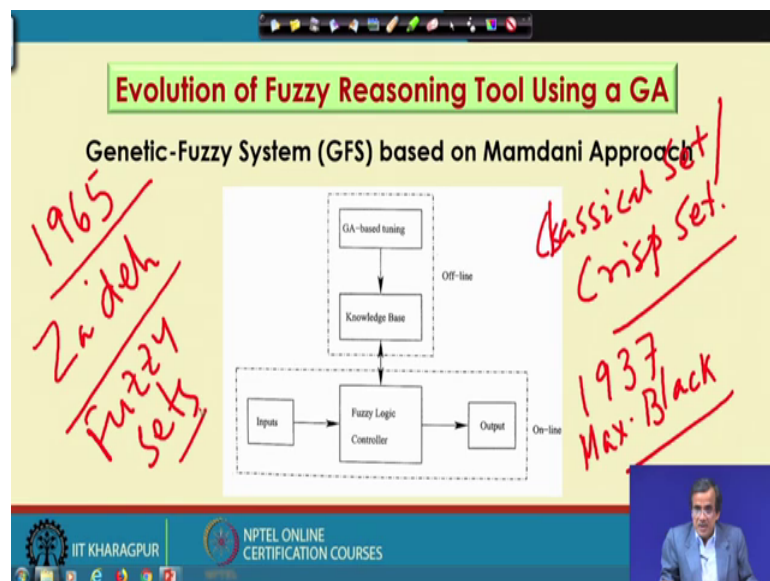
output, this is actually the input of the output neuron and this is the final output after using this transfer function that is the tan sigmoid transfer function.

And once you have got this particular the calculated value that is  $O_{01}$ , now I can find out this particular your the deviation in prediction. And we can see that the target value is 0.9 and the calculated value is 0.981265 so I can find out this particular the difference and here. So, this particular difference is coming to be negative. So, what you will have to do is either we will have to consider the mod value or I will have to find out that your like  $T$  minus this calculated output. So, this particular expression either I can use or I can I will have to use the mod value of  $T$  minus  $o$ .

And using this now, I will have to pass all the training scenarios one after another just to find out what should be the fitness of this particular the GA string and once you have got the fitness of the GA string now, the operations of the GA like the operators of the GA are going to operate and through a large number of iteration. So, it is going to evolve the most suitable and adaptive network so that I can make the prediction both in the forward direction and if required in a the backward direction also.

So, this is the way actually we can the evolved this network using the principle of your the genetic algorithm.

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Now, I am just going for the another thing another area where we can evolve the fuzzy reasoning tool using a genetic algorithm. Now, this fuzzy reasoning tool and this network the basic purpose is is exactly the same, here also we can find out the input output relationship for a particular process or for a particular the system. But here there is a basic difference the difference lies in the fact the way we represent the fuzzy reasoning tool and this particular the network.

Now, exactly in the same way using the principle of this genetic algorithm we can also evolve the fuzzy reasoning tool and as I told that truly speaking there are some similarities between the fuzzy reasoning tool and this particular network and both of them, both are learning tools and they are having the same property like they can predict the input output relationship accurately both in the forward as well as in the backward direction. Now, before I start with this fuzzy reasoning tool actually I will have to spend some time in fact, I will have to give little bit of introduction to the concept of so this particular the fuzzy sets and fuzzy reasoning tool, then only I will be able to discuss, this particular the thing.

So, to start with let me let me try to concentrate a little bit on the concept of the fuzzy sets. Now, if you see nowadays the fuzzy sets have become so much popular, but previously used people used to believe only the classical set or the crisp set. So, the classical set or the crisp set is very old and people used to believe that particular classical set or the crisp set and they used to think that it is the classical set which can tackle different types of uncertainties in this particular the world.

Now, if you see in this particular world the most of the things are uncertain there is a uncertainty and imprecision and people used to think that we can model this uncertainty and imprecision using the principle of the classical set only. Then actually the people are happy this classical set and the probability theory was also proposed based on this particular the classical set or the crisp set. Then in the year something happened that is 1937, one American Philosopher Max Black, Max Black actually he gave the concept of fuzzy sets and the moment he gave the concept of fuzzy set he was opposed by the traditional mathematicians who used to believe only the crisp set or the classical set.

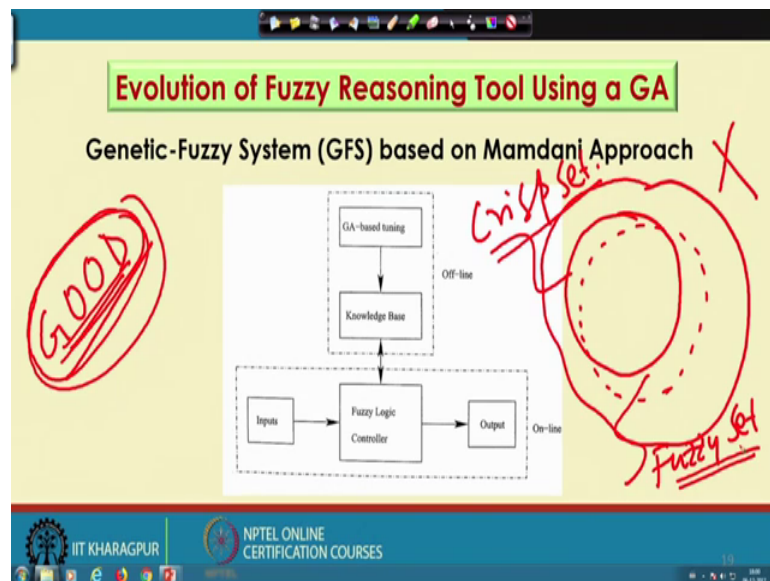
Now, Max Black actually he stopped then after a few years around 1965 professor Jadhe from University of California, USA. So, he reintroduced the concept of, this particular



the fuzzy sets. Now, if you see the fuzzy sets, now according to Professor Jadhe, actually there are some uncertainties which can be tackled using the principle of the classical set or the probability theory, but there are a few which cannot be tackled using the principle of, so this particular the crisp set or the classical set and he gave or he introduced the concept of, so this particular the fuzzy sets.

Now, the fuzzy sets are actually sets with imprecise boundary. So, there is there is no boundary here in this particular the fuzzy set. Now, let me let me take one example. Now, now supposing that like I am just going to investigate the technical universities in this universe, now, if I just tell you that can you please find out a set of technical universities which is having at least 10 departments each. Now, if I see the universe of discourse supposing that say this is nothing, but the universe of discourse is denoted by capital X. Now, if I tell that can you please find out a subset of technical universities having at least 10 department.

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So, there is a possibility throughout the universe. So, might be I will be getting a few such universities and I will be getting one set something like this. Now, these set is having the well defined bounding and this is nothing but the classical set or the crisp set.

Now, I am just making this particular the question be difficult. Now, second question is can you please find out a set of technical universities having at least 10 good departments each. The moment I use this particular adjective the good the problem becomes much

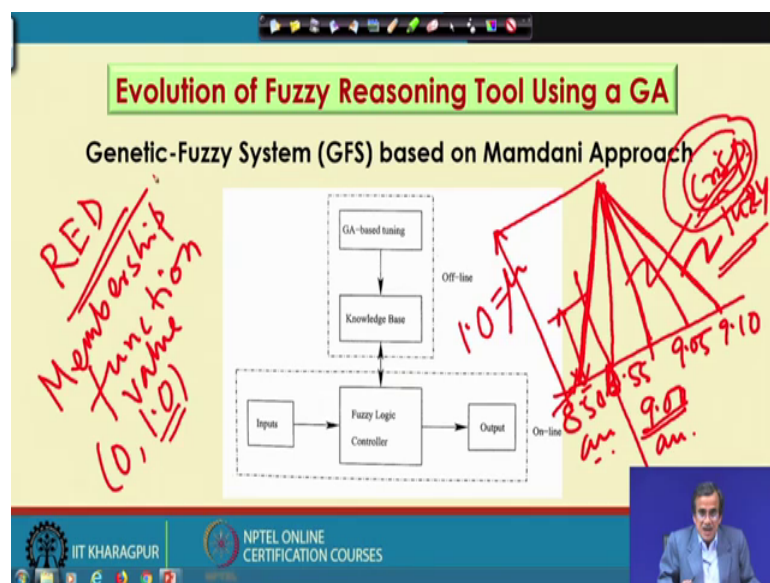
more difficult and in fact, the way Mister X will define good it may vary from the way Mister Y will define.

So, there is a variation of the definition and that is why for this type of definition if I just try to add this particular the good, so it is bit difficult to answer and it will vary from person to person. So, we may not get a set which is a well defined set and there is a possibility say you will be getting one subset having this type of dotted boundary and this particular dotted boundary means it has got no fixed boundary and this is nothing, but a fuzzy set. So, by definition fuzzy set is a set having no such well defined the fixed boundary.

Now, let us see how the, how does this particular fuzzy set can help us let me take one very simple example. Now, the example, now before I take that example let me tell you once again the moment you add this particular the word the good I am injecting some fuzziness and the problem becomes much more difficult, because how to define this particular the good department and this particular definition will vary from person to person. So, I hope you understood the difference between the classical set and this particular the fuzzy set. Let me take another very simple example.

Supposing that your class starts at say 9 and you are told to reach that particular class exactly at 9.

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Now, here, this shows your 9 am. Now, a few students will reach the class exactly at 9. So, this is exactly at 9, a few students may come before 9 say might be 8:55 or 8:50 something like this and a few other students will come might be 5 minutes past 9 or 10 minutes past 9. Now, so this is your 9:05, this is 9:10 this is 8:55 and this is 8:50 am. Now, the student who is coming exactly at 9, because 9 is the class class timing he is following, so a set which is nothing but the crisp set, the crisp 9.

And the students who are coming say 8:50 or 8:55 or 9:05 or 5 10 minutes past 9 they are following another set and that is nothing, but your the fuzzy set. So, this triangular representation is nothing, but the fuzzy set for the time 9:00 am and your this particular the vertical line is actually the crisp set representation for the 9 am. On the other hand, so this particular triangle is a fuzzy set representation for 9 am. So, in fuzzy set actually if somebody is coming say at 8:55 then also we say that he has reached the class at 9 with some membership function value or the degree of belongingness.

The maximum value of this particular the degree of belongingness is nothing, but 0 sorry 1.0 and that is denoted by  $\mu$ . So,  $\mu$  is the membership function value. So, this is nothing, but the membership function membership function value and it varies from 0 to 1, the range is 0 to 1. And here the student who is going to reach at 8:55 he said that he has reached at 9 with some membership function value less than 1, might be 0.5, 0.6 whatever may be. So, this is the way actually in fuzzy set we represent the time 9 am.

On the other hand increased set it is simply one vertical line that is nothing, but the crispiness for this particular the 9 am. Now, you can realize which one is more practical. See it is it is bit difficult to reach the class exactly at 9 it could be a few minutes after or few minutes before. So, fuzzy set is actually a more general concept more practical way of looking into the problem, where the crisp set could be little bit more theoretical.

Now, there are many such argument the difference between the classical set and the fuzzy set and what is the utility of this particular the classical set and the fuzzy set. Now, before I start with this the principle of the fuzzy reasoning tool and let me let me just try to take another example, that example is actually what is the utility of this particular the fuzzy set. Now, let us try to understand the utility of this particular the fuzzy set.

Now, there is one very popular example the same example I am going to take that is your the color red or the color green. Now, supposing that say one of your friends has gone to

the market and you have requested your friend to bring say a 10 apples the red apples. Now, depending on the situation, depending on the your season there is a probability of getting apples. So, there is some uncertainty and depending on the season, so your friends are going to bring 10 apples for you from the market and depending on the season there will be some probability of getting that particular apple.

So, this particular probability can be tackled using this particular the crisp set that is the probability theory. So, this uncertainty can be tackled. But if I say that supposing that you have got the apples 10 apples. Now, what is the guarantee that the apple is red? How can you define this color red? Now, to define the color red actually the different people will define the color red in a slightly different way and; that means, so if it is perfectly red. So, we can say that it is red with the membership function value 1.0. But if it is slightly red then also we will have to say that it is red with the membership function value say 0.4, if it is almost red then also we will have to say that it is red with some membership function value lying between 0 and a 1. That means, the color red that is defined with the help of one fuzzy set ok.

So, this is actually the practical utility of the fuzzy set. And let me just put one sentence like if you take the help of fuzzy set, so we can handle the different types of uncertainties available in these real world problems and we can also tackle some sort of imprecision.

Now, using the concept of this particular fuzzy sets if you see the literature a large number of problems of different types have been solved. And these problems are for examples like how to design and develop your the fuzzy reasoning tool to establish the input output relationship of a process, and we also use to solve some other real world problems. I am not going to discuss here all such things in details.

Now, here actually in this course I am going to concentrate only on how to model this input output relationship using the principle of this particular the fuzzy sets, and I am just going to explain the principle of fuzzy reasoning tool and how to evolve that particular the fuzzy reasoning tool using the principle of a genetic algorithm. So, that I am going to discuss in details with the help of one numerical example.

Thank you.