

Traditional and Non-Traditional Optimization Tools
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Lecture - 36
Genetic Algorithm as Evolution Tool

Now let me recapitulate our aim is to find out the optimal cross section of this particular single point cutting tool, subject to the condition that there is no mechanical breakage while doing that machining. Now here if we just try to see if we try to analyze the nature of these 2 objectives. So, we can find out that our aim is to find out that particular cross section of the cutting tool which corresponds to the minimum weight of the cutting tool and at the same time the developed stress should be less than equal to the permissible or the value of the developed stress. Now these 2 objectives actually are such that they are conflicting in nature.

Now, for example, say if I just go for the more weight of this particular cutting tool the developed stress the developed bending stress will be less and vice versa; that means, if I go for the lighter weight for this particular cutting tool, there is a possibility that the developed stress will be more and there could be some failure in terms of the developed stress.

So, our aim is to minimize both the weight of the cutting tool and at the same time the developed bending stress; however, these 2 objectives are conflicting and they are fighting with each other. Now that means, if the weight of the cutting tool increases the developed stress is going to decrease vice versa and using this particular the information I can formulate as a 2-objective optimization problem as follows.

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g) Using Distance-based Pareto-GA (DPGA) ✓

Solve this two-objective optimization problem as given below.

Minimize $f_1(b, h) = 1572bh$ ✓

Minimize $f_2(b, h) = \frac{300}{bh^2} \times \frac{1}{10^6} \text{MPa}$ ✓ N/mm^2

subject to

$0.005 \leq b \leq 0.20$ ✓

$0.005 \leq h \leq 0.10$ ✓

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Now, our aim is to minimize the weight of this particular the cutting tool and that is nothing but f_1 is a function of b and h that is the width and the height of the cutting tool and that is nothing but $1572bh$ this have already formulated. Now the second objective is nothing but minimize another function of b comma h is nothing but 300 divided by bh^2 multiplied by 1 divided by 10 raised to the power 6 , just to express this particular develop stress in MPa that is nothing but newton per millimetre square.

Now, here our aim is to minimize the weight of the cutting tool aim to minimize the develop stress like this and these are nothing but the range for the design variables. Now let us see how to tackle this type of problem having 2 objectives and where both the objectives are conflicting in nature and our aim is to minimize both the objectives. So, let us see how to find out the optimal solution or how to find out the parrot optimal front of solution.

Now I am just going to discuss a particular method that is called the distance based pareto GA, the working principle of this distance best pareto GA that is DPGA are discussed in detail and now I am just going to use the principle of this particular DPGA to solve these 2 -objective optimization problem. The principle as I told I have discussed in much more details, but let us see how to solve this particular the real-world problem using the principle of the DPGA.

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Solution:

Solution No.	b	h	f ₁	f ₂	D _{min}	Assigned/Calculated fitness	E ^k
1.	0.010	0.015	0.236	133.3	--	100.00	{1}
2.	0.030	0.006	0.283	277.8	1.102	98.89	{1}
3.	0.100	0.080	12.576	0.037	52.297	152.297	{1,3}

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Now, here if we remember we start with actually a population of solution and these solutions are generated at random, supposing that the total population size is denoted by N and this particular population is divided into 2 sub population that is N_1 and N_2 . So, N_1 is nothing but going to contain all the solutions on which the GA operators are going to operate and N_2 is another sub population that contains all the non-dominated solution with the help of which we can find out what should be the pareto optimal front of solution. Now the way to works so as I told that the whole population is generated at random.

And the design variable should lie within their respective ranges. Now we know the range for this b and h and the first solution is selected at random. Supposing that b is equal to 0.010 and h is nothing but 0.015. Now this b and h if I just substitute in the value of this particular the expression for h_1 , this f_1 and f_2 . So, I will be getting the numerical value for this particular f_1 and the numerical value for this particular the f_2 and once we have got the numerical value for this particular f_1 and f_2 . So, f_1 is coming to be equal to 0.236, f_2 is 133.3 and the D_{\min} do not we do not assign anything here and we assign some random fitness.

For these particular the solution that is the first solution and let me assign that this is 100.00. So, this is the random fitness we assigned to this particular the solution 1. And we put the first solution in the non-dominated set. So, this E_k indicates actually the non-

dominated set. The first solution I am going to put in the non-dominated set. Now once we have done it next we go for the second solution and once again b and h those values are selected at random lying within their respective ranges. Now using the expression for these 2 objectives we calculate this f_1 and f_2 and once we have got this particular f_1 and f_2 , let us discuss how to determine this particular D_{min} and how to find out the modified fitness.

And how to take the decision whether the second solution we will enter the non-dominated front or not. So, how to take all such situations I am going to discuss. Now if I compare the values of the objective function; that means, if I compare this f_1 corresponding to the first and the second. So, these 2 things if I compare corresponding to the second solution. So, f_1 is more compared to the first solution; that means, so corresponding the second solution. So, this 0.283 is a dominated solution there is the worse solution compared to this 0.236 because this is a minimization problem.



Similarly, if I compare the f_2 values corresponding to the first solution and the second solution, then the second solution in terms of x_2 f_2 is found to be worse compared to this f_2 in the first solution; that means, 277.8 is higher compared to 133.3, but as I told our aim is to minimize both the objectives f_1 and f_2 , but unfortunately corresponding to the second solution both f_1 and f_2 are found to be worse compared to the respective values of f_1 and f_2 for the first solution.

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Solution:

Solution No.	b	h	f_1	f_2	D_{min}	Assigned/ Calculated fitness	E^k
1.	0.010	0.015	0.236	133.3	--	100.00	{1}
2.	0.030	0.006	0.283	277.8	1.102	98.89	{1}
3.	0.100	0.080	12.576	0.037	52.297	152.297	{1,3}

dominated

That means the second solution is a dominated one corresponding to the first solution with respect to both f_1 and f_2 ; that means, the second solution is not going to enter the non-dominated set; that means, second is not going to enter. So, the non-dominated set will contain only solution 1.

Now, how to determine this D_{\min} and how to calculate this particular the fitness. So, that I am going to discuss. Now let us see how to find out that.

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2nd solution:

$$D_{K,X} = \sqrt{\left(\frac{0.236 - 0.283}{0.236}\right)^2 + \left(\frac{133.3 - 277.8}{133.3}\right)^2}$$

$$= \sqrt{0.039 + 1.175}$$

$$= 1.102$$

$D_{\min} = 1.102$

As solution 2 is dominated by solution 1, i.e., elite solution in terms of both the objectives, it is not selected in the elite set and fitness is calculated as $\max[0, 100.00 - 1.102] = 98.89$

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Now, to find out that we take the help of this particular calculation of the equilibrium distance between K and X. So, K indicates actually the non-dominated solution and X is nothing but the second solution, which I am going to take for which I am going to take the decision whether it will enter the non-dominated set or not. Now what I do is. So, in terms of f_1 and in terms of f_2 .

So, I am just going to find out this particular the equilibrium distance now 0.236 is nothing but the value of f_1 corresponding to the non-dominated solution and 0.283 is the value of f_1 corresponding to the second solution. So, we try to find out this particular the numerical value for this.

Similarly, this 133.3 is nothing but the value of the f_2 with respect to the non-dominated solution that is the first solution and these 277.8 is nothing but the value of f_2 with respect to the second solution. So, using this particular expression I can find out the

equilibrium distance between the non-dominated solution that is K and the second solution that is nothing but X. So, I can find out what is this particular the D KX.

Now, here I have got only one equilibrium distance value, the reason is I have got till now I have got only one non-dominated solution that is K and that is why there is only one equilibrium distance value and as there is only one that is nothing but D minimum and D minimum is 1.102

Now, as solution 2 is dominated by solution 1 that is the elite solution in terms of both the objectives. So, this particular solution 2 is not selected in the elite set and its fitness is calculated as follows according to the rule of DPGA, there is a maximum between 0 comma the fitness of the non-dominated solution that is 100.00 minus this D minimum 1.102 that is nothing but 98.89. That means, using this I can find out like what should be the value for this particular the calculated fitness that is 98.89.

And now I am just going to concentrate on the third solution. Now the third solution the b and h values are selected at random that is b equals to 0.100 and h is nothing but 0.080 and the value of the objective function that is f 1 is calculated like this and f 2 is calculated like this.

Now, let us see how to take the decision where the third solution will entered the elite set or not and how to calculate this particular fitness and how to find out this D minimum. Now to take that particular decision I will have to concentrate on the non-dominated set; that means, I will have to compare this particular value of f 1 with this and this particular value of f 2 with this. Now if I compare the value of f 1 corresponding to the first non-dominated solution that is 0.236 and corresponding to the third it is 12.576.

Now, if I compare these 2 definitely this is higher compared to this 0.236 and this is the minimization problem. So, this is not a very good solution. So, the solution 3 is worse compared to the non-dominated solution in terms of f 1. Now let me compare in terms of f 2 now if I compare so this 133.3 with 0.037 and as this is a minimization problem. So, in terms of f 2 the solution 3 is better compared to the solution 1 there is a non-dominated solution because this is a lower value compared to 133.3; that means, solution 3 is a non-dominated solution with respect to f 2. At least one objective function, but not with respect to f 1 now as it is found to be better compared to the already found elite solution in terms of at least one objective function that is copied in the elite set.

So, the solution 3 is copied in the elite set. So, the elite set will contain 1 comma 3 and once we have selected here let us see how to find out this D minimum and the calculated fitness. Now to find out that actually what I will have to do is I will have to calculate.

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3rd solution:

$$D_{K,X} = \sqrt{\left(\frac{0.236 - 12.576}{0.236}\right)^2 + \left(\frac{133.3 - 0.037}{133.3}\right)^2}$$

$$= 52.297$$

$D_{\min} = 52.297$

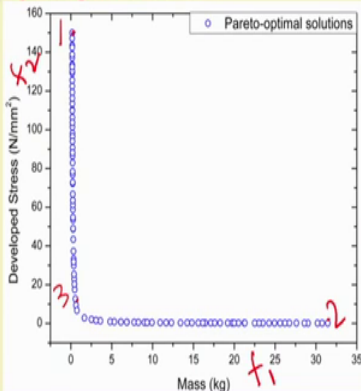
As solution 3 is non-dominated with respect to f_2 (in comparison with the elite set), it is included in the elite set. Its fitness is calculated as $100.00 + 52.297 = 152.297$.





For the third solution the equilibrium distance D_{KX} is nothing but the equilibrium distance between the K th non-dominated solution and the third solution that is capital X and this is nothing but square root 0.236 there is the value of f_1 corresponding non-dominated 1.

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f) Using NSGA-II



Sl No.	b (m)	h (m)	Mass (kg)	Stress (N/mm ²)
1	0.005	0.2	0.157	150
2	0.2	0.1	31.4	0.15
3	0.005	0.094	0.739	6.79
4	0.0106	0.1	1.66	2.83

The value of f_1 corresponding to the non-dominated 1 and 12.576 is the value of f_1 corresponding to the third solution

Next 133.3 is the value of f_2 corresponding to the non-dominated solution then 0.037 is the value of f_2 with respect to the third solution and I can find out the equivalent distance values. And as there is only one equilibrium distance value till now so this is nothing but D minimum now as the solution 3 is a non-dominated with respect to f_2 with respect to at least f_2 in comparison with the elite set.

So, this will be included in the elite set; that means, solution 3 will be included in the elite set and its fitness will be calculated as the 100.00 there is the value of the elite set the fitness value of the elite set plus this D minimum that is 52.297 and that is nothing but one 52.297. So, this is actually the calculated fitness for this particular the third solution and once we have got the fitness for this particular the third solution

Now, I am in a position to write down this particular fitness for the third solution there is one 52.297 and the solution 3 is going to enter. So, this particular the non-dominated set. The same procedure I will have to follow for the other solution of the population. So, similarly I go for the 4th fifth 6th and so on for the whole population and iteratively we can find out that a large number of solutions are going to enter this non-dominated front. And as I told that the size of the non-dominated front is going to increase and consequently the size of this N_1 on which the GA operators are going to operate is going to be reduced.

Now, on N_1 the GA operators are going to work just to bring diversification in the population and on N to sub population; that means, N is nothing but N_1 plus N_2 . So, on N_1 the GA operators are working and N_2 indicates the size of or the sub population of your the non-dominated set. Now these particular non-dominated set is going to determine what should be the pareto optimal front of solution and if you follow this principle. So, there is a possibility that this DPGA can find out ultimately the non-dominated set of solution and if we just plot them, like if I plot f_1 versus f_2 . So, this is f_1 this is f_2 and our aim is to minimize both f_1 f_2 .

And if I just concentrate on this non-dominated front solution and if I plot. So, there is a possibility that I will be getting this type of pareto optimal front of solution. So, this possibility is there and using this DPGA. So, I can find out this pareto optimal front of

solution for the single point cutting tool. So, this is the way actually we will have to use the principle of DPGA to find out like what should be the optimal values for this particular the width of the cutting tool and the depth of the cutting tool so that it can ensure the minimum weight and your the minimum develop stress.

So, this is the way actually we can find out the pareto optimal front of solution using the principle of your DPGA that is distance best pareto GA. Now I am just going to discuss the principle of another efficient tool to obtain the pareto optimal front of solution and the name of the algorithm is nothing but your NSGA that is non-dominated sorting genetic algorithm the second version NSGA 2 I am just going to discuss.

Now the principle I have already discussed and now I am just going to see using the principle of this NSGA 2 how to find out the pareto optimal front of solution. Now for this NSGA 2 as I have already discussed that we are going to use the principle of the crowding distance just to ensure the diversity of the population of solution, and at the same time to ensure the selection pressure we take the help of some set of reproduction scheme like say the tournament selection.

And using this the tournament selection as a reproduction scheme and the crowding distance the concept just to find out the diversification we try to keep a proper balance between the population diversity and the selection pressure, and using the principle of this crossover and mutation we try to bring some are the new solutions in the population. And if I follow this particular principle there is a possibility that I will be getting the pareto optimal front of solution. Now here along this I am just going to plot the mass that is nothing but f_1 and the develop stress is nothing but f_2 and our aim is to minimize both f_1 and f_2 , but the fact is if f_1 increases then f_2 is going to be reduced and vice versa.

So, that is the fact that means these 2 objectives are going to conflict each other and there is a possibility if I use this NSGA 2 through a large number of iteration. So, gradually it will try to find out this type of pareto optimal front of solution. Now here on this pareto optimal front of solution actually we have got a number of possible optimal solution. Now the user can select any one out of these large number of solutions line in the pareto optimal front according to his choice. Now here on this particular table so we are going to show some of the values of the design variables corresponding to the optimal solution,

for example, if you see the solution 1 b is 0.005 h is 0.2 mass is found to be equal to 0.157 and the develop stress is 150 MPa newton per millimetre square. Now here if I see. So, this corresponding to this solution 1 might be this is the point this is the solution corresponding to the solution 1.

So, this is one extreme solution lying within the feasible range for this pareto optimal front. Now another the borderline solution could be this and might be this corresponds to the second solution here that is 0.2 h equals to 0.1 mass is 31.4 and the develop stress is 0.15. So, this could be second solution. Then comes solution 3, b is 0.005, h is 0.094 mass is 0.739 and the stress is 6.79, the 6.79 it could be here might be I am here.

So, this is your point 3. Next from the solution 4 that is b is 0.010,6 h is 0.1 mass is 1.66 and the stress is nothing but 2.83. So, stress is 2.83 so might be here. So, this could be your point 4. Similarly, we have got other points now the user can choose any one according to his choice and all the solutions are optimal solution with respect to the different weightage or different weights on the 2 objective functions like f_1 and f_2 . Now out of all the possible optimal solution the user can select any one.

So, this is the way actually we can find out the pareto optimal front of solution; that means, different optimal solution for this particular the single point cutting tool and as I told several time. Once it got the optimal values for this particular b and h the length of the cutting tool is kept fixed so I can find out what should be the optimal design or the several optimal design now I can choose any one depending on the choice.

And this is the way actually we can find out the optimal multiple optimal solution for this type of the 2-objective optimization problem; that means, using this the different algorithms like some traditional optimization algorithm and some non-traditional optimization algorithm I can find out the optimal solution for the practical problem that is the optimal cross section or the optimal design for this particular the single point are turning tool.

And as I told so this problem the same problem has been tackle using the principle of single objective optimization and multi objective that is 2 objective optimization. So, the optimization algorithm whose working principle we have discuss in much more detail can be used to solve the real-world optimization problem in a very efficient way.

Thank you.