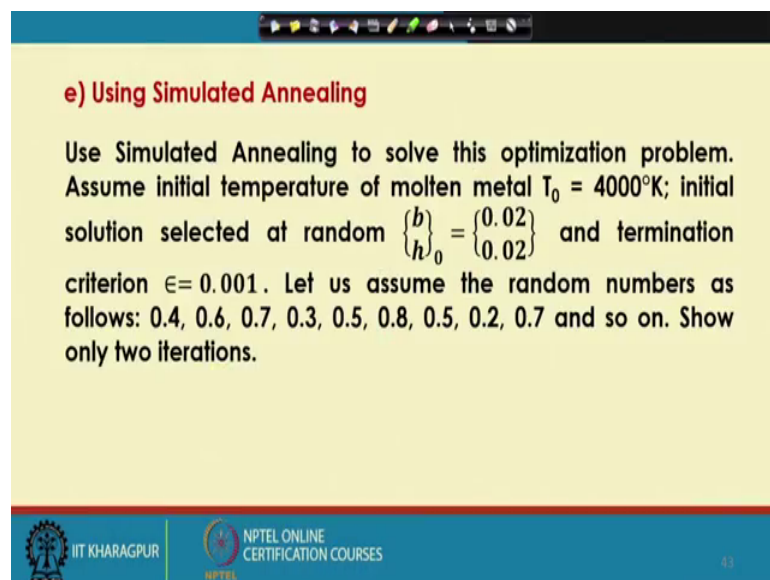


Traditional and Non-Traditional Optimization Tools
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Lecture - 34
A Practical Optimization Problem (Contd.)

Now I am going to discuss how to use another very popular Non-Traditional tool for optimization known as simulated annealing to solve the same problem, like how to find out the optimal design of this particular the single point cutting tool?

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e) Using Simulated Annealing

Use Simulated Annealing to solve this optimization problem. Assume initial temperature of molten metal $T_0 = 4000^\circ\text{K}$; initial solution selected at random $\begin{Bmatrix} b \\ h \end{Bmatrix}_0 = \begin{Bmatrix} 0.02 \\ 0.02 \end{Bmatrix}$ and termination criterion $\epsilon = 0.001$. Let us assume the random numbers as follows: 0.4, 0.6, 0.7, 0.3, 0.5, 0.8, 0.5, 0.2, 0.7 and so on. Show only two iterations.

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Now, this simulated annealing it is working principle I have already discussed in details unlike, the genetic algorithm it starts with only one solution and try to check it is feasibility.

Now, let us see the statement of this particular problem. Now here actually I am just going to use simulated annealing to solve the same optimization problem. And as we have discussed that it starts with a process of how to model the cooling process of this particular the molten metal and on principle it can serve the minimization problem.

Now, here we assume some initial temperature at random of the molten metal and here we have consider T_0 in the initial temperature is nothing, but 4000 degree kelvin, an initial solution selected at random lying within the respective ranges. So, b, h_0 is

nothing, but 0.0 2 0.0 2. So, this is the initial solution which I have selected and the termination criteria epsilon is nothing, but 0.0 01

Now, let us assume the following random numbers, because in in while working with the iteration I will have to use these particular random numbers several times. And I am just going to saw only 2 iterations; how to proceed with this particular algorithm to determine the optimal solution?

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Solution

Given $T_0 = 4000^\circ\text{K}$; $\begin{Bmatrix} b \\ h \end{Bmatrix}_0 = \begin{Bmatrix} 0.02 \\ 0.02 \end{Bmatrix}$

Value of objective function corresponding to $\begin{Bmatrix} b \\ h \end{Bmatrix}_0$, $m'_0 = 1572 \times 0.02 \times 0.02 = 0.629\text{kg}$

Iteration 1:
 $T_1 = 50\% \text{ of } T_0 = 2000^\circ\text{K}$

Corresponding to the random numbers 0.4 and 0.6,

$$\begin{Bmatrix} b \\ h \end{Bmatrix}_1 = \begin{Bmatrix} 0.005 + 0.4(0.20 - 0.005) \\ 0.005 + 0.6(0.10 - 0.005) \end{Bmatrix} = \begin{Bmatrix} 0.083 \\ 0.062 \end{Bmatrix}$$

Handwritten notes: $b = b_{\min} + r(b_{\max} - b_{\min})$

Handwritten notes: 0.4 and 0.6 are circled.

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Now, given this T_0 is the initial temperature that is 4000 degree kelvin and b, h_0 , that is the initial solution is 0.2 0.2 the value of the objective function corresponding to this b, h_0 is nothing, but m'_0 is 1 5 7 2 multiplied by b multiplied by h , that is 0.6 2 9 kg, we concentrate on iteration 1. Now what I do is. So, the temperature T_1 is kept equal to 50 percent of T_0 that is your 50 percent of 4000 degree kelvin that is nothing, but 2000 degree kelvin.

Now, corresponding to the random numbers 0.4 and 0.6; now if I see the values of the random numbers which we have considered. So, I am just going to consider this 0.4 and 0.6, now corresponding to the value of this particular the random numbers 0.4 and 0.6, I will have to find out the value for this b and h .

Now how to find out to find out the value for this particular b corresponding to 0.4 we use this particular the formula that is b is nothing, but the b minimum plus the random

number r multiplied by b maximum minus b minimum. So, this particular formula we are going to use to find out what should be the value for the b now here b minimum is 0.005 the random number is 0.404 b maximum is 0.20 and this is b minimum.

So, I am getting the value of b is 0.083 . Similarly I can find out the value of h is h minimum plus random number that is 0.6 multiplied by h maximum minus h minimum, that is 0.062 . So, we can find out what should be the possible values for b and h for the next.

Now, once you have got these numerical values now I will have to check their feasibility whether, these particular solutions are feasible or not that I will have to find out.

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Value of objective function corresponding to $\begin{Bmatrix} b \\ h \end{Bmatrix}_1$,

$$m'_1 = 1572 \times 0.083 \times 0.062 = 8.089 \text{ kg}$$

As $m'_1 > m'_0$, we cannot select $\begin{Bmatrix} b \\ h \end{Bmatrix}_1$ as the next point now, and go for the next checking.

The next random number $r=0.7$

Change on m' , i.e., $\Delta m' = |m'_1 - m'_0| = |8.089 - 0.629| = 7.46$

We calculate $\exp\left(\frac{-\Delta m'}{T_1}\right) = \exp\left(\frac{-7.46}{2000}\right) = \exp(-0.00373) = 0.99$

As $r < 0.99$, we accept $\begin{Bmatrix} b \\ h \end{Bmatrix}_1$ as the next point.

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Now, to find out that ah before that actually what we do is we try to find out the value of this particular objective function corresponding to b h 1. And we substitute in the expression of objective function that is 1572 multiplied by b into h . So, I will be getting this as actually the value of the objective function.

Now, initially your m m 0 1, now this m 0 1 actually it was equal to, if I see this m 0 1. So, m 0 1, was 0.629 and m 1 prime m 1 prime is a 8.089 kg. Now if I compare then m 1 prime is greater than m 0 prime; that means, it is not a good solution, because our aim is to actually to minimize that.

Now, based on this particular condition we cannot select $b h 1$ as the next solution. And just to give another chance to this particular solution, what we do is we go for the next checking? And the next checking as we discussed while discussing the principle of simulated annealing this is based on that Boltzmann probability distribution so, that particular expression, which I have already discussed in detail.

Now, the next random number r is 0.7 , now what I do is we try to find out the change in m prime that is Δm prime is nothing, but the mod value of the difference between m_1 prime and m_0 prime and this is becoming equal to 7.46 . And now we try to calculate exponential of minus Δm prime by T_1 that is nothing, but exponential minus Δm is 6.46 divided by T_1 is 2000 and if you calculate this this will become equal to 0.99 .

Now, this random number r is found to be less than 0.99 . So, we accept this $b h 1$ as the next point. So, this is the principle which you use although this particular $b h 1$ is not selected according to this particular condition. So, we give a another chance using this Boltzmann probability distribution that particular concept and according to this particular point that is $b h 1$ we are going to accept, these completes actually one iteration.

Now, we go for the next iteration, but before I go for actually the next iteration. So, what you will have to do is?

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Iteration 2

$$T_2 = 50\% \text{ of } T_1 = 1000^\circ K$$

Corresponding to the random numbers 0.3 and 0.5 ,

$$\begin{Bmatrix} b \\ h \end{Bmatrix}_2 = \begin{Bmatrix} 0.005 + 0.3(0.20 - 0.005) \\ 0.005 + 0.5(0.10 - 0.005) \end{Bmatrix} = \begin{Bmatrix} 0.063 \\ 0.052 \end{Bmatrix}$$

Value of objective function corresponding to $\begin{Bmatrix} b \\ h \end{Bmatrix}_2$

$$m'_2 = 1572 \times 0.063 \times 0.052 = 5.149 \text{ kg}$$

As $m'_2 < m'_1$; we select $\begin{Bmatrix} b \\ h \end{Bmatrix}_2$ as the next point.

Handwritten notes: $\Delta m = |m'_1 - m'_0|$ and $\Delta m \ll T$

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You will have to check whether it has reached that, particular termination criteria. Now this termination criteria that is actually is very simple.

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Change in m' ,

$$\Delta m' = |m'_2 - m'_1|$$
$$= |5.149 - 8.089|$$
$$= \underline{2.940} \gg \epsilon$$

We go to the next iteration.

$\epsilon = 0.001$

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The termination criteria is something like this. So, what you will have to do is actually we will have to find out the change in m prime the mod value that is nothing, but m_1 prime minus m_0 prime and if this particular value is found to be less than is found to be less than epsilon that is; that means, it is a very small value then we say it has reached that particular the termination criteria otherwise actually we do not we proceed to the next iteration.

So, now I am just going to start with the second iteration and in the second iteration this T_2 is nothing, but 50 percent of this particular T_1 that is 1000 degree kelvin. Now before I proceed with iteration this checking has to be done, whether I should go for second iteration or not. Supposing that we have taken the decision that we will have to go for the second iteration, this is how to proceed with the second iteration.

Corresponding to the random numbers 0.3 and 0.5, we try to calculate b h corresponding to second solution, exactly following the same principle. I can find out that b is 0.0 6 3 and h is 0.5 2. Now corresponding to this b h_2 , we will try to find out what is the value of the objective function that is m_2 prime.

Now m_2 prime is 1.572 multiplied by $b \cdot h$. So, this is nothing, but 5.149 . So, this is nothing, but m_2 prime.

Now, if I compare m_2 prime with a m_1 prime, we see that m_2 prime is less than m_1 prime; that means, your this is a good solution. So, what do you do we select $b \cdot h^2$ as the next point. And once again we will have to go for that particular checking, that is whether it has reached that termination. And to check that we try to find out the change in m prime that is Δm prime is nothing, but mod value of m_2 prime minus m_1 prime and this is nothing, but this particular thing.

So, we can find out the change in this particular your the m , here m stands for energy and as it is found to be much higher compared to this α this ϵ , which has been taken to be equal to 0.00 . Now we should go for the next iteration and this process will go on and go on and ultimately after running this particular algorithm for a large number of iteration there is a possibility that it is going to hit that particular optimal solution.

Now, this is another very popular or nontraditional tool for optimization and using the principle of this particular algorithm I can also find out the optimal design of this particular the practical problem that is single point cutting tool. Now one thing I just want to mention regarding this particular that the technique that is a simulated annealing.

Now this simulated annealing works based on the principle of one physical phenomena that is the cooling process of molten metal at this particular cooling process of molten metal, can be modeled mathematically. And that is why a mathematical convergence proof for this particular algorithm exists which is still missing for the genetic algorithm. So, many people actually relied on this particular algorithm also to solve the optimization problem.

Now, another thing I will have to mention here, because this particular practical problem which we are considering, this is one constrained optimization problem and that particular checking is not yet done. Now what you can do is before we declared the finally, acceptable optimal solution we will have to check the feasibility.

Whether it is going to violate that particular functional constraint or not, because we will have to find out 1 optimal design after satisfying the condition of that particular

functional constraint otherwise the solution is very simple because b and h are having some minimum value each.

So, if you just substitute those minimum values in the value of the objective function expression, we will be getting the optimal design there is no such actually pressure from other side and that is why that functional constraint has to be maintained there should not be any violation of this particular the functional constraint and we will have to find out the optimal design of this single point cutting tool.

So, that it becomes a light in weight at the same time up it will have to satisfy the condition that there will be no mechanical breakage during the machining.

Thank you.