Traditional and Non-Traditional Optimization Tools
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Lecture - 32
A Practical Optimization Problem (Contd.)

Now, I am just going to discuss, how to use the principle of another genetic algorithm.

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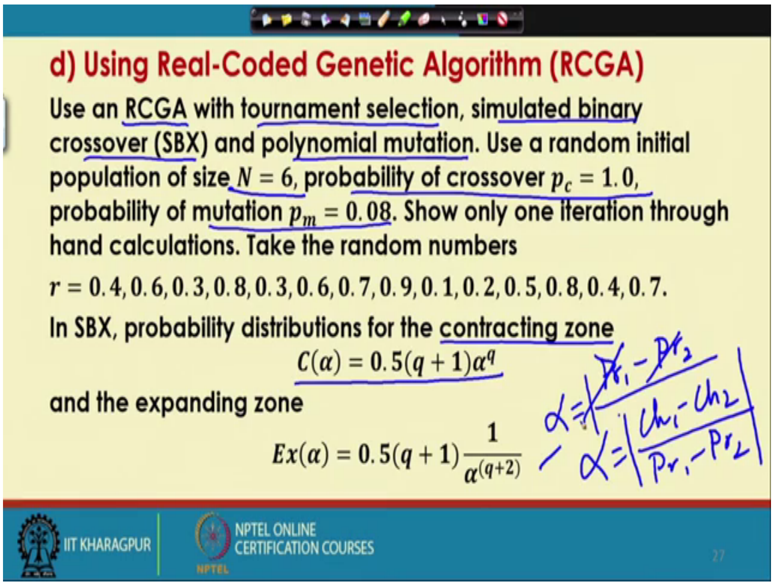

That is called the real coded genetic algorithm in sort RCGA. In order to solve the same optimization problem, the constrained optimization problem related to optimal design of single point cutting tool. Now, here the statement of the problem is as follows. Now, I am just going to use one RCGA; that is, the real coded genetic algorithm with tournament selection, simulated binary crossover and polynomial mutation to solve this particular the problem. We are going to use a random initial solution of size N equals to 6, probability of crossover PC equals to 1.0, the probability of mutation pm is 0.08 and I am just going to show you one iteration through hand calculation.

Now, to proceed with the calculation, I will have to take the help of some random numbers. Now, let us assume that, these are the random numbers like 0.4, 0.6 and so on. And I am just going to use SBX; that is, your simulated binary crossover, the principle of which I have already discussed in detail. And here, for the contracting zone, the probability distribution is given by C alpha is nothing but 0.5 multiplied by q plus 1 multiplied by alpha raise to the power q and alpha is actually nothing but the spread factor and that is defined as the ratio of parent 1 minus parent 2 or child let me just remove it.

So, it is nothing but, alpha in the spread is nothing but the difference between child 1 child 2 divided by parent 1 parent 2. So, this particular the alpha is actually the spread factor and q is nothing but a positive exponent and it value may vary from say 1 2 3 up to a very high value.

So, this is actually the expression for this contracting zone. Similarly, for the expanding zone, the expression for this is ex alpha is nothing but 0.5 multiplied by q plus 1 multiplied by one divided by alpha raise to the power q plus 2. So, this is the expression for this particular the expanding the zone. Now, using this particular expression, actually I will have to define that particular the probability distribution of creating the children solution from the parent that I am going to discuss in much more details after some time.

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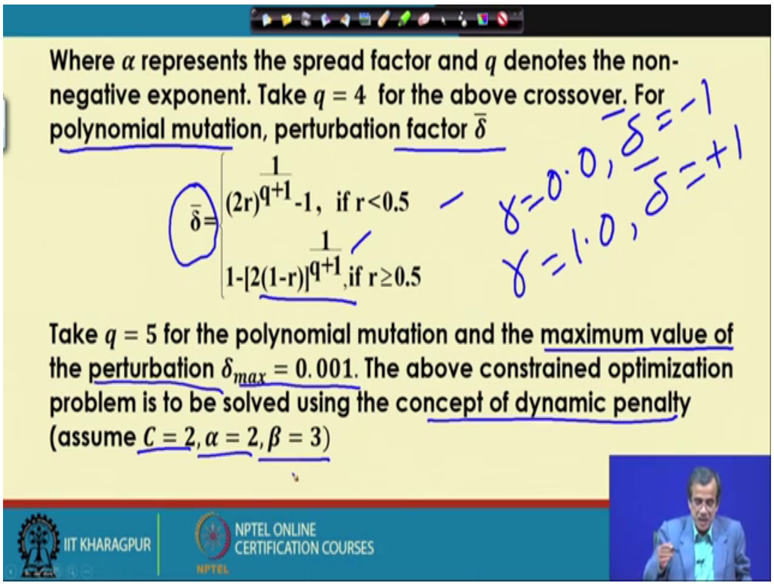

Now, here alpha represents the spread factor as I told and q is nothing but non-negative exponent. We take q equals to 4 here and for the crossover. Now, for the polynomial mutation, so actually, we will have to use the distribution function for the perturbation factor; that is, delta bar. Now, delta bar is nothing but 2 r raised to the power 1 divided by q plus 1 minus 1, if r that is a random number is found to be less than equals to 0.5.

Otherwise, this delta bar is nothing but 1 minus 2 into 1 minus r raised to the power 1 divided by q plus 1.

Now, here this particular expression has been taken using some philosophy. Now, the reason is something like, how to select this type of distribution? Now, let me put r equals to 0 here in the first expression. Now, if I put r equals to 0, this delta bar will become equal to minus 1. Similarly, if I put r equals to 1.0 here in this particular expression, so I will be getting delta bar is equals to plus 1.

So, the value of this particular perturbation factor is going to vary from minus 1 to plus 1 in the normalized scale. And accordingly, this particular expression has been considered. But remember, this is not the unique expression, we can also find out some other mathematical expression to represent the perturbation factor. Now, perturbation means that dissimilarity I am just going to discuss in detail.

Now, for this polynomial mutation, we will take q equals to 5 and the maximum value of this particular perturbation there is a maximum value of the dissimilarity is taken to be equal to 0.001. And to handle the constrained optimization problem, we are going to use the concept of dynamic penalty and for which the constant C alpha is assumed to be equal to 2, alpha C is 2, alpha is assumed to be equal to 2 and beta has been taken to be equal to 3. So, let us see how to tackle this particular constraint optimization problem or the constraint minimization problem using the concept of this real coded GA. Now, let us let us try to concentrate.

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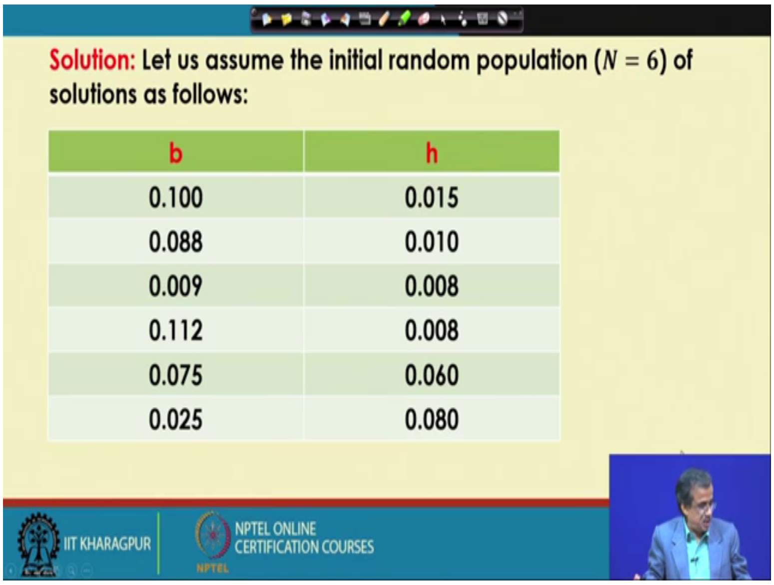

Now, just like the binary coded GA, the initial population has to be generated at random using the random number generator. Now, in a binary coded GA, we used to generate some binary codes consisting of 1s and 0s, but here, in real coded GA, the values of the variables like your this b and h, there is a width of the cutting tool. And the height of the cutting tool and these are having the real values. And those real values are directly generated at random using the random number generator lying within their respective ranges.

Now, if you see the range for this particular b and h, all such values are generated within their respective ranges for the first solution the b is 0.100 and h is 0.015. For the second solution, it is 0.088 and for h it is 0.010. Similarly, for the 3rd 4th 5th and 6th the values of this, b and h have been generated at random lying within their respective ranges.

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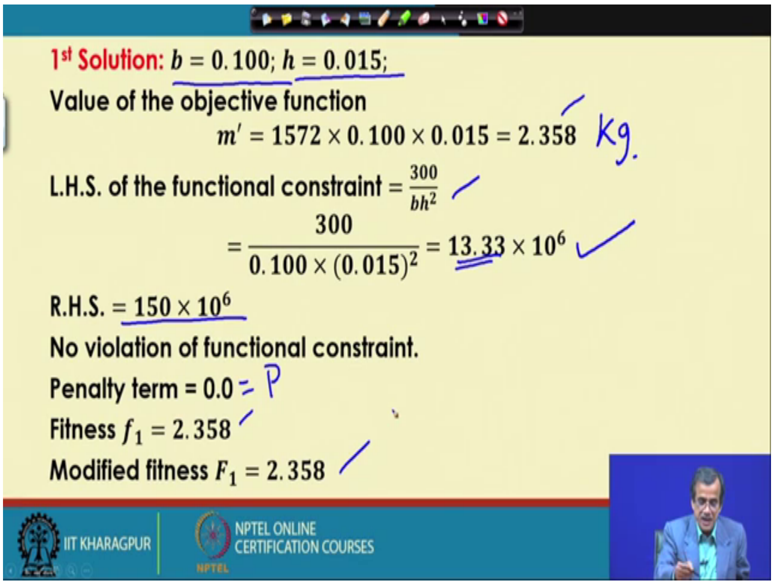

Now, once you have got the initial population of solution, let us see how to proceed to find out the optimal solution. Now, let me concentrate on the first solution; that is, b equals to 0.100 and h equals to 0.015. So, I am just going to concentrate on the first solution and corresponding to the first solution, let us try to find out the value of the objective function; that is, the fitness and let us try to find out whether there is any such violation of the functional constraint.

Now, first solution, now, here actually corresponding to the first solution, b equals to 0.1 and h equals to 0.015. So, value of the objective function can be determined very easily. That is, 1572 multiplied by b multiplied by h. So, 2.358 so much kg.

Now, the left-hand side of the functional constraint is 300 divided by b h square and if we substitute, we will be getting this as the developed stress and the allowable stress is this much. So, the developed stress is less than the allowable stress. So, there is no violation. So, penalty term P is equals to 0. So, the original fitness is 2.358. So, the modified fitness will remain same as original that is, 2.358.

So, this is the way for the first, we can find out like what should be the value of the objective function and what should be the fitness information.

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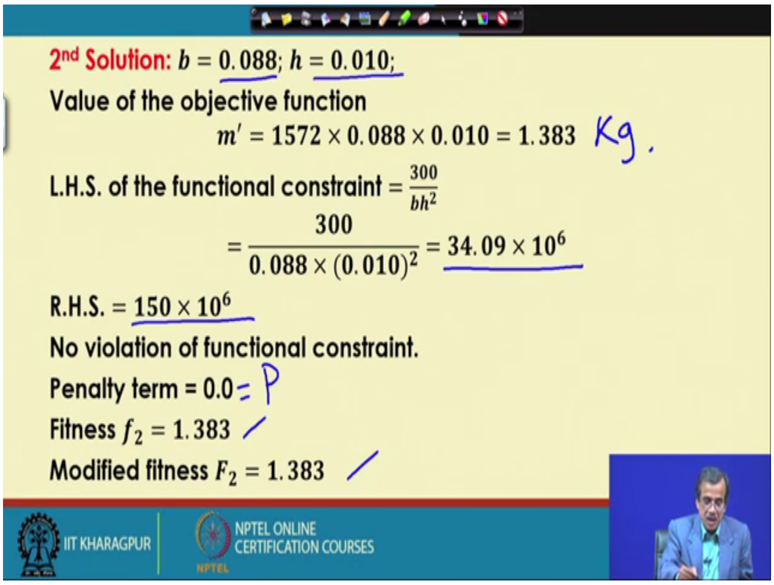

Now, I concentrate on the second solution. Now, corresponding to the second solution, like b is equals to this h is equals to 0.010, the value of the objective function exactly in the same way I can find out 1.383 kg the left-hand side of the functional constraint is coming to be equal to 34.09 into 10 raised to power 6 and this particular the developed stress is less than the allowable stress. So, once again there is no violation of the functional constraint. So, penalty term P is equals to 0 the original fitness is 1.383 modified fitness will remain same as the original the fitness.

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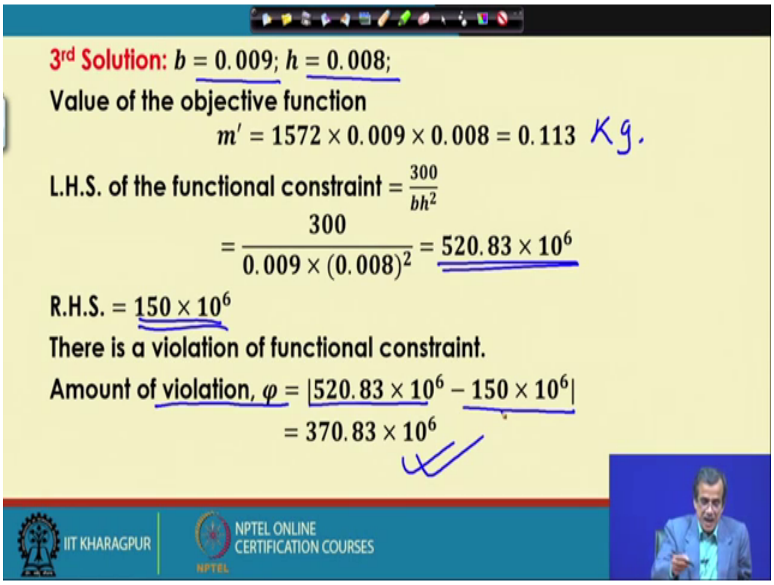

Now, we go for the third solution. Now, for the third solution, like b is equal to 0.009 equals to 0.008. The value of the objective function is 0.113 and let us try to find out, whether there is any violation of the functional constraint. The left-hand side of the functional constraint is 300 divided by b h square; that is nothing but this and this is more than the allowable value. So, there is a violation of the functional constraint and if there is a violation of the functional constraint we try to find out how much is the amount of violation. So, the amount of violation is nothing but phi that is the mod value of the difference between 520.83 into 10 raised to the power 6. So, this is the developed stress minus the allowable stress. So, this is the amount of violation.

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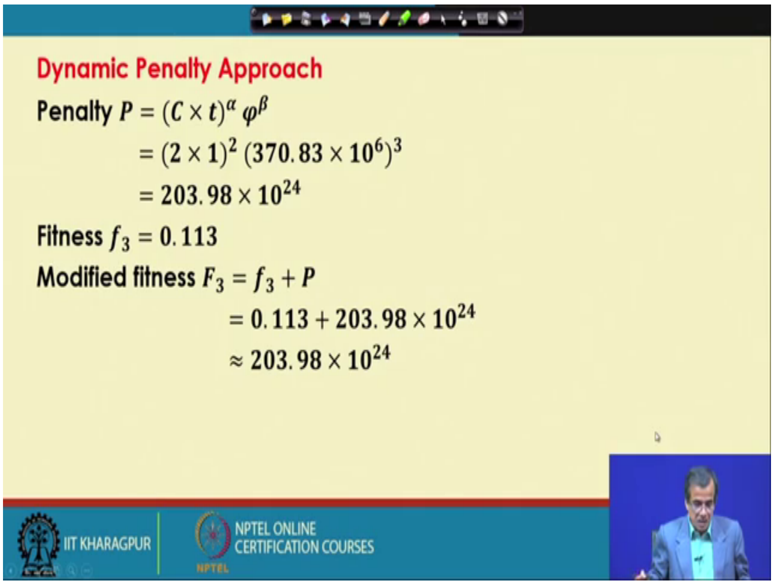

And once we have got the amount of violation, following the same principle, I can find out what should be the penalty value for this dynamic penalty approach. The penalty is C multiplied by t raised to the power alpha, phi raised to the power beta and if you assign the numerical values, so I will be getting your the penalty term that is equal to 203.98 into 10 raised to the power 24 a very high value. And here, the original fitness is 0.113 and the modified fitness will be F 3 will be f 3 plus P and this is approximately equal to 203.98 into 10 raised to the power 24. Now, as it is a minimization problem, this particular solution is not a good solution because, it is going to violate the functional constraint.

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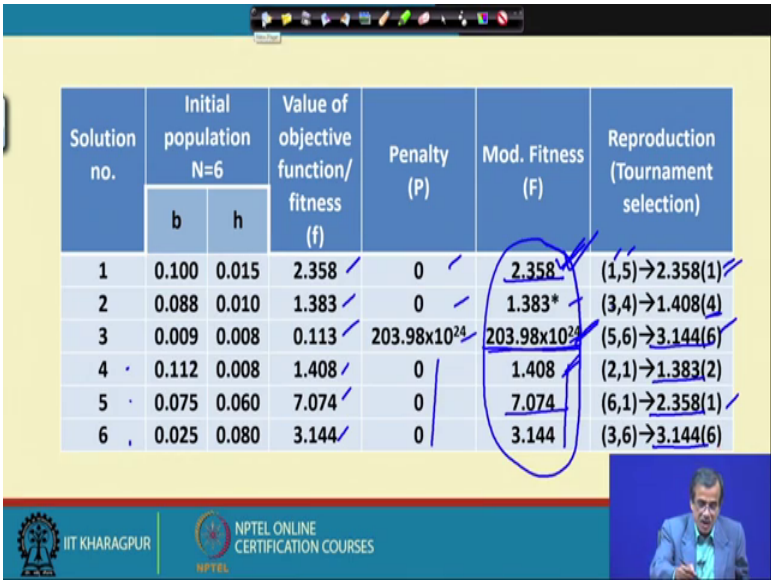

Now, following the same principle, actually I can find out the fitness information for the other solution. Now, the way I discussed, I have already discussed how to find out the fitness for the first solution, the penalty term for the first solution and the modified fitness. Similarly, I have already discussed for the second solution, for the third solution. Now, following the same principle, I can also calculate for the 4th, 5th and 6th what should be the value of the fitness, what should be the penalty and there is no violation. So, all the penalty terms will be 0. So, I will be getting this particular the modified fitness.

So, I am just going to get the modified fitness for the whole population. And you can see that, is not a very good solution because, it has been paralyzed and in the mating pool, definitely there should not be any copy of this particular solution that is the third solution. And once again, we take the help of the rip the reproduction that is the tournament selection.

Now, once again for this tournament, will have to select the solution at random, using the random number generator supposing that in the first tournament. So, the first one and the fifth one have been selected because, the tournament size we have taken is equal to 2. Now, out of this first and fifth, so this is the first one and this is the fifth one. If I compare their fitness values, the first one is better and that is 2.358. So, I have selected the first one and it is corresponding fitness is 2.358. Next, we go for the second tournament and in the second tournament, I am just going to consider the third and 4th. So, this is the third one and this is the 4th one. And if I compare in terms of fitness, I will be getting the 4th one is the better. So, I am going to select the 4th one in the mating pool.

Then, third tournament between 5 and 6, the fifth and the 6th and definitely the 6th is found to be better. So, 60 is copied here. The 4th tournament between first and second; second is found to be better. Second is selected here. The fifth tournament between 6th and the first; the first one is better. So, the first one is selected here. Now, the last tournament is between 3 and 6. Now, out of these 3 and 6, 6th is better. So, the 6th one is copied here.

Now, you can see the first one has been selected how many times once, twice. So, this particular solution has been selected twice. Then comes your second, has been selected only once; 4th has been selected only once and 6th has been selected only once.

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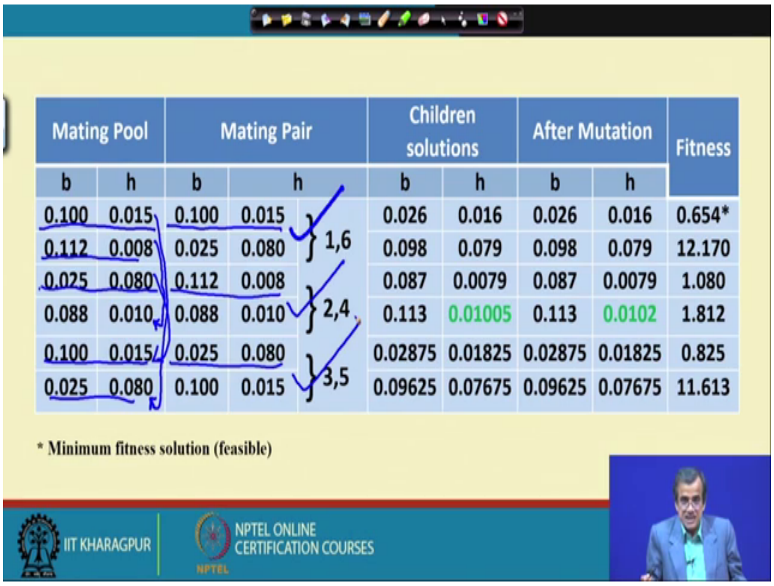

So, by doing this, so I will be getting the mating pool so, this is nothing but the mating pool. And once you have got this particular mating pool, now you will have go for the mating paired selection. And once again, we will have to select this particular mating pair at random using the random number generator.

Now, supposing that, I am just going to decide corresponding to this particular solution, what should be the mating pair? So, in the mating pair, directly we copy this the first one and supposing that, first one is found to meet with the 6th one. So, this is going to meet with the 6th one. So, I will have to copy the 6th one here; that is, 0.025, 0.080. So, it will constitute the first mating pair.

Next, we go for the second mating pair. So, here, this is the second solution. So, let me just copy it here 0.112, 0.008 and this solution is going to meet with the 4th one. So, this is actually the 4th one. So, 0.088, 0.010. So, these constitute the second mating pair. The next is the third mating pair. So, I am just going to find out the third mating pair. So, what you do is, the third one you directly copied here 0.025, 0.080 and this third one is going to meet with the fifth one; that is nothing but 0.100, 0.015. So, this is actually the third mating pair.

Now, I will have to check whether there is a repetition of 2 parents in a particular mating pair. Now, if I just concentrate on the first mating pair, so there is no repetition. So, this is a valid mating pair. Next, I will have to go for the second mating pair. So, this b and b, h and h are different. So, this is a valid mating pair. Next, I will have to compare b and b here; h and h here. So, this is a valid mating pair. So, all 3 mating pairs are found to be valid and now they are going to participate in crossover operator.

Now, how to participate in crossover operator and how to get the children solution? That I am going to discuss in details before I concentrate here on this particular the table. So, let me try to find out the detailed calculation, like how to get the children solution and let us first concentrate on the first mating pair; that is, b equals to 0.100 and second b equals to 0.025. So, these 2 things are going to participate in SBX; that is simulated binary crossover. Similarly, these 2 parents are going to participate in the simulated binary crossover and let us see how to implement that.

Now, to implement that, actually what you do is, I am just going to show you that particular detailed calculation now. So, till now, I am here. So, up to the mating pair, we have got and now, I am just going to discuss how to find out the children solution and once again, I will be coming back to this particular the table after some time.

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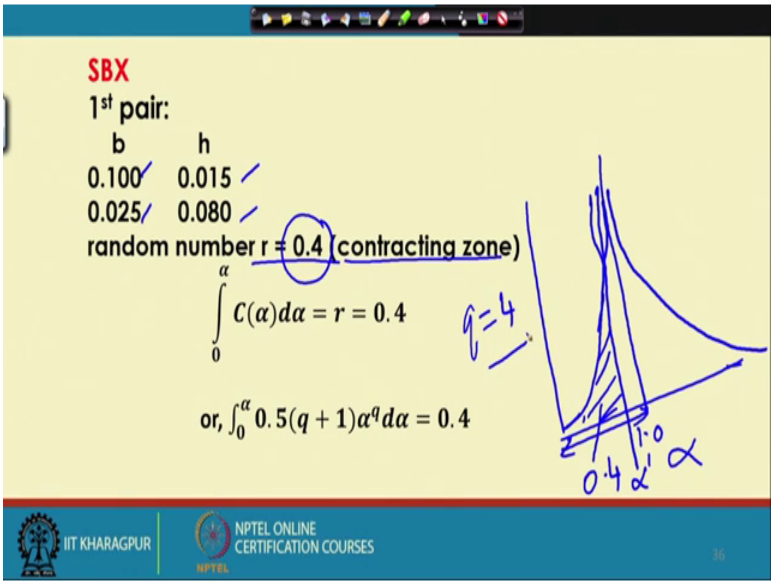

Now, let us concentrate on this particular the first mating pair. So, b equals to 0.100 and h equals to 0.015 and for the second parent, b equals to 0.025 and h is 0.080 and if you see, while stating the problem, I took the help of a few random numbers. And if you see that particular list, the first random value was 0.4. So, let me consider r equals to 0.4.

Now, if I consider r equals to 0.4, now the principle of SBX, I have already discussed in much more details. Now, r corresponds to 0.4 that r is nothing but the area under the probability distribution curve and if it is found to be less than equals to 0.5, this is in contracting zone. And in the contracting zone, I will have to find out the value of alpha. There is the spread factor corresponding to this random number value or corresponding to this particular area which is equals to 0.4. Now how to find out?

Now, to find out that, I will have to use integration 0 to alpha in the contracting zone C alpha d alpha is equals to r and that is nothing but r equals to 0.4. Now, to recapitulate like, if I just want to see that probability distribution, say, this is actually the alpha corresponds to 1 here, this side is more than 1 less than 1 and this is the probability distribution.

Now, if you see the distribution corresponding to a particular the exponent that is q, I will be getting different types of expression. For example, if q is too high, I will be getting this type of steeper distribution for this particular probability distribution function. So, this type of I will be getting. So, the moment I say, r equals to 0.4, now area under this particular curve is 0.5 and the total area is 1.0. The moment I consider r equals to 0.4, I am in the contracting zone; that means I am here. This is the expanding zone and 0.4 means the area under the curve I will have to find out. So, this particular area is 0.4 and corresponding to this area I will have to find out what is alpha prime. So, that is actually what I am doing here.

Now here, if I just substitute the values of the expression, integration 0 to alpha C alpha is 0.5 multiplied by q plus 1 into alpha raise to the power q d alpha and that is nothing but 0.4. Now, here this q is equals to 4. So, I am just going to substitute q equals to 4. Now, if I substitute q equals to 4, I will be getting the modified expression of this particular is as follows.

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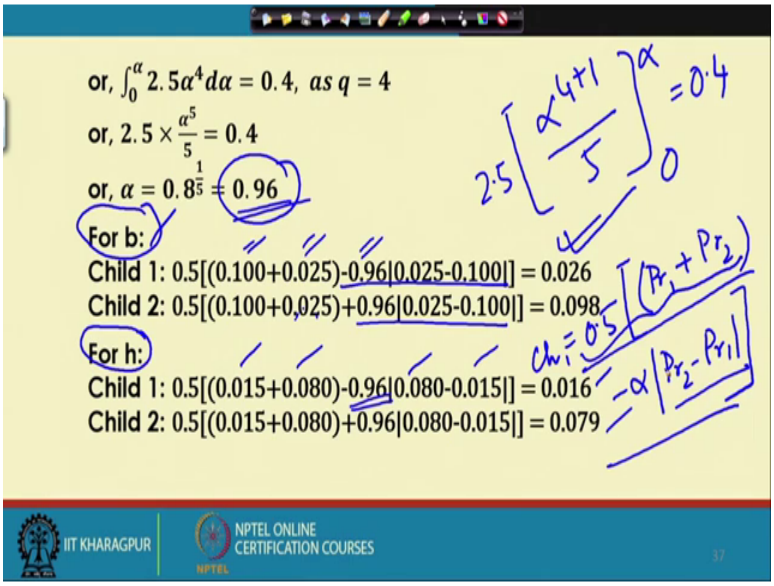

So, it is nothing but integration 0 to alpha 2.5 alpha raised to the power 4 d alpha and that is equals to 0.4 and very easily, I can find out. So, this particular integration that is 2.5 then comes your alpha raised to the power 4. So, it is 4 plus 1 divided 5, 0 to alpha and this is nothing but your 0.4.

Now, if we simplify, then you will be getting actually this particular thing; that is, alpha is equals to 0.96 and as expected, the alpha is found to be less than 1; that means, in the contracting zone. Now, corresponding to this alpha equal to 0.96, now I am going to find out the children solution for b. Child 1 is 0.5 the parent 1 plus parent 2 minus actually this alpha, then the mod value of the difference between the parent 2 minus parent 1 and this is something like this.

Now, here I am just going to write down the expression. So, this child 1 is nothing but is 0.5 multiplied by that is your parent 1 plus parent 2 minus alpha the mod value of the difference between parent 2 minus parent 1. So, this is actually the expression for the child 1. So, if you substitute all such numerical values here, for example, parent 1 is 0.100, parent 2 is 0.025, alpha is 0.96. This is parent 2, this is parent 1, then very easily I can find out this numerical value for child 1.

Similarly, the child 2 in place of this minus this will be plus other things are the same. So, 0.5 into 0.100 plus 0.025 plus 0.96 mod value of the difference 0.025 minus 0.100 that is 0.098. Now, here the philosophy behind using this particular expression is as follows. Now, to determine the child solution, actually what we do? First, we consider the average of the 2 parents and that is nothing but 0.5 multiplied by parent 1 plus parent 2, this will give the average of these 2 parents.

Now, I try to find out the difference between the 2 parents; that is, pr 2 minus pr 1 it could be either positive or negative consider the mod value, so that this becomes a positive quantity multiplied by alpha that is the spread factor multiplied by 0.5 that is 50 percent of that and that 50 percent you subtract here and you add here. So, you subtract to get child 1 and you add to get child 2 that is all. So, by using this, I can find out the expression for child 1 and child 2 for this particular the b.

Similarly, for h, that is the height of that particular single point cutting tool, use the same principle to find out child 1 and child 2 and use the same value of alpha. Now, if the same value of alpha and this is actually for h this is your parent 1, parent 2, this is parent 2 parent 1 and this is the alpha. So, following the same principle, I can find out this child 1 and child 2.

So, using this particular principle, I can find out what should be the value for this particular child 1 and child 2 in terms of b and h corresponding to a particular solution.

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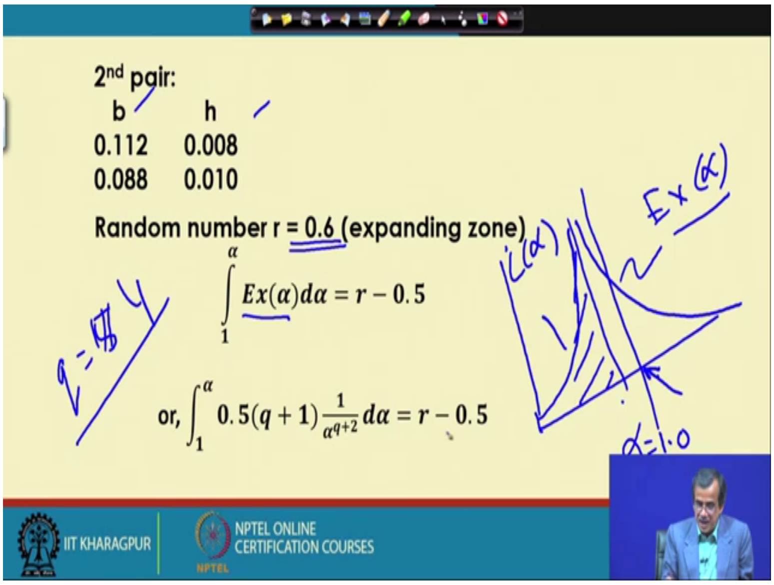

Now, if I follow this, for the second pair exactly in the same way. So, let us try to explain how to find out the second one. The second one actually the b is equal to 0.112, h is equal to 0.008 and the second parent b is 0.088 and h is 0.010. Now, here the random number from that particular list is coming to be equal to 0.6 and that is more than 0.5; that means, I am in the expanding zone of the distribution which I showed earlier. And if it is on the expanding zone; that means, my alpha that is spread factor will be more than 1.0 and to get that actually what we do is, we try to take the help of this integration that is, integration 1 to alpha e x alpha that is, I mean the expanding zone d alpha and that is nothing but r minus 0.5.

Now, if you remember, once again this particular expression let me draw it one second. So, this is corresponding to alpha equals to 1.0 and corresponding to a value of q. So, this is the contracting zone and this is say expanding zone. So, here actually I am here. So, up to these the area is 0.5 and this will give the remaining 0.5 the total is actually 1.0. Now, I am getting the area this r is what r is 0.6; that means I am in the expanding zone; I am in the expanding zone.

And now, I will have to find out what should be value of this particular alpha? To get this value of this particular alpha, actually what we do is, I am just going to start the integration alpha equals to 1 to alpha prime ex alpha d alpha. So, this is nothing but your ex alpha and this is nothing but C alpha contracting zone and this is the expanding zone.

I am in the expanding zone now. So, ex alpha d alpha is nothing but r minus 0.5 because up to this this is 0.5. So, r minus 0.5 and if you substitute the values of q. So, q we have assumed here is 0 point I think q is assume to be equal to 5 here for this, no I am sorry this is 4 here now. So, this integration one to alpha 0.5, this is the expression for ex alpha into q plus 1 into 1 divided by alpha raise to the power q plus 2 d alpha is r minus 0.5.

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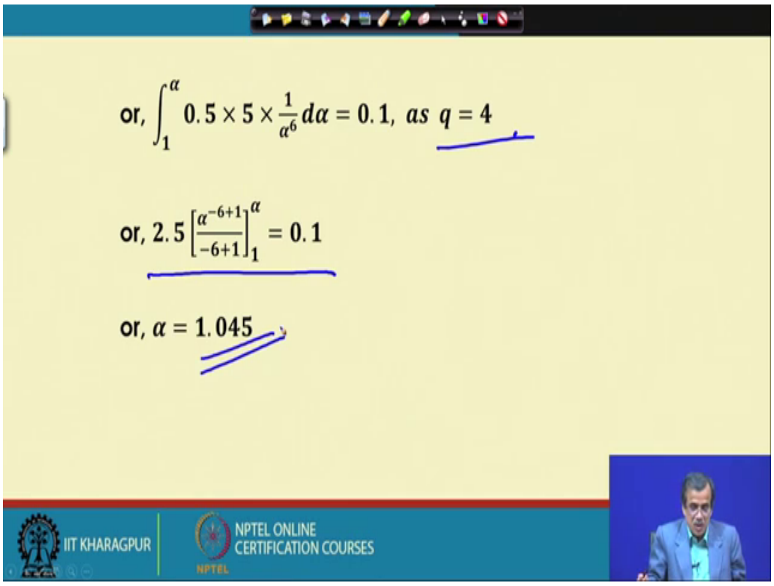

Now, here if you substitute the value of this particular q equals to 4 here, then I will be getting this particular the expression. So, integration 1 to alpha 0.5 multiplied by 5 into 1 divided by alpha raise to the power 6 d alpha is nothing but 0.1. And if you just carry out this integration, you will be getting this and if you solve for alpha. So, alpha will become 1.045. So, slightly more than 1.0 and this is obvious because, I am in the expanding zone.

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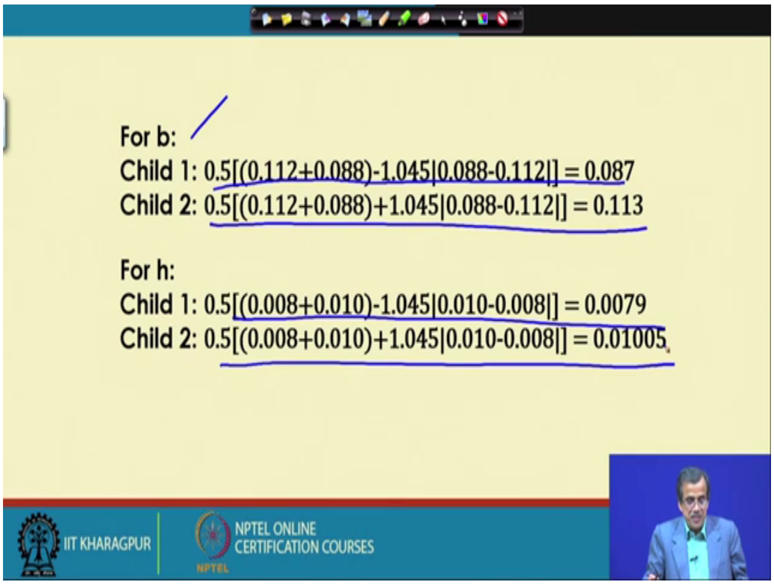

Now, corresponding to this alpha equals to 1.045, following the same principle, I can find out for this b for b, I can find out what should be this particular the child solution and child 1 and child 2. I can find out similarly corresponding to h. So, I can find out this is child 1 and this is child 2. So, I can find out the both for this b and h child 1 and child 2 corresponding to this particular the second solution.

Thank you.