Traditional and Non-Traditional Optimization Tools
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Lecture - 28
A Practical Optimization Problem

Now I am going to start with how to solve the different optimization tools which I have already discussed to solve a real world problem now till now we have seen we have discussed the working principle of a number of traditional and nontraditional tools for optimization and we took the help of a few examples. Now those examples were actually in the form of some sort of mathematical expression like in the form of say y is a function of a few variables say x 1 x 2.

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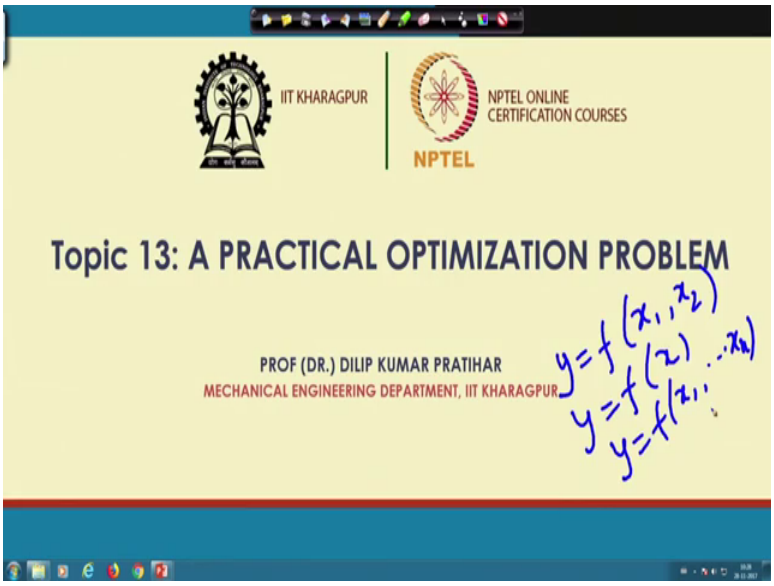

Let me consider only 2 variables or we have considered y is a function of only one variable or we have considered y is a function of a large number of variables x n. So, this type of problem we have solved and we have discussed how to find out the optimal solution using different algorithms. Now I am just going to take a very simple the practical problem and how to how to use the algorithms which I have already discussed to solve this optimization the problem.

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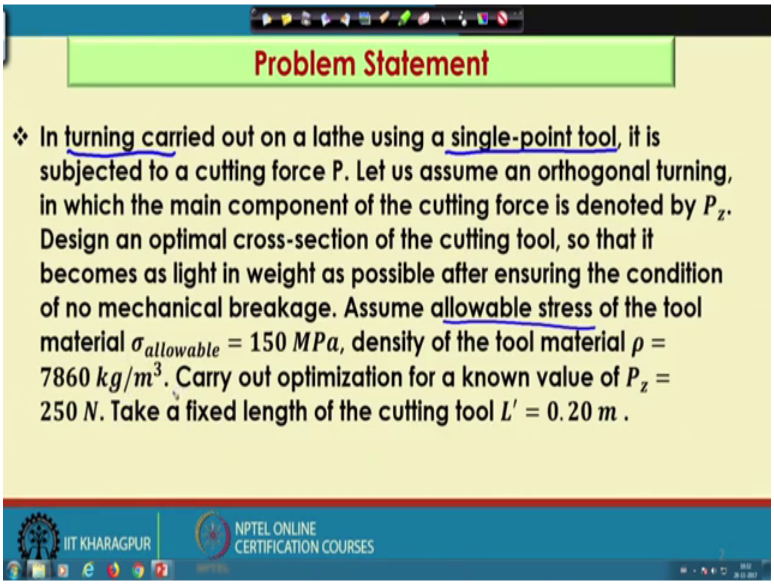

Now, let us try to concentrate on a very practical example and this is not very difficult to understand and this is a very simple example. Now let me see the statement of this particular the problem, in turning carried out on a leg so, the turning operation supposing that I have got a steel cylinder of say diameter d 1, I reduce the diameter from d 1 to d 2 throughout this particular the length. So, what we do is we just hold this particular job on a lathe between the headstock and the tailstock and this particular job is rotated.

Now, we use one single point cutting tool to do that this particular the turning, now the purpose of this particular turning is actually just to reduce the diameter of this cylindrical job from say d 1 to d 2, d 2 is less than d 1 throughout this particular length. Now this is the problem of the state turning. So, this is the problem of straight turning and this is carried out on a lathe using a single point cutting tool.

I am just going to discuss what do you mean by a single point cutting tool it is subjected to a cutting force P, now while cutting while do this particular machining the tip of this particular cutting tool will be subjected to some amount of load I am just going to show you one the sketch or the diagram might be in the next slide. Now let us assume an orthogonal turning in which the main component of the cutting force is denoted by P z.

Now, if you see the literature of metal cutting. So, this particular turning can be considered either orthogonal turning or oblique turning, orthogonal turning is simpler compared to the oblique turning let me concentrate on the orthogonal turning only. Now we will have to design an optimal cross section of the cutting tool. So, that it becomes as light in weight as possible so; that means, the weight of the classic cutting tool or the mass of the cutting tool should be as minimum as possible; that means, it should be as light in weight as possible after ensuring the condition of no mechanical breakage.

So, there should not be any mechanical breakage of this particular cutting tool while doing this particular cutting operation that is the turning operation. Assume allowable stress of this particular tool material that is sigma allowable is 150 mega Pascal MPa, density of the tool material rho is 7860 kg per meter cube carry out optimization for a known value of P z; that means, the main component of cutting force and that is 250 Newton approximately 25 kg take a fixed length of the cutting tool that is L prime is 0.20 meter.

Now, I am just going to show you that particular picture and this particular statement will be very clear, now I am just going to concentrate more on defining this particular the problem.

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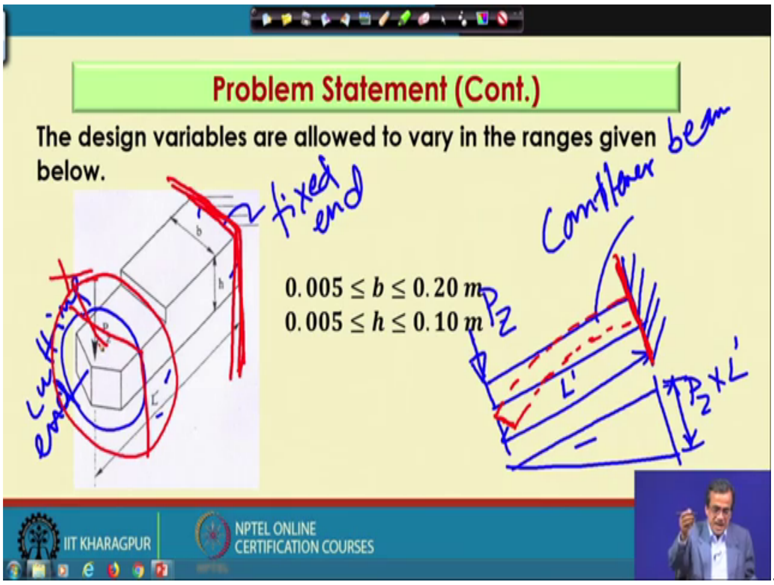

Now, this is actually a single point cutting tool now if you see this particular single point cutting tool at this particular end. So, this will be gripped by the tool post and this is actually the cutting edge.

So, this is the gripping end and this is the cutting end we can say and this is actually known as the cutting point and that is why this is called the single point cutting tool I am not going to discuss more on the tool geometry, but one fact I want to just tell you that during the machining there should not be any mechanical failure of this particular the cutting tool. The tool is having the dimension the length of the tool is L prime then b is nothing, but the breadth of this particular tool h is nothing, but the height of this particular tool and this is actually the fixed end of the tool.

This is the fixed end and this is actually the cutting end this is the cutting end of this particular the tool and this is the load which is acting during this particular the turning operation so, this is nothing, but the period. Now how to ensure a shape design of this particular the cutting tool, now I am not going to much complexity of this particular tool geometric complexity of the tool geometry and I just want to simplify and to simplify the problem let me draw a very simple schematic view here supposing that it is almost similar to the situation that I have got a beam this end is fixed and here I have got a load.

Now, this load is nothing, but P z and the total length of this particular the beam is nothing, but L prime and it has got the cross section that is denoted by b and h and this is the fixed end now this type of beam is known as all of us we know this is known as actually the cantilever beam. Now this cantilever beam this end is the fixed end and here actually I am just putting this particular concentrated load. So, there could be some failure here.

Now, let us try to find out which one is the weakest section of this particular the beam now if I draw the bending moment of this so, I will be getting this type of picture the bending moment will be something like this and it is negative and here actually I will be getting the maximum bending moment and the maximum bending moment is nothing, but P z multiplied by L prime and why this is negative.

Now, if I use this type of the load concentrated load there is a possibility due to this particular load this end is fixed. So, this will take the deflected position something like this, the beam will take the deflected position something like this and this is actually called the hogging nature and that is why so, this is taking this type of shape. So, this indicates the negative moment now the bending moment at this particular section will be the maximum that is nothing, but P z multiplied by L prime and here it is the minimum;

that means, if I try to find out that which section is the most dangerous the weakest one it is not this particular section, but this particular section is the weakest.

And that is why here if there is any mechanical failure during this particular machining the mechanical failure may come here in this particular section, but not in this particular section. So, there will be no such mechanical failure here and while designing this cutting point the single point cutting tool we actually take that particular advantage and that is why we are able to do little bit of grinding here at this particular end just to incorporate this particular the tool these tool angles the different types of tool angles here. So, that it will be able to do this particular cutting very efficiently.

Now, we are able to reduce this particular dimension little bit just to provide the different angle the reason is this is not the weakest region, the weakest region is coming here and that is why this region the gripping end there is no change of this particular geometry, but we make also changes here by incorporating also changes in the geometry. So, there is no chance that failure will come here because the weakest section is this.

Now, how to solve or how to find out the optimal dimension of this particular cutting tool using the principle of some traditional and some nontraditional tools for optimization those things I am going to discuss. Now let us see how to proceed with this step of the problem. So, once again let me repeat our aim is to find out an optimal design corresponding to the minimum mass of this particular cutting tool or the minimum weight of this particular cutting tool.

Now, minimum weight means there will be saving of material the cost of the tool will be less. So, our aim is to minimize the cost of the material the weight of the material and at the same time there should not be any mechanical breakage because if I reduce the dimension just to minimize the mass there is a possibility some failure will come during the machining. So, there should not be any failure of this particular cutting tool during the machining.

So, here the design variables are this b the design variables are b and h L is kept fixed P z that is the maximum amount of cutting force that is also known. So, there are 2 unknowns one is the, this particular b and h and here we are defining this particular range for b and h. So, b is lying between 0.005 and 0.20 meter, h is lying between 0.005 and 0.10 meter.

So, these 2 are actually the ranges of the variable these are the variable bounds now I will have to mathematically write down what is the objective function and what is the functional constraint.

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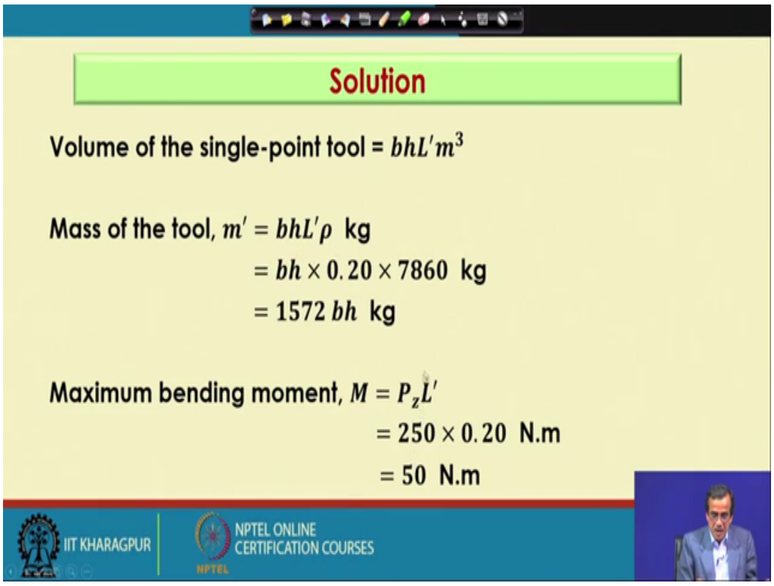

Now, to do that actually what we do is we proceed further in this particular the direction we try to find out the volume of this single point cutting tool and that is nothing, but b multiplied by h multiplied by L prime meter cube. Next we try to find out the mass of the cutting tool denoted by m prime that is nothing, but b h L prime multiplied by rho is nothing, but the density. So, we will be getting by putting the numerical values like L is L prime is 0.20 meter it is pre specified rho is 7860 kg per meter cube that is the density and if you just substitute all such values then we will be getting 1572 multiplied by b h kg. So, this is nothing, but the mass of the tool.

So, our aim is to minimize the mass of this particular the tool and we try to find out what is the maximum bending moment, the maximum bending moment M is nothing, but capital M is nothing, but P z multiplied by L prime which I have already discussed P z is 250 and L prime is 0.20 and if you multiply you will be getting 50 Newton meter. So, this is nothing, but the maximum bending moment.

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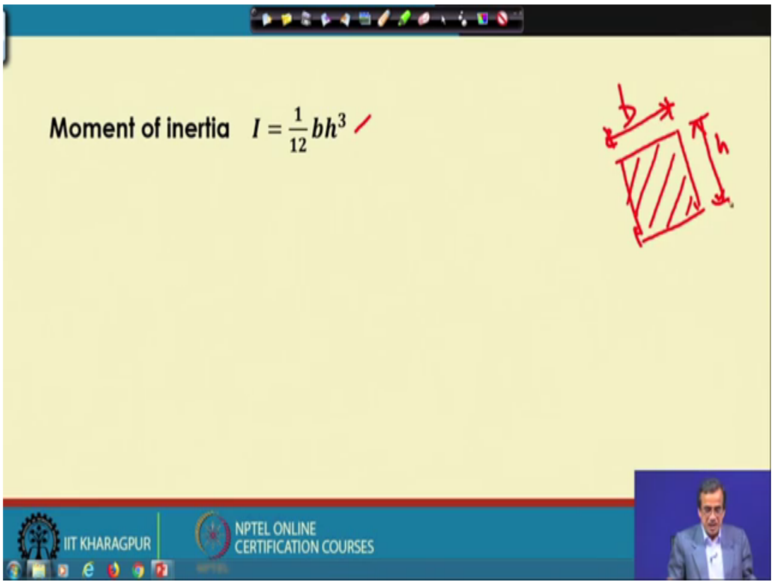

Now the moment of inertia of this particular the section now, if I just draw this particular section the cross section of this particular tool supposing that this is b and h say this is the height of this particular tool and supposing that this is the width that is b. So, this is the rectangular cross section of this particular your the cutting tool and for this the moment of inertia is nothing, but 1 by 12 b h cube is nothing, but the moment of inertia for this type of rectangular cross section.

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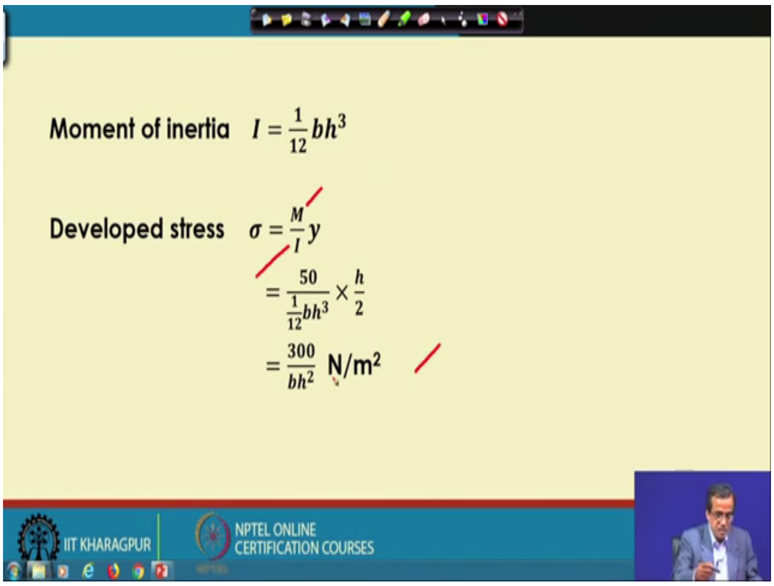

Now, once you have got the moment of inertia expression the develop stress that is nothing but the bending stress, now bending stress we know the formula like the first year mechanics the bending formula is M divided by I into y M is nothing, but the moment M is nothing, but actually the maximum bending moment I is the moment of inertia and y is nothing, but h divided by 2 that is the half of this particular height that indicates actually the location of that particular the centerline.

Now, M is 50 Newton meter I is one 12 bh cube and y is nothing, but h by 2 and if you simplify we will be getting 300 divided by bh square and the unit is Newton per meter square. So, this is actually the developed the bending stress and this developed bending stress should not exceed the allowable stress of the tool material.

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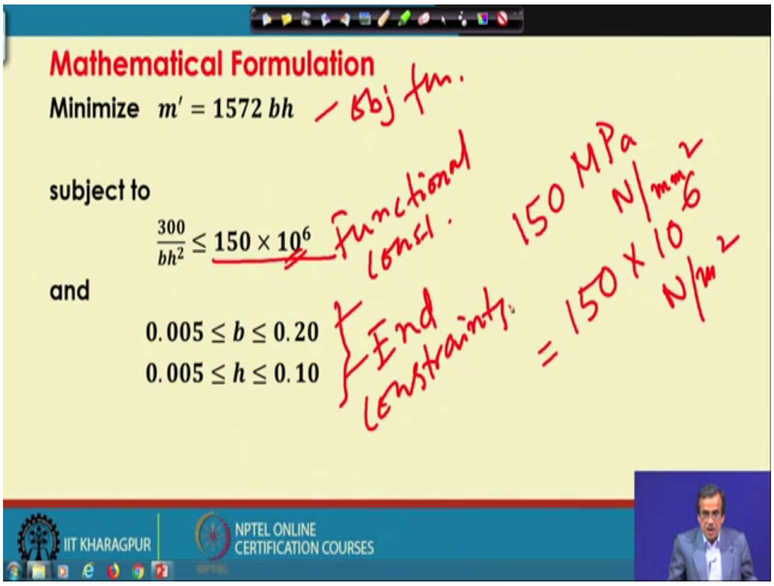

So, very easily we can mathematically formulate so, this particular optimization problem. So, our aim is to minimize m prime that is the mass of this particular cutting tool and that is m prime is 1572 bh subject to the developed bending state that is 300 divided by bh square is the less than equals to the allowable that is 150 raised to the multiplied by 10 raised to the power 6.

Now, here the allowable stress is 150 MPa is Mega Pascal that is Newton per millimeter square now this has to be converted to Newton per meter square. So, it is your 150 into 10 raised to the power 6 Newton per meter square that is why we have put these 10 raised to the power 6. So, the developed stress 300 divided by bh square should be less than equals to the allowable stress there is 150 into 10 raised to the power 6 and b is lying within this particular range h is lying within this particular the range.

So, this is nothing, but objective function, this is the functional constraint, if you remember. So, this is the functional constraint and these are nothing, but the variable bound or the end constraint or the geometric constraint. So, these are nothing, but the end constraints. So, this is the way actually we can mathematically state this constrained optimization problem.

And let us see once you have stated now how to solve it using some traditional tools for optimization and some nontraditional tools for optimization, the working principle of whom I have already discussed. So, I am just going to apply those tools which I have already discussed to solve this real world problem.

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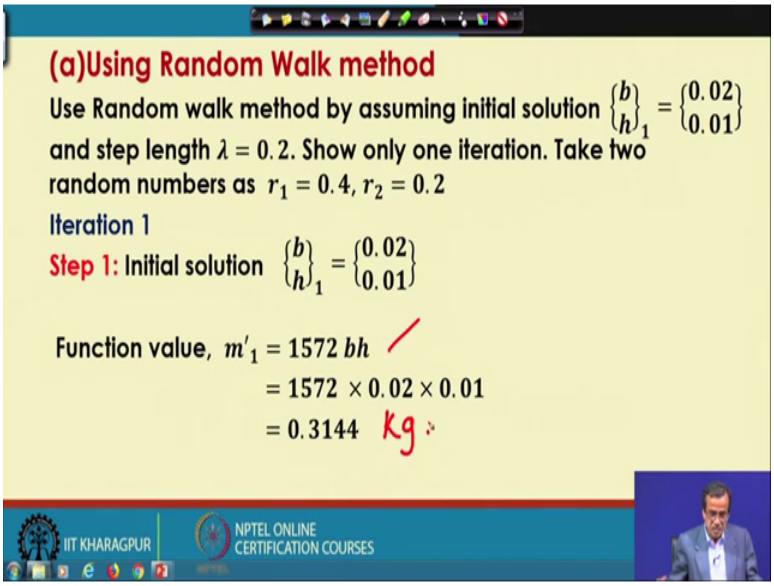

Now, let me start with the Random walk method which I discuss at the beginning of this particular course. Now, using this Random walk method actually the statement is we will have to use the random walk method to solve this minimization problem by assuming the initial solution which is selected at random b h 1 that is the initial solution is 0.02, 0.01 lying within the respective ranges we assume that the step length lambda equals to 0.2 and we are going to show you one iteration through hand calculation we take 2 random numbers r 1 equals to 0.4 and r 2 equals to 0.2.

Now, let us see how to proceed now these have discussed in much more details, but here I am just going to show you how to use that particular principle to solve one real the practical optimization problem. Now iteration one step one the initial solution is b h 1 is 0.02 0.01, now the function value m 1 prime corresponding to this b h 1; that means, I will have to take the help of the expression of the objective function that is 1572 multiplied by b h.

So, you just insert or substitute the value of b and h. So, you will be getting there is m 1 prime 0.3144 and the unit should be kg. So, this is actually the mass or the weight of this particular your; the cutting point cutting tool now once you have got this particular expression.

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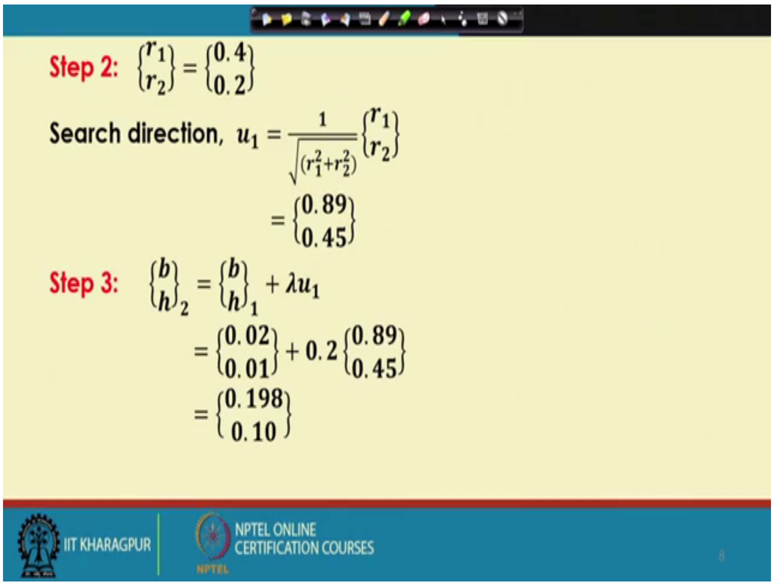

Now, we go for step 2, how to find out the step 2, we will have to find out the search direction by using another random numbers we will have to decide the search direction and the step length we have already assumed. Now, how do you find out the search direction? The search direction u 1 is nothing, but 1 divided by square root of r 1 square plus r 2 square multiplied by r 1 r 2 and if we just substitute the numerical values for r 1 and r 2. So, it will be getting.

So, this is nothing, but the search direction. So, the search direction is actually a vector decided by this now once you have got the search direction I can find out step 3 that is b h 2 that is what should be the second solution is nothing, but b h 1 plus lambda into u 1 now b h 1 is 0.02 0.01 lambda is 0.2 u 1 in the search direction and if you simplify. So, you will be getting this as your b h 2; that means, the next possible solution.

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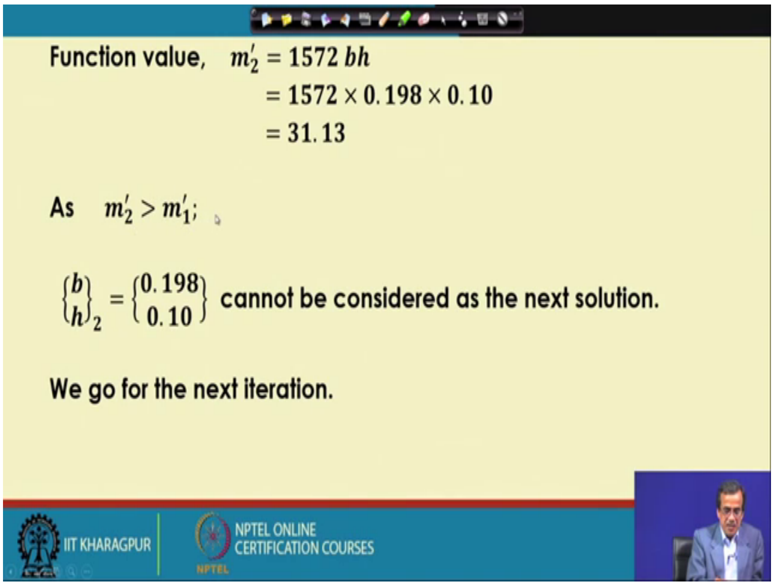

Now, we will have to take the decision whether we should select this as the next solution or not now to take this particular the decision actually what we do is we try to calculate the function value, the value of the objective function corresponding to b h 2 now if you substitute the values for this b and h. So, we will be getting 31.13 kg now if I just compare this m 1 prime and m 2 prime. So, this m 2 prime is greater than m 1 prime, but this is a minimization problem.

So, the next solution that is b h 2 is not a good solution and it is not acceptable and that is why we take the decision that b h 2 that is nothing, but 0.198 0.10 cannot be considered as the next solution and next is we go for the next iteration and once again what you will have to do, you will have to find out the search direction you will have to assume the values of the random numbers calculate the search direction and as this solution b h 2 is not selected.

So, what will have to do is. So, you will have to start with the b h 1 that is the initial solution and you will have to find out the random search direction and once again you will have to calculate what should be the b h 2 and you will have to go for checking and this process will go on and go on through a large number of iteration and there is a possibility that you will be getting some optimal solution.

But of course, there is no guarantee that if this algorithm will be able to hit this particular the globally optimal solution and moreover another thing which are not considered that is the functional constraint in this particular algorithm till now, now supposing that I am getting some feasible values for the b and h.

Now, if I get some feasible solution corresponding to which the value of the objective function is found to be less compared to the previously found based solution we accept provided the functional constraint is not violated; that means, we will have to check the functional constraint related to your the developed bending stress and if that particular condition is fulfilled then only finally, we can select that b h 2 as a the next solution.

But if there is a failure of the functional constraint we should not select because there is a failure. So, this is the way actually we can tackle this type of constrained optimization problem to solve using the principle of this particular the random walk method now as I discussed long back while discussing the principle of. So, this particular the random walk method. So, this random walk method has some merits and demerits.

The demerit is there is no guarantee that it is going to hit the globally optimal solution and unnecessarily it may take a large number of iteration to reach that particular optimal solution and it has got a merit what is that supposing that the objective function is having some discontinuity now if there is any discontinuity of the objective function. So, it is bit difficult to locate that optimal solution or the globally optimal solution.

But there so, this type of algorithm like random walk method may help to find out what should be the optimal or the globally optimal solution. So, this is the merit of this particular the algorithm and we have seen that the principle of random walk can be applied to solve a real world optimization problem.

Thank you.