**Traditional and Non-Traditional Optimization Tools**
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**Lecture - 26**
**Multi-Objective Optimization (Contd.)**

NSGA could achieve some popularity, but it has got a few drawbacks. Now to overcome these particular drawbacks the concept of this NSGA 2 came.

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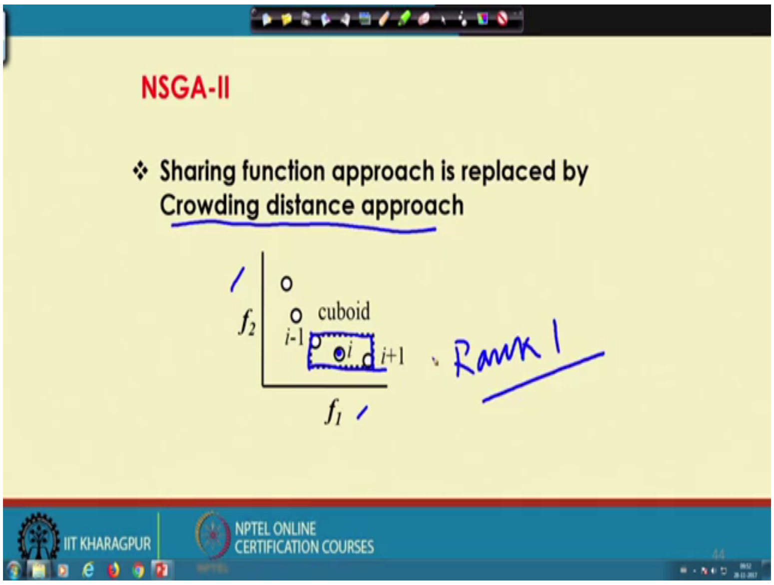


Now, in NSGA 2 actually what it do is the concept of sharing function approach that is replaced by the crowding distance approach.

Now, let us try to explain what do you mean by this crowding distance approach. Now what you do? So this crowding distance approach, we use in place of sharing function approach to maintain the diversity in the population..

Now here let me take one example supposing that I am just going to solve, the minimization problem involving 2 objectives f 1 and f 2. And supposing that this is the front one solution or the rank 1 solutions we have got.

So, these are all rank one solution. Now I will have to find out the solution for the mating port. Now here what you do is supposing that I am just going to concentrate on a particular solution and the i t h solution. And I want to find out what should be the crowding distance value corresponding to this particular the i th solution. Now what I do is. So, with respect to this i 1 with respect to this point i. So, whatever is lying towards my right that is i plus 1 and whatever is lying towards my left that is i minus 1?

So, these 2 points are considered. So, what I do is we try to form 1 cuboid. So, this is actually the cuboid surrounding this particular the point i and we try to find out the side lengths of these particular cuboid or we try to find out the perimeter of this particular the cuboid..

And actually what I do is the crowding distance value for the i th solution is nothing, but the sum of the side lengths of this particular the cuboid and by following the same principle, we try to calculate the crowding distance value for each of these particular the points.

And then we do the proportionate selection based on the concept of this crowding distance value. Now let us see let us see how to implement so, this particular the concept.

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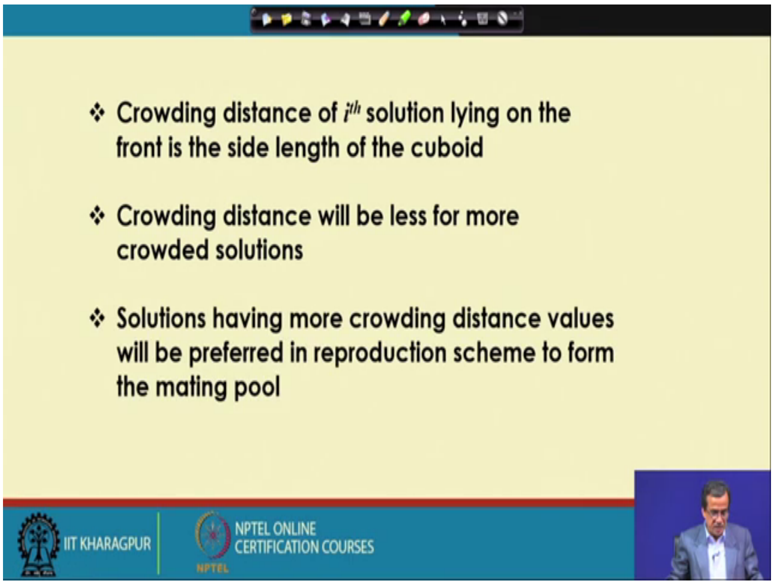


Now, if I consider the points which are too far the crowding distance value, the crowding distance value will be more for the points which are too far and the points, which are very close to each other the crowding distance value will be less; that means, the crowding distance value will be less for more crowded solution.

Now, if I just go back to the situation.

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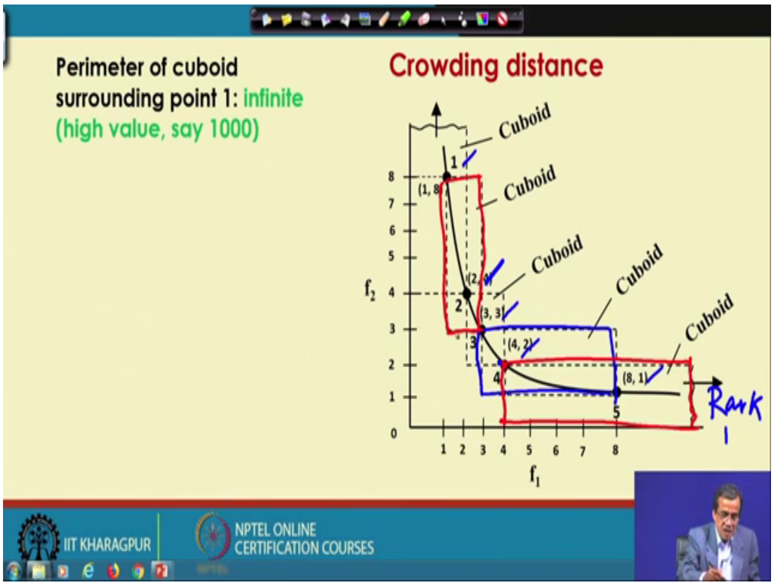


So, this particular situation let me concentrate. Now here I am just going to consider there are only 5 solutions here on this rank 1. Now the crowding distance value for this particular the point 1. So, that will be more because if I want to just construct 1 cuboid. So, this is actually the cuboid and this will go towards infinity. So, the crowding distance value for this particular point 1 will have very high value compared to this particular point 4.

Now, if this for this particular 4 we have got this particular keyword something like this whose crowding distance value will be less. Now this particular example I will take after some time, but before that let me try to concept in on this particular the idea. The crowding distance value of i th solution lying on the front is the side length of the cuboid, which I have already discussed and the crowding distance value will be less for more crowded solutions. And for the endpoints of the crowding distance value will be very high.

So, the solutions having more crowding distance values will be preferred in the reproduction scheme to form the mating pool. And let me let me try to summarize whatever I told, supposing that I have got say 13 points on a rank 1. So, for each of the 13 points we try to find out the crowding distance values, now the points which are too far or the end points the crowding distance value will have high value and the points which are very close to each other the crowding distance value will be less.

Now, during this reproduction the proportionate selection, if I give more preference to the solution which are lying on the side of this particular front; that means, which are too far there is a possibility there will be multiple copies of these particular points in the mating pool. And if I can ensure a large number of end points in this particular the mating pole, there is a possibility that the diversity of this particular population will be maintained.

Now, this is the way actually, we maintain this particular the diversity. Now if I compare the concept of this particular crowding distance with that of the sharing functions. Now here the computational complexity will be much less compared to that of this concept of the sharing function. So, using this concept of your the crowding distance this algorithm will be much faster.

Now, to explain the working principle of this particular crowding distance, let me solve one numerical example. Now this is a very simple example and once again the problem is I am just going to minimize the 2 objectives f 1 and f 2 and supposing that this is the rank 1 solutions, this is the rank 1 solution and here I have got only 5 solutions like 1 2 3 4 and 5 and I have shown here the coordinates also.

Now, if I want to find out what should be the cuboid corresponding to a particular solution say solution 4? Now surrounding the solution 4 the cuboid will be something like this. So, this is the cuboid, because here I have got solution 5 solution 3 here. So, this particular solution 4 will be surrounded by this particular the cuboid..

And once I have got this particular cuboid. So, very easily I can find out the side lengths and we sum all the side lengths values. So, I will be getting the crowding distance value for this particular the point 4.

Similarly, if I want to find out the crowding distance value for this particular the crowding distance value for this particular point 5. So, what I do is we try to find out whatever is lying towards the left of this particular point 5 that is nothing, but point 4 and towards the right. So, there is nothing..

So, we can assume that there is a point in infinity. So, the crowding distance value if I want to calculate. So, I will have to consider. So, this type of actually the cuboid and here actually there is no end because this is moving towards the infinity. That means, the side length values for this particular cuboid the sum of the side lengths value will be very high and let me assume a very high value 1000 2000 something like that. Now similarly following the same principle I can find out the crowding distance value for point 1.

Once again it will have a very high value; because this side the next point it is assumed that it is in infinity. So, it will have a very high value for the crowding distance, then if I concentrate on point 2. So, I will have to find out the crowding distance value by considering this particular the cuboid.

So, following this I can find out the crowding distance values for each of these particular the 5 points. Now whatever I discussed the same thing actually I have put it here.

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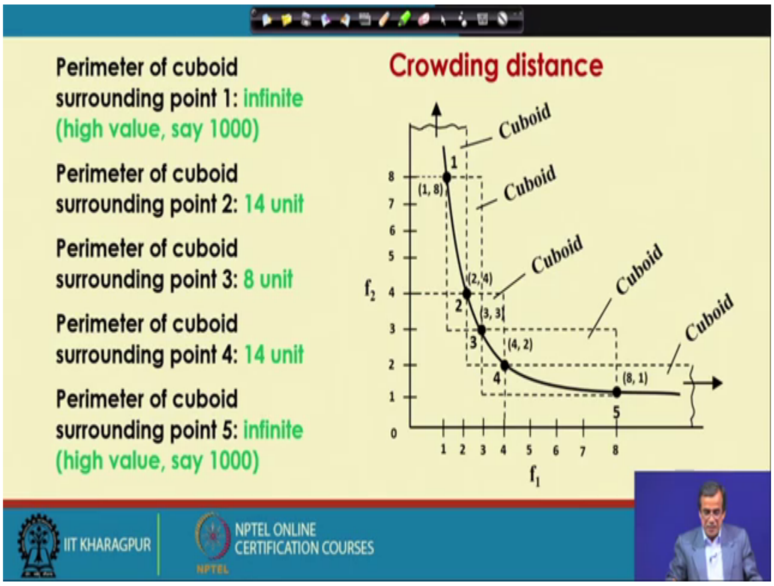


Now, for the point 1 the crowding distance value will be very high and let me consider it is infinite do it will have infinite value let me consider this is 1000 for simplicity. Now perimeter of cuboid surrounding point 2 surrounding point 2 the perimeter if we calculate this will become equal to 14 unit.

Similarly, corresponding to cuboid surrounding point 3, this is point 3. So, this particular cuboid and it is perimeter is 8 unit. Similarly the perimeter of cuboid surrounding point

4, this will become equal to 14 units and the perimeter of cuboid surrounding point 5, will have very high value and let me assume that this is once again equal to 1000. So, these are all actually the crowding distance values and as I told the points which are having the more crowding distance values like point 1 and point 5.

So, if we give more preference to those points in the mating pool during the proportionate selection, there is a possibility, there will be more copies of point 1 and point 5 in the making pole; that means. So, this particular part and these particular points will not be lost from the population and there is a possibly that I will be getting I will be able to maintain, the whole stretch of this particular the Pareto optimal front of solution.

And there is no chance that a part of this particular Pareto optimal front will be lost and we will be able to maintain actually, the diversity of this particular solution or the stretch of the Pareto optimal front of solution; that means, we will be getting more options more number of feasible optimal solutions we will be getting.

So, this is the way actually the concept of the crowding distance works just to maintain the diversity in this particular the population.

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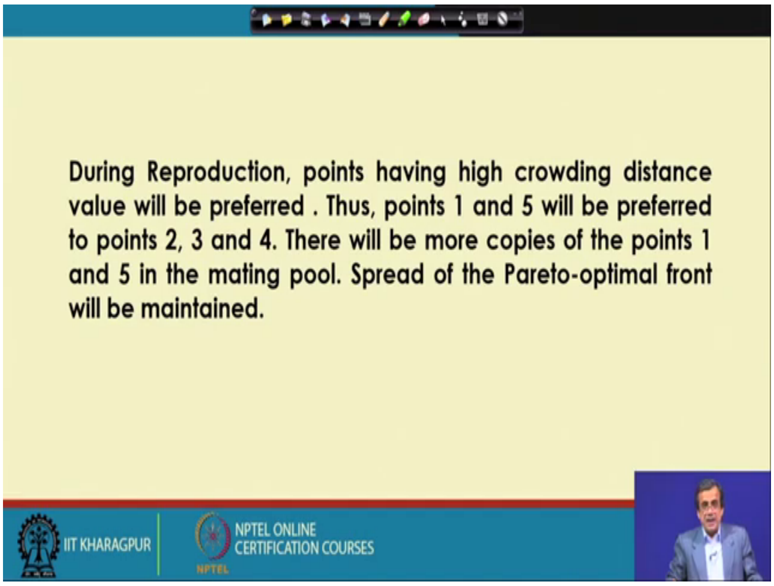


Now, if I compare the computational complexity of this particular crowding distance calculation and that of sharing function calculation. It is obvious that this particular calculation of crowding distance values will be much faster compared to that of sharing function value..

And that is why this particular algorithm will become faster compared to your the algorithm using the sharing function value this is the concept of in NSGA 2.

And NSGA 2 is expected to be faster compared to this particular NSGA and moreover there is another thing which was missing in NSGA that has been incorporated here that is the principle of elitism. So, in NSGA 2 the principle of this elitism has been used and moreover, it has been made faster compared to NSGA and that is why actually NSGA 2 become more popular compared to NSGA and nowadays this NSGA 2 is in use many people have used and it has been further modified also.

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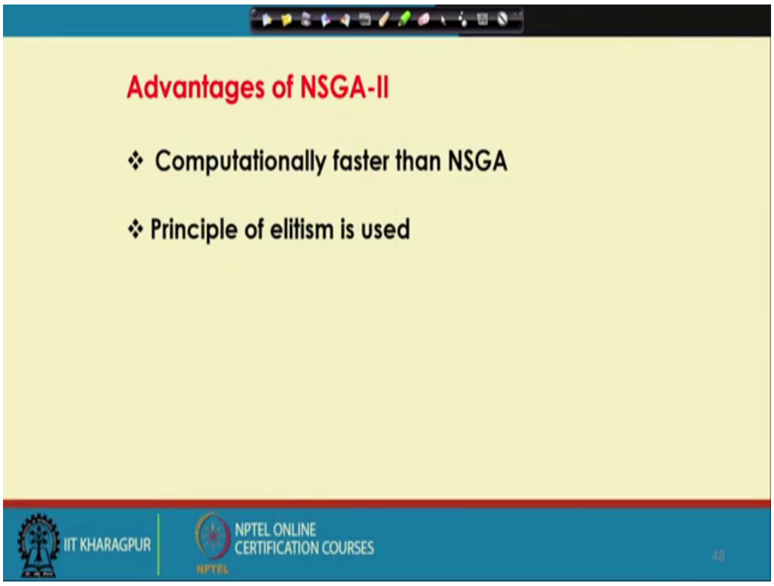


Now, the advantages of NSGA 2 I have already mentioned that it is computationally faster than NSGA and the principle of elitism has been used in this particular your NSGA 2. Now using this NSGA 2 there is a possibility that we will be able to find out the Pareto front of solutions..

Now actually if you see the literature, there are a few other algorithms just to tackle this multi objective optimization problems particularly the problem having either 2 or 3 objectives.

For example say we have got the algorithms like multi objective party particle swarm optimization that is called AMPSO, then we have got multi objective simulated annealing MOSA, we have got SPA algorithm, we have got PAES algorithm and there are some other algorithms..

So, this particular literature is huge, but the basic principle which I have already discussed. So, those principles only with little bit of modifications have been used to design and develop the tool for multi objective optimization.

So, as I told there are a large number of algorithms available to solve the problem involving either 2 or 3 objectives there is of course, a chance of further improvement of these particular the algorithms.

Thank you.