Traditional and Non-Traditional Optimization Tools
Prof. D. K. Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 24
Multi-Objective Optimization (Contd.)

Now, I am going to start discussing the principle of another approach which is known as distance based pereto GA in sort DPGA.

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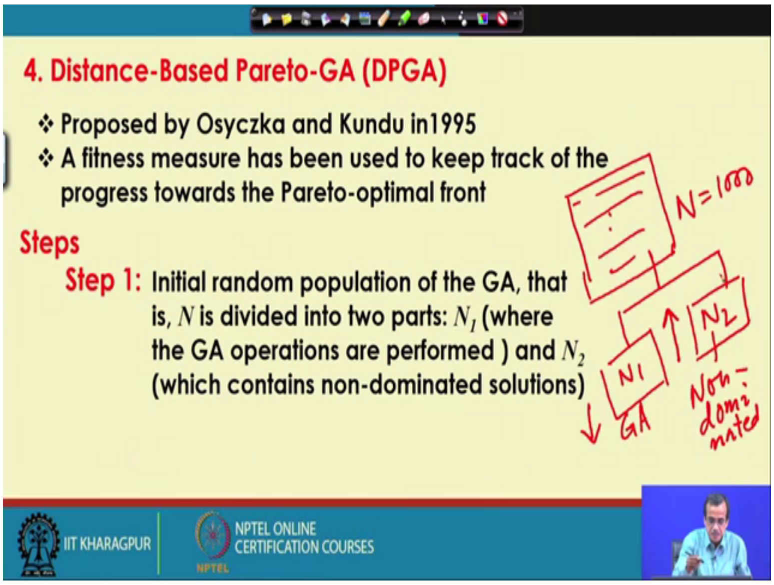

Now this DPGA is actually found to be efficient to find out that the pareto optimal front of solution and we can also solve this particular multi objective optimization problem efficiently.

Now, let us try to see the principle of this particular the DPGA. Now this DPGA it was proposed by Osyczka and Kundo in the year 1995 and here we use a fitness measure to keep track of the progress towards the pareto optimal front. Now here what he do is if you see step 1. The initial random population of the GA that is capital N is divided into 2 sub populations N1 and N2.

Now, N1 is a sub population where we use all the GA operators and N2 is nothing but a sub population which contains all the non-dominated solution. Now to explain it let me let me just draw this particular the population, supposing that this is the initial population

N, capital N might be N equals to 1000 and this is generated at random the solution is generated at random.

So, what I do is so this is divided into 2 parts, one is N1 one is sub population N1 and another is sub population N2. Now this N1 actually all the GA operators like reproduction crossover and mutation they are going to walk on this particular sub population. And N2 contains all the non-dominated solution, non-dominated solutions are contained in the your N2.

Now, N1 as I told the GA operators are going to work; that means, through this GA operation it will try to find out some diversification in the population and there is a possibility with the iteration, the size of this particular N1 is going to decrease and the size of this particular N2 is going to increase. How to do that I am going to discuss now, but this is the thing what we do in this particular the DPGA.

So, once again let me repeat we start with N that is divided into 2 sub population in one and N2 on N1 the GA operators will be working to find out diversification and it is going to supply some solution to N2, the size of the N2 is going to increase with the number of iterations and gradually the size of N1 is going to decrease and N2 the size of N2 is going to increase and we will find that this N2 will contain all the non-dominated solution.

So, we will be getting at the end a large number of non-dominated solution and with the help of this non-dominated solution we can draw the pareto optimal front of solution. Now let us see how does it work..

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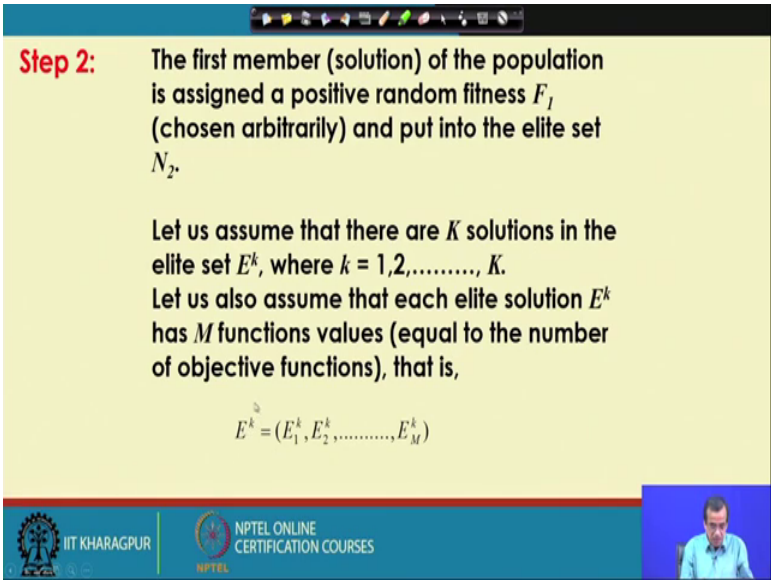

now step 2 the first member or the solution of the population is assigned a positive random fitness f 1. So, capital f 1 the random fitness that is chosen arbitrarily at random say might be it is 10 20 30 or it could be 100 1000 something like this and that is put into the elite set that is N2 and that is nothing but the non-dominated set.

Now, as I told that the first solution is selected at random from the initial population and that is just put in the elite set there is a non-dominated set that is N2 after assigning the fitness f 1. Now let us suppose that after some time or after a few iteration there are capital k number of solutions in the elite set E k or k varies from small k varies from 1 to up to capital K.

And let us assume that I am solving one multi objective optimization problem having capital N number of objectives and if that is the case then this particular E k will have capital M number of numerical values. So, E k is nothing but the k th solution lying on the non-dominated front and this small k varies from 1,2 up to capital K and to represent a particular non-dominated solution that is the k th one I am using capital m number of numerical values because it has got capital N number of objective functions. So, this is corresponding to the first objective, corresponding to the second objective, corresponding to the M th objective and supposing that we have got this particular the information.

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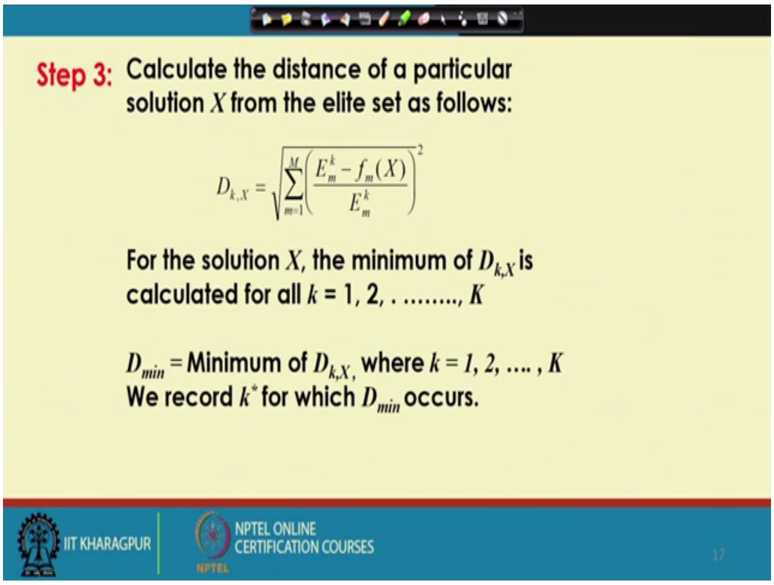

Now, step 3 we calculate the distance of a particular solution X from the elite set as follows supposing that I have got a few elite solution in the non-dominated bin. Now I am just going to select a particular solution from the initial bin the initial population of solution and I am just bringing it to the non-dominated bin for the purpose of comparison and what is the purpose of comparison? The purpose of comparison is whether I should select this particular solution for the non-dominated bin or not.

Now, what it is we try to find out the Euclidean distance between the k th solution lying on your the non-dominated bin and the new solution that is capital X using this particular formula that is square root of summation small m equals to 1 to capital M E m k minus f m X divided by E m k square and using this particular expression actually what I can do is so I can find out what should be this particular the Euclidean distance values between a solution capital X and one elite solution.

Now this f m X is nothing but is actually the value of the objective function. Now an E m X is nothing but it is corresponding value which is there in the. Elite now what do you do is supposing that I have got only one elite solution. So, there will be only one distance values.

But if I consider that we have got more than one solution in the elite set. So, I will be getting more than one Euclidean distance values. Now supposing that I have got a 10 solution in the elite set for the non-dominated set. So, I will be getting 10 such numerical values for this Euclidean distant.

And after we have got 10 such values what we do is we try to find out Euclidean distance and compare those particular the numerical values. Now we compare the numerical values of this particular D k X and we try to say for a which value of this particular k I am getting the minimum of the Euclidean distance values; that means, we try to identify what is D minimum that is the minimum of all D k X values where k varies from 1 2 up to K and we try to locate that particular value of k for which I am getting this particular the D minimum.

Let me take one example supposing that I have got 10 such solution in the non-dominated bin 1 2 up to say 10, I am just selecting a particular solution from the initial population that is brought for the purpose of comparison we calculate Euclidean distance values I will be getting 10 values for this particular D k X, we compare those numerical values and supposing that out of this particular 10 say corresponding to the 6th elite solution or the non-dominated solution I am getting the minimum D k X that is nothing but D minimum and k star will be equal to 6 solution.

So, I think it is clear now.

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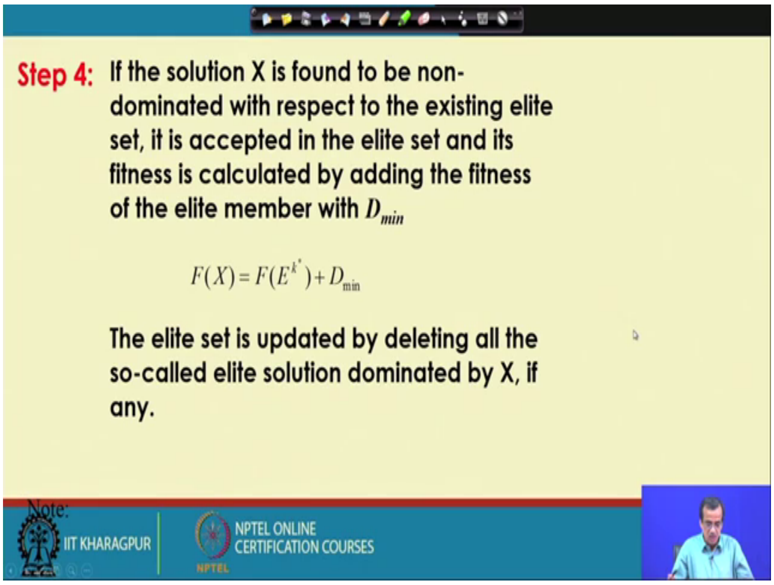

Now, step 4 if the solution X is found to be non-dominated with respect to the existing elite it is accepted in the elite set and it is fitness will be calculated as follows. So, this is the formula which is generally used to find out the fitness of the solution which has been selected for the non-dominated bin. So, if capital X if it is found to be suitable for the non-dominated bin. So, it will be put in the non-dominated bin and it is (Refer Time: 11:29) fitness will be calculated using this particular the formula.

So, how to select that particular how to get this particular formula. So, this F E k star is nothing but the value of the objective function corresponding to that particular elite which gave that particular the minimum distance value and this particular fitness is added to the D min that is the minimum distance value just to find out what should be the fitness for the solution X which has been selected for the non-dominated bin.

Now, on the other hand if this particular solution is not selected let us see what happens.

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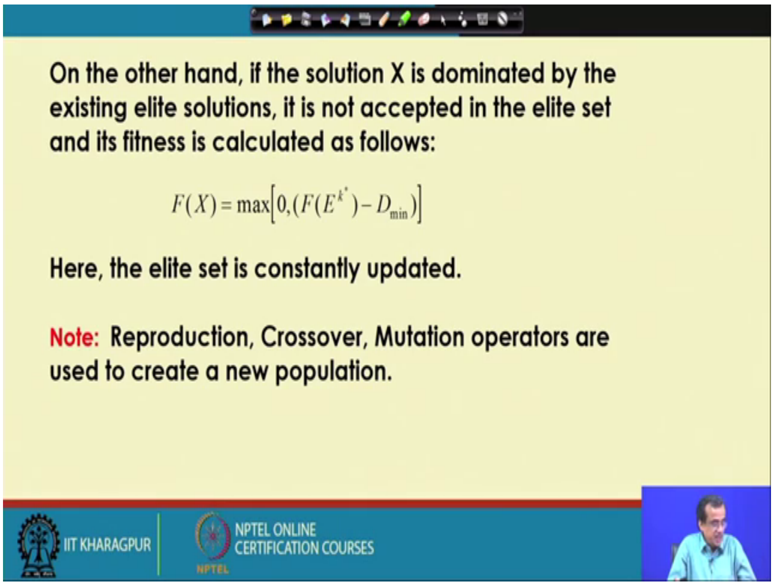

If the solution x is dominated by the existing elite solution; that means, it is not accepted in the non-dominated set or the elite set then how to find out or how to calculate the fitness. So, if x is nothing but the maximum between 0 comma F E k star minus D minimum.

So, what do you do is we try to calculate this and we compare and we find out the maximum between these 2, that will be actually the fitness assigned to the capital X if it is not selected in the elite set. Now just to summarize whatever I discuss let me just put in this particular format. Now I have started with a large number of population of size say capital N, say N could be one thousand the first solution you select at random from this particular population and you just put it in the non-dominated bin that is N2.

And we assign some arbitrary fitness say 1000 or 100 whatever maybe, now we select the second one we bring it for the purpose of comparison and we try to find out the Euclidean distance between the second solution and the non-dominated solution. And we try to find out by comparing it is fitness value f 1 and f 2 whether it is a dominated solution or a non-dominated solution.

If it is found to be non-dominated solution so we put it in the elite set or the non-dominated set and it is fitness is calculated using this particular the formula F X is F E k star plus D minimum. On the other hand, if the capital X is found to be dominated; that means, there is a what solution compared to the existing non-dominated solution. Now in that case what we do is we do not select X for the non-dominated bin and it is fitness is calculated using this particular the formula and this process will go on and go on and we will be getting some N1 subpopulation solution and some other solution in the N2 that is the non-dominated set.

And as I told that this process will go on and go on through a large number of iteration. And of course, we use the operators. So, we use the operators like cross over mutation to modify the population of solution and this completes one iteration and this particular iteration will go on and go on and there is a possibility the size up N1 sub population is going to decrease as I mentioned and consequently the size of N2 which contains the non-dominated solution that is going to increase; that means, if I run this particular algorithm for a large number of iteration there is a possibility that this algorithm will become slower and slower with the number of iterations.

Because to store that particular N2 sub population more storage space is required in the computer and gradually. So, the later iteration will take more CP time compared to the previous one. So, if you run this particular algorithm for a large number of iteration there is a possibility the algorithm will become slower and slower this is actually one of the drawback of this particular the algorithm. Otherwise there is a possibility that ultimately you will be getting a very good collection of N2 sub population which contains all the non-dominated solution and we can plot just to get that particular pareto optimal front of solution.

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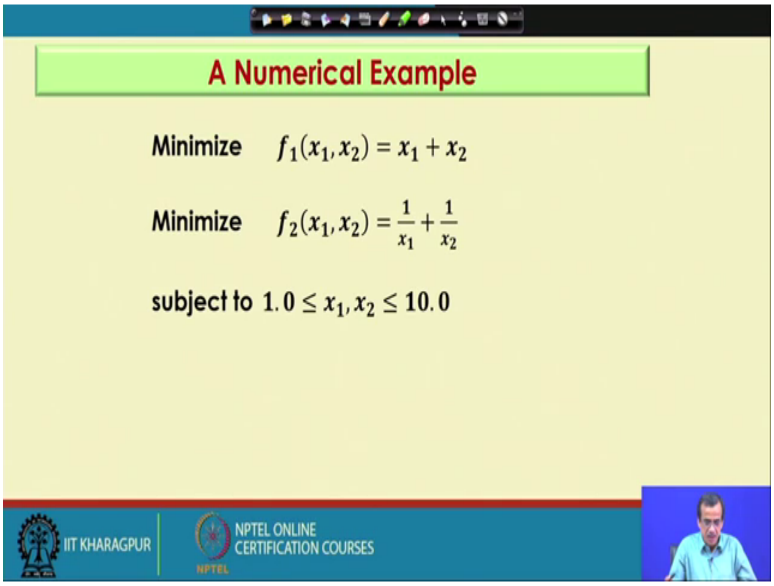

Now, I am just going to discuss so this particular algorithm with the help of one numerical examples. Now let us see how does it work. Now supposing that I have got a 2-objective optimization problem the aim is to minimize the function f 1 function of 2 variable x 1 and x 2 and that is nothing but x 1 plus x 2 and the second objective function minimize f 2 the function of the same design variable x 1 and x 2 is nothing but 1 by x 1 plus 1 by x 2 and x 1 x 2 are lying between 1 and 10.

Now, let us try to examine the nature of these 2 objectives now the method is such. So, this is the past is x 1 plus x 2 and the second objective is 1 by x 1 plus 1 by x 2; that means, for a particular set of value I will be able to calculate f 1 and f 2. Now if I just increase the values of this x 1 and x 2 there is a possibility f 1 is going to increase, but at the same time f 1 f 2 is going to decrease. So, what is happening is these 2 objectives are fighting with each other they are conflicting.

So, these are ideal problem to get the pareto optimal front of solution. Now let us see how to solve this type of 2 objective optimization problem with the help of this DPG. Now what do is lying within the range for the design variables.

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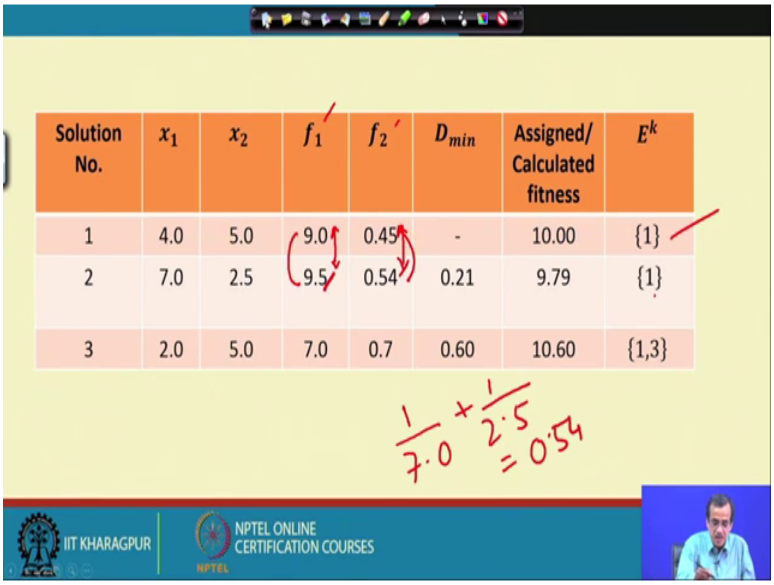

Solution No.	x_1	x_2	f_1	f_2	D_{min}	Assigned/ Calculated fitness	E^k
1	4.0	5.0	9.0	0.45	-	10.00	{1}
2	7.0	2.5	9.5	0.54	0.21	9.79	{1}
3	2.0	5.0	7.0	0.7	0.60	10.60	{1,3}

Let me assume some numerical value for this x 1 and x .2 now x 1 and x 2 are lying between 1 and 10.

Now, let us consider x 1 equals to 4.0 and x 2 equals to 5.0. Now what we do is we just substitute the values of x 1 and x 2, here values of x 1 and x 2 here. So, I will be getting the numerical value for this particular f 1 and f 2. And as I told the first solution is selected at random for this particular the non-dominated bin.

So, what I do is. So, here we select this particular solution for the non-dominated bin. So, it is going to lie in that particular E k subpopulation. So, this is nothing but the non-dominated subpopulation or the non-dominated bin. So, one is going to enter here and we assign some fitness value arbitrarily.

Let me put it is 10.00 I can also put 100 also there is no problem, but let me put 10.00 here. So, the first solution is selected at random and that is put here in the non-dominated bin, now we go for the second solution. Now once again these are all generated at random. So, I am selecting at random a particular value for x 1 that is 7.0 another value for x 2 that is 2.5 and using the same expression of the objective function I can calculate f 1 that is nothing but x 1 plus x 2. So, this is 9.5 and f 2 is nothing but one divided by x 1 plus 1 divided by x 2; that means, your one divided by 7.0 plus 1 divided by 2.5 and if you calculate we will be getting 0.54.

So, this is nothing but the value of the second objective. Now if I compare our aim is to minimize both f 1 and f 2. Now if I compare in terms of f 1 so the elites is having 9.0 this is the elite solution, now non-dominated solution and the second solution is having 9.5; that means, second solution is a worse solution compared to the non-dominated solution that is one now I compare the objective function value f 2 and corresponding to the second solution it is 0.54 corresponding to that elite or the non-dominated it is 0.45.

Once again second solution is a worse solution in terms of f 2; that means, solution 2 is found to be dominated or the worse compared to the non-dominated solution that is 1 in terms of both f 1 and f 2. So, this is a bad solution this would not enter in the non-dominated bin and that is why the second solution I did not put it here and you see the elite set will contain only the first solution.

Now, once I have not put then how to find out the D minimum? And how to find out the fitness? That I am going to discuss to find out the D minimum as I told. So now, I will have to compare these 2 these 2 in terms of f 1 and f 2; that means, I will have to compare 9.0 is 9.5, 0.45 with 0.54 and I will have to find out what is that particular the Euclidean distance value.

How to get it? So, I am just going to discuss that to get that particular the Euclidean distance values actually what we do is.

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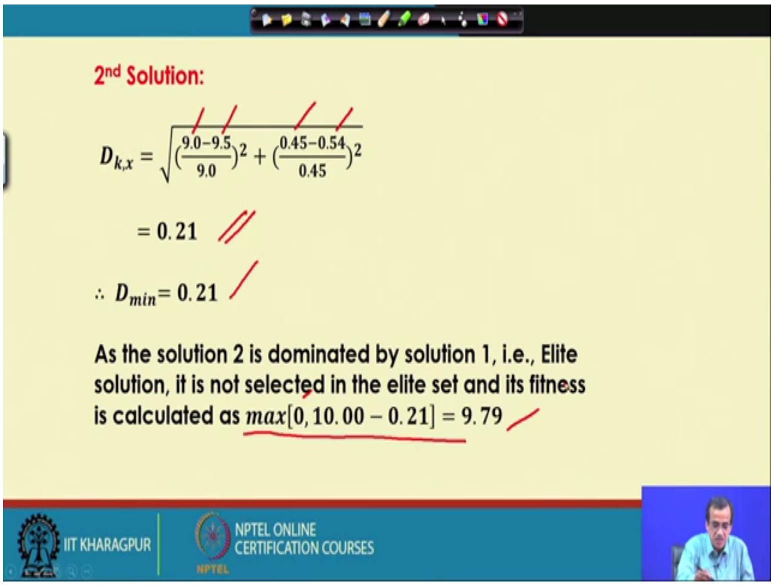

So, we try to find out this D k X that is nothing but square root 9.0 minus 9.5 divided by 9.0 square, the formula I have already discussed. So, 9.0 is corresponding 2 actually the elite one that is a non-dominated one and 9.5 is corresponding to the solution capital X the new solution that is the second solution.

Similarly, 0.45 is corresponding to the non-dominated and 0.54 the value of the second objective this is corresponding to the second solution. So, we try to find out the Euclidean distance value and it is coming to be equal to 0.21. Now as I told there is only one elite solution till now. So, there will be only one Euclidean distance value and that is nothing but the D minimum..

So, D minimum we are able to calculate that is 0.21 and as this particular solution is not selected for the elite set it is a dominated solution and it is fitness will be calculated using that particular formula which have already discussed that is nothing but the maximum between 0 comma E k star if we remember that is nothing but the fitness corresponding to the non-dominated or the elite minus D minimum.

Now if I calculate this. So, I will be getting 10 minus 0.21 is 9.79 and the maximum between 0 and 9.79 is nothing but 9.79. Now I can assign this particular the fitness to the second solution; that means, I will be getting so this particular 9.79 here.

So, this particular fitness I will be able to calculate and I will be able to calculate the D minimum and as I told this is not going to enter the non-dominated bin. Now let me concentrate on another solution that is the third solution and once again this is selected at random the values of x 1 and x 2 lying within the respective ranges and very easily I can find out f 1 is x 1 plus x 2 f 2 is 1 by x 1 plus 1 by x 2, 0.7.

And now I will have to compare the values of corresponding to the solution 3 with that of solution 1 that is the non-dominated one in terms of both f 1 and f 2; that means, I will have to compare this 9.0 and 7.0, similarly I will have to compare 0.45 and 0.7 and both are minimization. So, in terms of f 1 solution 3 is a slightly better solution compared to solution 1; that means, solution 3 is a non-dominated solution compared to solution 1 in terms of f 1.

But in terms of f 2 solution 3 is the worse compared to the solution 1. Now if this is the situation. So, in with respect to one objective solution 3 is a non-dominated, but with respect to another objective solution 3 is a dominated one there is a worse compared to solution 1. Now in that case we give a chance to solution 3 to be there in the non-dominated front. So, that I just we just want to give some chance initially.

Because there is a possibility through a large number of iteration this particular solution is going to improve itself and we do not know ultimately it could be a very good solution in future, that is why we just want to give some chance to this particular solution although it is not found to be non-dominated compared to solution 1 in terms of both the objectives. So, what we do is we give a chance because it is found to be better compared to solution 1 with respect to at least one objective and we put it in the non-dominated mean; that means, solution 3 is put in the non-dominated bin.

So, there is updating of this particular the non-dominated bin. Now how to calculate the Euclidean distance and the fitness? That I am going to discuss..

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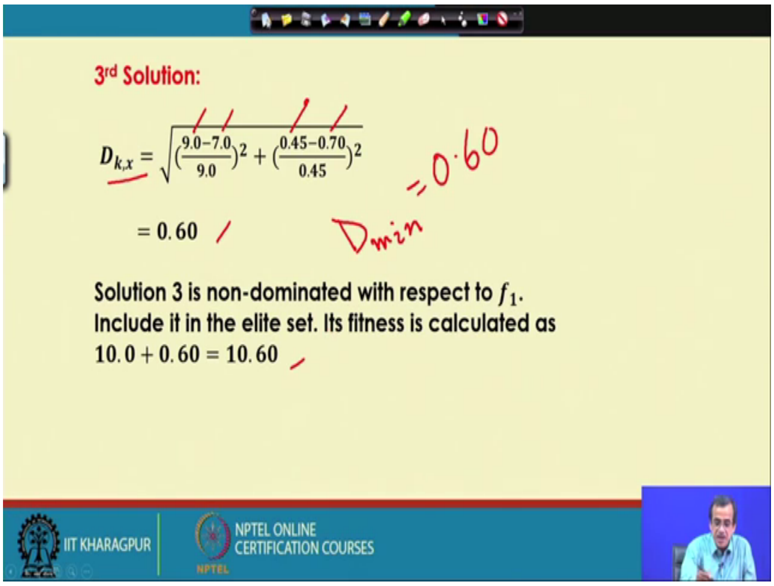

So, here as I told that we will have to compare with the elite that is nothing but your this 9.0 corresponding to the first solution and 7.0 is corresponding to the third solution. So, 0.45 is corresponding to the first solution value of f 2 and 0.70 is the value of f 2 corresponding to the third solution and if we just calculate. So, I will be getting this particular D k X and I will be getting once again only one value for the Euclidean distance that is coming to be 0.60.

So, this is nothing but the D minimum. So, the D minimum we can declare is nothing but equals to 0.60 and the solution 3 is found to be non-dominated with respect to at least one objective. So, we put it in the non-dominated set or the elite set and it is fitness will be the fitness of the elite plus the D minimum there is 0.60. So, this will become 10.60.

So, what do you do is we put this particular.

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We put this particular your the values of the Euclidean distance and the fitness in this particular the table. Now what do you do here I have considered only 3 solution, similarly this method will go on and go on and it will go to all the solutions checking for all the solutions lying in the population and gradually you can find out. So, this particular sub population is going to size of this is going to increase, and the solutions which are not selected for the non-dominated bin they will be put in another sub population there is a N1 sub population which will participate in cross over. So, this is the crossover and mutation.

Now, this particular process will go on and go on through a large number of iteration and ultimately, we are going to get a very good non-dominated front that is N2 sub population and if I just plot this N2 sub population at the end of a large number of iteration there is a possibility you will be getting the pareto optimal front of solution. So, this is the way actually using this particular method..

So, we can find out the pareto optimal front of solution, but one demerit which I have already mentioned it is computationally expensive and with the as the iteration proceeds the complexity computational complexity of this particular algorithm is going to increase, that is actually the major drawback of this particular the algorithm.

Thank you.