Traditional and Non-Traditional Optimization Tools

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Lecture - 22
Multi-Objective Optimization

I am going to start with another topic when that is Multi Objective Optimization. Now till now we have discussed a number of optimization problems, and how to solve it and each of these optimization problem is having only 1 objective. Now these problems are known as single objective optimization problem, now I am just going to concentrate on a few others optimization problems in which we have got 2 or more than 2 objectives. Now these problems are known as the multi objective optimization problems.

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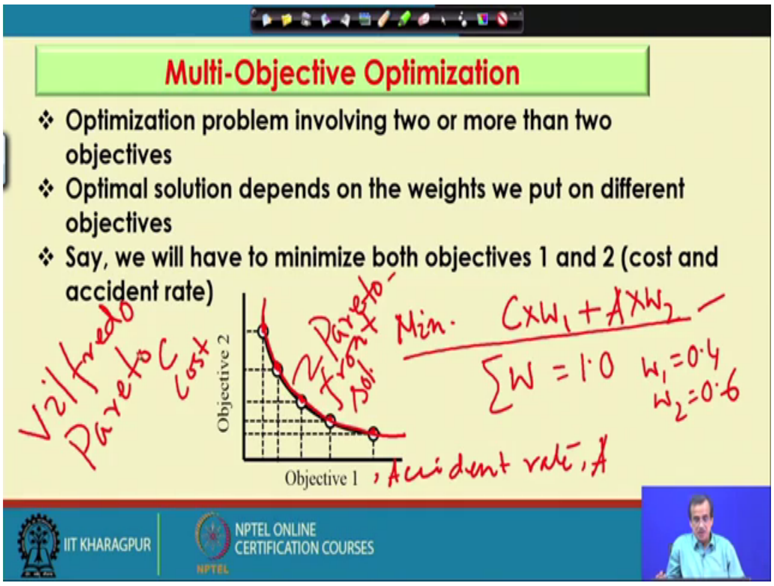

So, multi objective optimization problems are problems involving 2 or more than 2 objectives, now let me take 1 example, supposing that mister X is going to purchase a car. Now, before he takes the decision which model to be purchased, he generally tries to find out some information of the different models available, and while collecting information so, he will try to find out the information regarding, the cost of a car and its accident rate..

Now supposing that so, here I am just going to consider 2 objectives, now Mister X will try to select a car at the cost of a minimum amount, and at the same time the accident rate should be minimum. So, if I consider. So, these 2 objective optimization problem, now let me let me just concentrate supposing that I am just going to plot, the first objective that is say the accident rate accident rate, and the second objective is a cost of a car. So, aim is to minimize both the cost as well as the accident rate.

Now, the cost supposing that this is denoted by C, and accident rate its denoted by A, now aim is to minimize both cost as well as the accident rate. So, what he will do is he will try to formulate another optimization problem, single objective optimization problem, and that is nothing, but minimize the cost multiplied by a particular factor W 1 plus the accident rate A multiplied by another factor W 2 where W 1 W 2 will lie in the range of 0 to 1, and sum of the w values will be equal to 1.0. Now if I just set say 1 set of values for this W 1 and W 2, let be considered say W 1 is a 0.4, and W 2 equals to 0.6, and if I solve this single objective minimization problem. So, there is a possibility that he will be getting a particular solution say this particular solution say..

Now, if you changes the values of this particular objective the values of the like W 1 W 2 to some other numerical values, there is every possibility by solving this single objective minimization problem, he will be getting another solution. Similarly by selecting the different sets of values of W 1 and W 2 so, he will be getting different such solution. So, these are all optimal solutions in some sense now after than what he does he will try to put or he will try to feet a car, now that particular car is nothing, but the locus of all optimal solutions. Now this is nothing, but is actually the pareto front, this is known as the pareto front of solution.

Now, this pareto front of solution is actually a front consisting on a large number of solutions or each of the solutions are nothing, but the optimized solutions, and now the user can select any 1 depending on his choice. Now each of these particular optimal solution is nothing, but the model of a particular car, now mister X is going to select any 1 out of these optimal solution..

So, this is the way actually he can solve, this type of 2 objective optimization problem. now this front is known as the pareto front of optimal solution or pareto optimal front of solution, according to the name of its proposer he is he is a vilfredo pareto vilfredo pareto, and according to his name. So, this particular the front is known as pareto optimal front of solution or pareto front of optimal solution.

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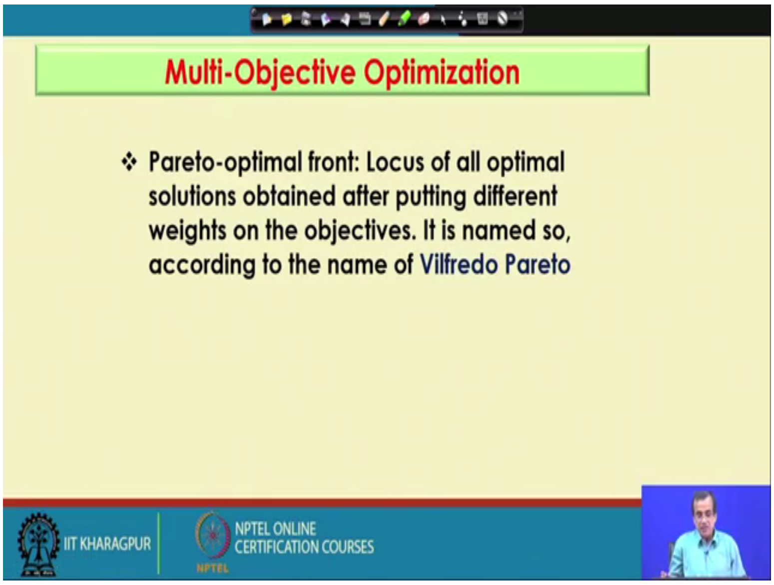

Now, here the same thing I put it the pareto optimal front of solution is nothing, but the locus of all solutions, the locus of all solutions obtained after putting different words of the objectives, and this has been named according to the name of the proposer that is vilfredo pareto.

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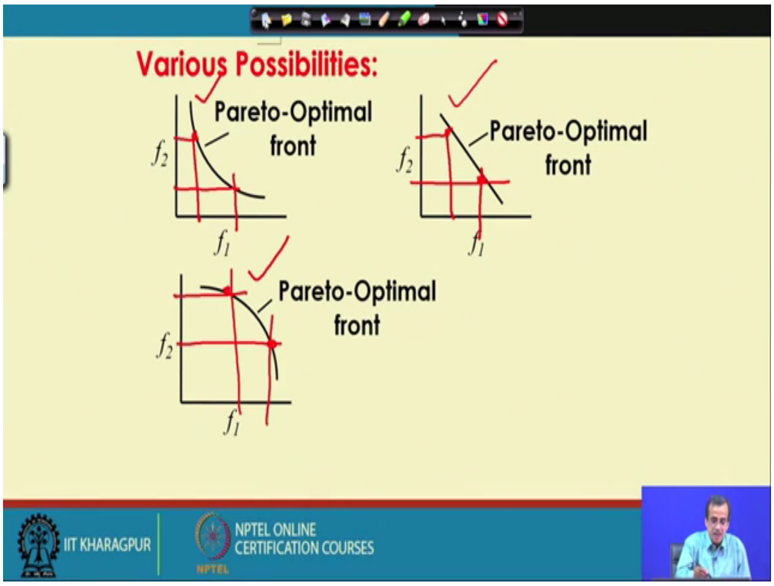

Now, here so I have considered only 2 objectives, and our aim is to be minimize both objective 1 that is f 1 and objective 2 that is f 2. Now here actually what you do is we try to minimize both f 1 and f 2, supposing this is cost and accident rate, now if I select e particular point on this particular front. So, this is my f 2 and this is f 1, now if I consider the higher value of this particular f 1.

So, I will be getting the smaller value for this particular f 2 now; that means, these 2 objectives are fighting with each other they are conflicting, and if they are found to be conflicting, and if there is a tradeoff between f 1 and f 2 so, then only we will be getting so, this type of pareto optimal front of solution..

Now let me take another example, now here I am just going to show a another variation for this particular f 1 and f 2, now corresponding to this particular poin. so, this is my f 2 this is f 1 corresponding to this particular point so, this is my f 1 and this is f 2. so as f 1 increases. So, f 2 is going to decrease so, they are conflicting. So, this is another valid pareto optimal front of solution. So, this is the valid pareto optimal front of solution, this is another valid pareto optimal front of solution, now here I am just going to show you another possibility. Supposing that so, this type of car, now if I select in particular point lying on this particular car. So, I can find out f 1 and f 2.

Similarly, another point if I consider I can find out f 1 and f 2. So, here as f 1 increases f 2 is going to be reduced, and once again they are fighting with each other, and this is another valid pareto optimal front of solution..

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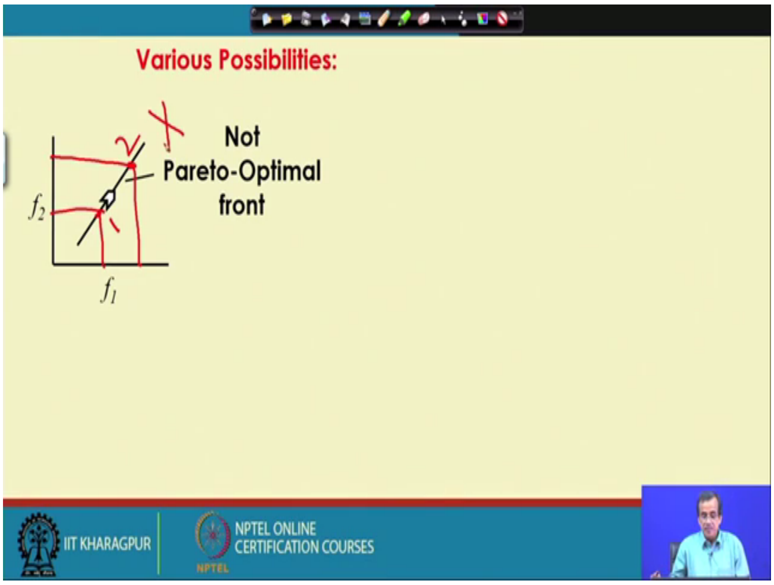

Now I am just going to take a few other examples, and where we can see that they are not the valid pareto optimal front of solution for example, if I take so this particular the solution. So, this is f 1 this is f 2. So, our aim is to minimize both f 1 and f 2. Now if I take a particular point here. So, this is my f 1 and this is f 2m, if I take a second point here so, this is my f 1 and this is f 2. Now from point 1 point if I move, so, both f 1 and f 2 are increasing.

So, there is no fighting between f 1 and f 2. Now by definition this is not a valid pareto optimal front of solution. So, this is not a valid pareto optimal front of solution. Now similarly if I take another example so, you can find out.

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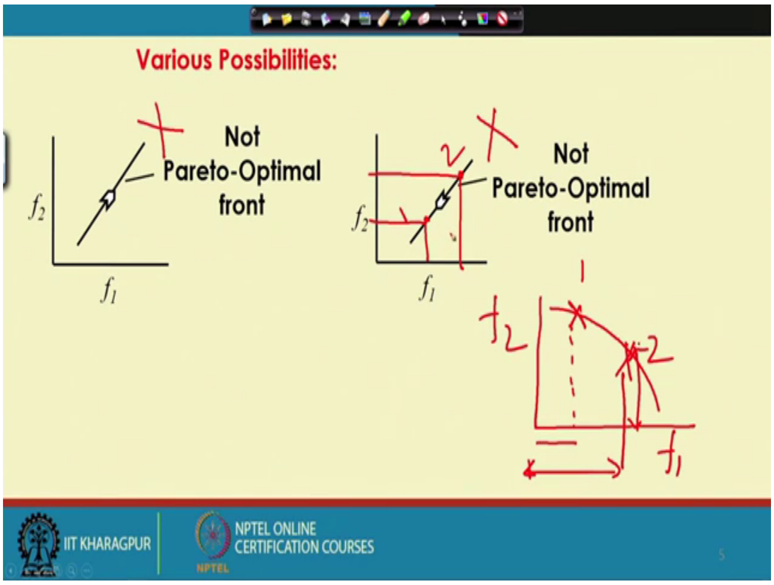

Now, this is another for this is f 1 and f 2 and our aim is to minimize both f 1 and f 2, and once again if I take say e particular point say point 1 here so, this is my f 1 and f 2 another point. So, this is my f 1 and f 2, and in this particular direction sat 2 to 1 both f 1 and f 2 are decreasing. So, there is there is no fighting at each with between f 1 and f 2..

This is once again not a valid pareto optimal front of solution, this also a not valid pareto optimal front of solution. Now here we have considered 2 objectives and our aim is to minimize both the objectives. Similarly I can consider another problem, where my objectives will be to maximize both the objectives f 1 and f 2, and if that is the case now in that case. So, there is a possibility I will be getting some pareto optimal front of solution, now let me take a very simple example. So, here my aim is to minimize both f 1 and f 2.

Now, let me consider another problem, now I am just going to maximize both f 1 and f 2 2 objectives f 1 and f 2. So, our aim is to maximize both f 1 and f 2, and supposing that I am getting a front like this. Now let me check whether this is valid pareto optimal front or not, now let me consider a particular point here, now corresponding to this particular point, and this is my f 1 this is f 2 another point is here say 0.2. So, this is my f 1 and this is my f 2, now here you see here f 1 is less f 2 is more, and here f 1 is larger compared to the previous f 1, but f 2 is smaller..

So, here as f 1 increases f 2 is decreasing. So, that fighting at each is there; that means, this is also a valid pareto optimal front of solution, if I want to maximize both f 1 and f 2. Now to summarize actually I just want to say that if there are 2 objectives I can consider, either 2 maximize both objectives or to minimize both the objectives in order to find out the valid pareto optimal front of solution.

Now, here I just want to put a note that any optimization problem, having 2 objectives may not be a candidate problem for obtaining the valid pareto optimal front of solution. Now these are the examples now this is also 2 objective optimization problem f 1 and f 2, and supposing there I am just going to minimize both f 1 and f 2, now here there is no conflict in nature between these 2 objects they are not fighting with each other..

So, this is not a valid problem for obtaining the pareto optimal front of solution although, this is an optimization problem having 2 objectives. So, once again let me repeat any problem having 2 objective function may not be a candidate problem for obtaining the pareto optimal front of solution. So, this is actually what you mean by the 2 objective optimization problem, and how to find out the pareto optimal front of solution for the problems having 2 objective functions.

Now, I am just going to extend this particular problem to a more complicated situation, now the situation is something like this, supposing that in place of 2 objectives I have got 3 objectives..

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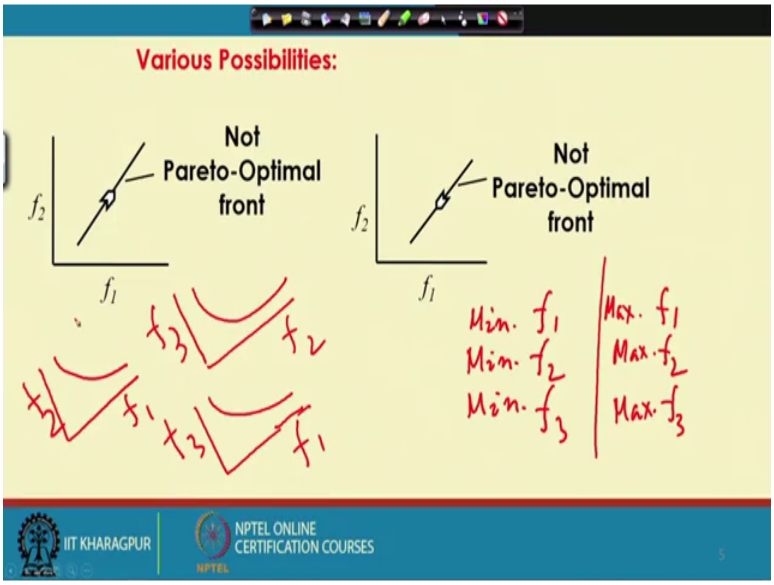

Now the objectives are as follows supposing that the first objective is minimize f 1, second objective is minimize f 2, and the third objective is minimize f 3, now I can also state in place of minimization like in the form of maximization for example, I can write down maximize f 1, maximize f 2, maximize f 3. So, either we can state as a minimization problem or as a maximization problem and supposing that the objectives are such that they are fighting with each other..

Now if this is the situation. So, I can find out 1 pareto optimal front, if I consider like f 1 versus f 2 so, there possibility I will be getting some pareto optimal front. Now if I consider say f 2 and f 3 there is a possibility that I will be getting another pareto optimal front, and if I consider like your f 1 and f 3. So, there is a possibility I will be getting another pareto optimal front, now what will have to do is there are 3 objectives. Now these 3 pareto optimal front I will have to super impose just to imagine what should be the pareto optimal surface, now as there are 3 objectives so, I will have to mutualize the pareto optimal surface.

Now, what I can do is I can super impose these 3 pareto optimal front, and I can visualize the pareto optimal front of surface, because this will be in 3 dimension. Now we human being we can visualize only up to 3 dimension, so, this particular the pareto front of surface we can visualize, now this is called actually the 3 objective optimization problem. Now let me take a very simple example very practical example, now we the faculty members we have 3 3 distinct responsibilities for example, we should be a good teacher we should be a good researcher we should be a good administrator.

So, we have got 3 objectives that all 3 will have to maximize. So, this is nothing, but a 3 objective optimization problem, and there are some functional constraints for example, say a particular day consists of say twenty 4 hours, and a person cannot walk f efficiently more than say 13 hours or 14 hours in a day, at the same time actually there will be a few constraint said by the surroundings these are all the functional constraints, and what at the variables what are the variable founds for example, a person will have some capability to work hard, he will have some level of intelligence, and he will have some level of a physical strength, and mental strength, these are all designed variables..

So, I have got 3 objectives maximize the teaching performance, maximize the research performance, maximize the performance of administrative work, and I have got the variable bounds, and there will be some functional constraint this is a very good example of a constraint optimization problem.

Now, the responsibility of the faculty member is to visualize that pareto optimal surface. So, that he can perfume in a optimal sense. So, this is the example of the 3 objective optimization problem, now I am making it more complex. Now if I just add another objective like the same faculty member should also do little bit of social work, now he will have to maximize the 4 objectives, the moment he consider 4 objectives. So, there will be some pareto optimal surface, but it will very difficult or it is quite impossible to visualize, because it is in 4 dimension. So, if this particular multi objective optimization problem is having like 4 or more than 4 objectives those problems are been difficult to tackle, and these are actually popularly known as many objective optimization problems.

Now, to sum this many objective optimization problem, we will have to use a slightly different type of approaches, and that particular study on many objective optimization is beyond the scope of this particular course. So, here what I am just going to is, I am just going to handle the 2 objectives, and 3 objective optimization problem, and I am just going to show how to tackle. So, this type of multi objective optimization problem. Now if you see the literature, we have a got a large number of tools or the methods available to solve this type of multi objective optimization problem, now if you see the oldest approach now this oldest approach is actually the waited some approach, and this is a very simple approach the principle I am already discussed little bit, but I am just going to discuss in much more details, there are some traditional other approaches. And after that the principle of the nontraditional optimization tools have been used to tackle this type of multi objective optimization problem. So, gradually actually I am just going to discuss this methods this tools 1 after another.

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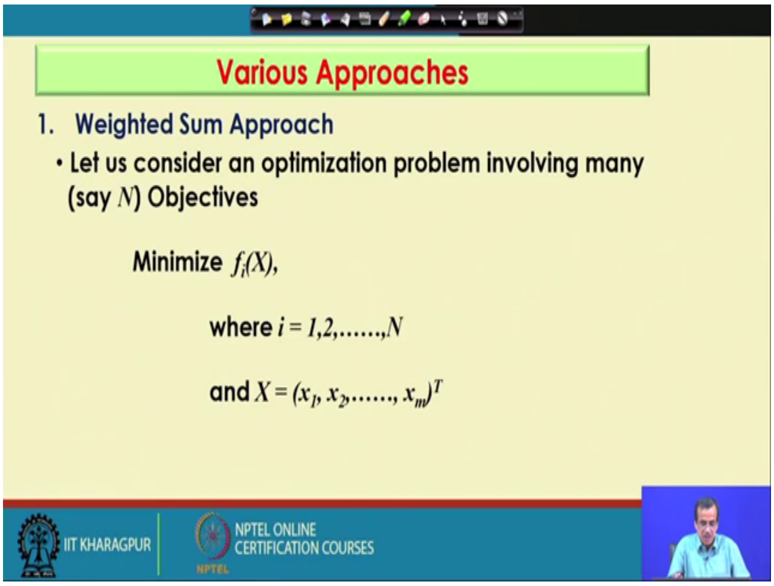

Now, let me start with the first approach that is called the weighted some approach, this weighted some approach actually as I told that we try to formulate this multi objective optimization problem as a single objective optimization problem. Now let us consider that an optimization problem has got capital N objectives. So, there are capital N number of objectives. So, I can formulate like this minimize f I capital X here, I varies from 1 2 up to N there are capital N number of objectives, and this capital X is nothing, but the collection of small mmm number of small X values.

So, this is in it is having N numerical values, now our aim is to minimize this particular multi objective optimization problem. Now let us see how to tackle this.

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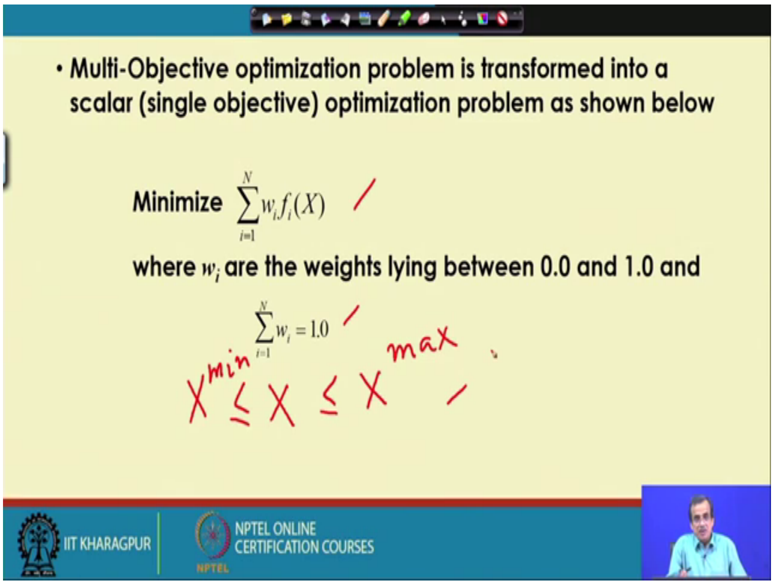

Now, to tackle this actually what you do is. So, this multi objective optimization problem is transferred into a scalar or a single objective optimization problem, as shown below. Now the same problem that is re written in the form of minimize, summation i equals to 1 to N so, capital N is nothing that the total number of objectives w i multiplied by f i X where this w i are nothing, but the weights lying between 0 and 1, and the sum of this w i values that is summation w i, i varies from 1 to N is equal to 1.0 and of course, the deign variables will have some range.

Now; that means your if I just want to put it this particular design variables that is X should be greater than equals to X minimum, and it should be less than equals to X maximum. So, this is nothing, but a constrained optimization problem for example, say so this is nothing, but the objective function, now this is the functional constraint and this is the variable bound.

So, this is nothing, but a constrained optimization problem. Now to solve this particular problem, if you see some literature many people are be used the traditional method of lag ranch multiplier now, I am not going for that so using the lag ranch multiplier. So, this particular optimization problem can be solved now this is 1 way of solving. Now, what I am going to do is? I am not going to follow this lag ranch multiplier here..

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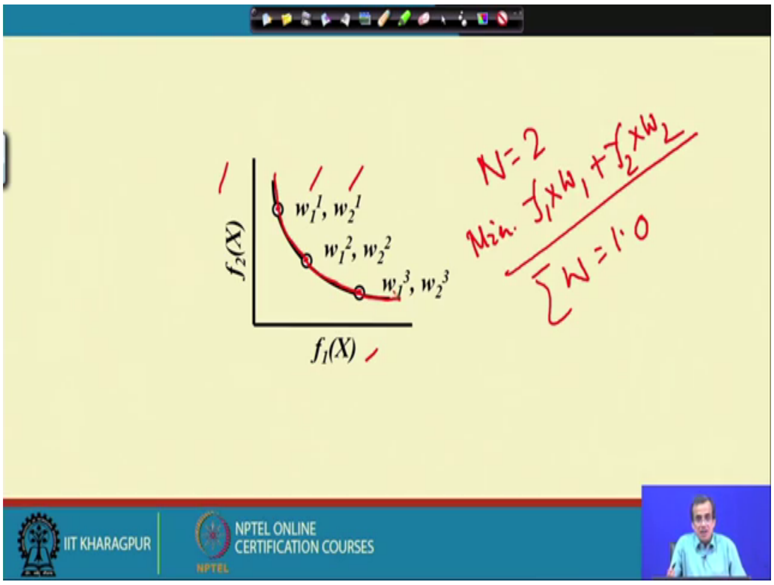

So instead what you do is I can convert it into a single objective, and by setting a proper values for this w 1 and w 2 if it is a 2 objective problem so, I can find out the pareto optimal front of solution. Now this particular multi objective optimization problem, let me consider this is a 2 objective optimization problem, and our aim is to minimize say both f 1 and f 2, Now for simplicity; so, I am considering that capital N is equal to 2 here, now what you do as I told that we try to find out a single objective thing that is nothing, but your so I will have to minimize in single objective form that is nothing, but f 1 multiplied by w 1 plus f 2 multiplied by w 2.

Now, what you do is we select 1 set up w values that is nothing, but w 1 1 w 2 1, and solve these as a single objective optimization problem, and if you solve this as a single objective optimization problem. So, I will be getting a particular solution, now if I change this particular w values to add the at the of course, the sum of w values will be equal to 0.

So, if I take another set of w values there is a possibility I will be getting these as the optimal solution, another set up w values this as the optimal solution, and as I discussed already I will try to find out the locus of this particular optimal solution, and this is nothing but the pareto optimal front of solution. So, this is actually the oldest of all the approaches to handle so, this type of multi objective optimization problem.

Thank you.