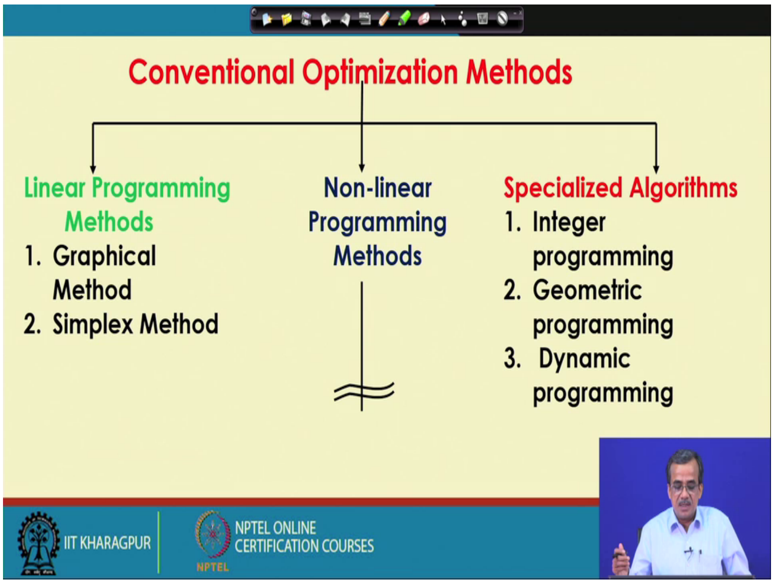
Traditional and Non-Traditional Optimization Tools
Prof. D. K. Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 02
Traditional Methods of Optimization

Let us start with topic 2 of this course that is Traditional Methods of Optimization.

(Refer Slide Time: 00:30)

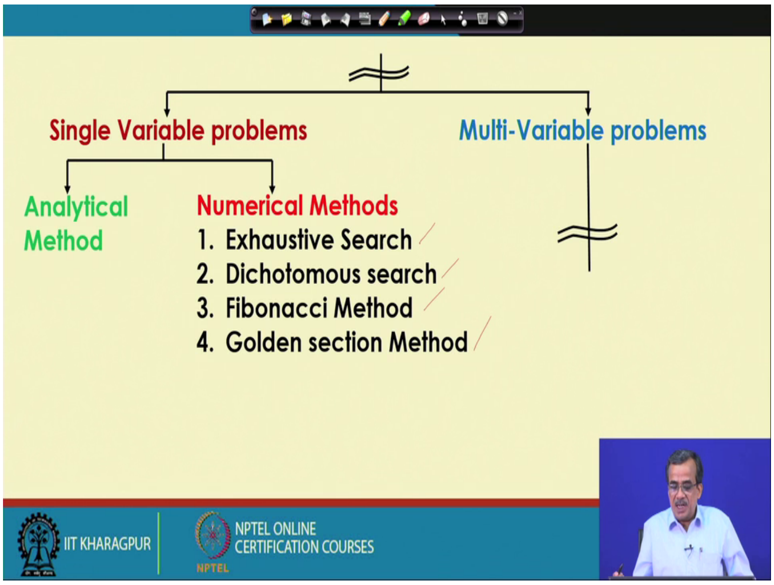


Now, the Traditional Optimization Methods are also known as the Conventional Optimization Methods. Now these methods can be broadly classified into 3 groups 1 is called the Linear Programming Method.

Next is a Non-Linear Programming Method and we have got the Specialized Algorithm. Now this non-linear programming method it could be either Graphical Method or Simplex Method. Now supposing that I have got a problem having only 2 variables, now for this problem I can go for the graphical method of linear programming, but supposing that I have got more than 2 variables. So, there is no way out, but we will have to go for the simplex method.

Now, then comes the specialized algorithms now these algorithms are required to solve some special problems and these algorithms are known as integer programming geometric programming, dynamic programming, and so on and now if I concentrate on the non-linear programming methods those are further classified into 2 subgroups.

(Refer Slide Time: 02:02)

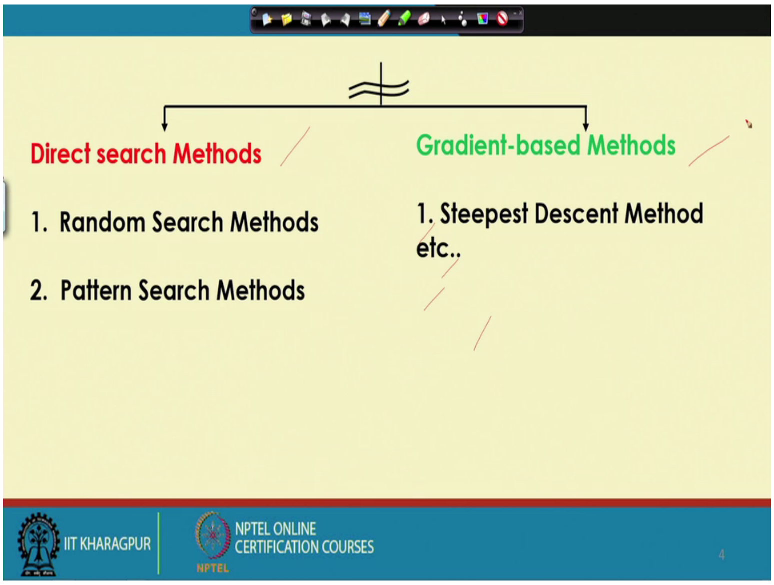


Now 1 is called the Single Variable problem another is called the Multi-Variable problems.

Now, single variable problems can be solved either by using analytical method or some numerical methods. Now analytical method I have already discussed that is the method of calculus; that means, we will have to find out the derivative of the function with respect to the variable and we will have to put equals to 0 and we will have to solve and find out the optimal solution, which I have already discussed.

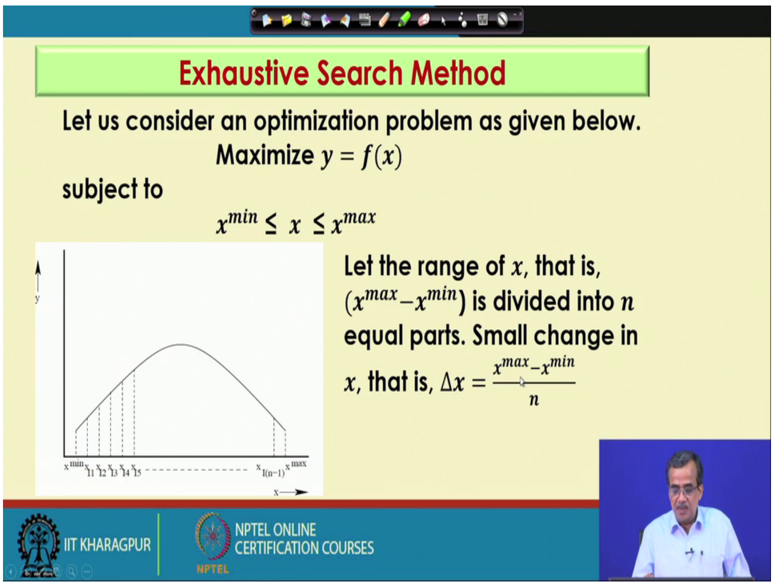
Now, I am just going to concentrate little bit on the numerical methods. Now this numerical method includes exhaustive search method Dichotomous search, Fibonacci search, Golden section method and so on. Now, similarly if we if I see the multi-variable problems, now these multivariable problems can be further classified into 2 subgroups 1 is called the direct search methods 1 is called the direct search method another is called the gradient based method.

(Refer Slide Time: 03:13)



Now, in direct search method we use the value of the objective function to help that particular optimization tool, where as in case of gradient based method we take the help of gradient direction to find out the search direction of this particular optimization tool. Now we have got a few direct search methods for example, random search method pattern search method and so on. Similarly we have got a few gradient based methods like steepest descent methods and others, now out of all the traditional tools for optimization this steepest descent method is found to be the most popular one.

(Refer Slide Time: 04:23)

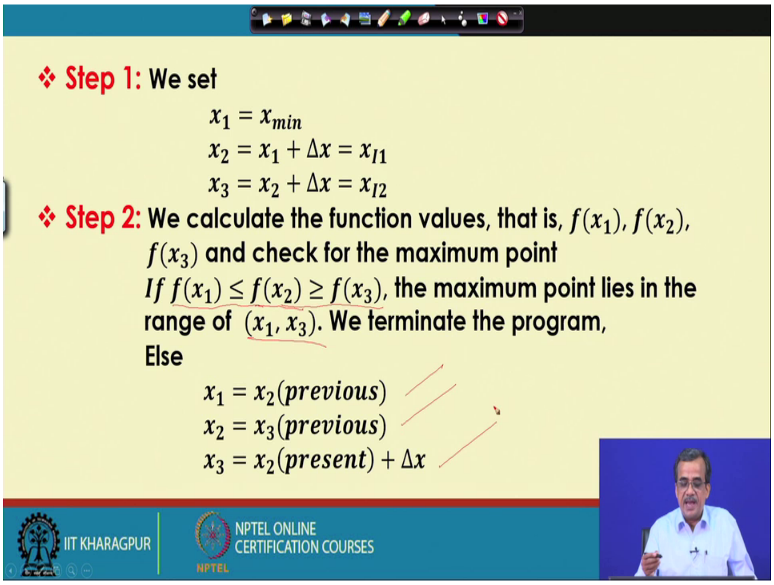


Now, here in this particular course I will try to explain the working principle of a few optimization tool and let me start with 1 numerical method which is popularly known as the exhaustive search method. Now let us consider an optimization problem of the form maximize y equals to f x. So, this is the function which I will have to maximize and let me assume that so this is a unimodal function.

So, maximize y equals to f x subject to x is lying between x minimum and x maximum and let be considered 1 unconstrained optimization problem and this is actually the plot of the objective function. So, why is a function of x and as I told I am considering 1 unimodal function like this.

Now, how to proceed with this particular algorithm to find out the optimal solution to find out the optimal solution actually what I will have to do is. So, I will have to find out the range for x that is nothing, but x maximum minus x minimum. Now this range is divided into some equal parts and let me consider small n equal parts then the small x change in x that is delta x is nothing, but x maximum minus x x minimum divided by n.

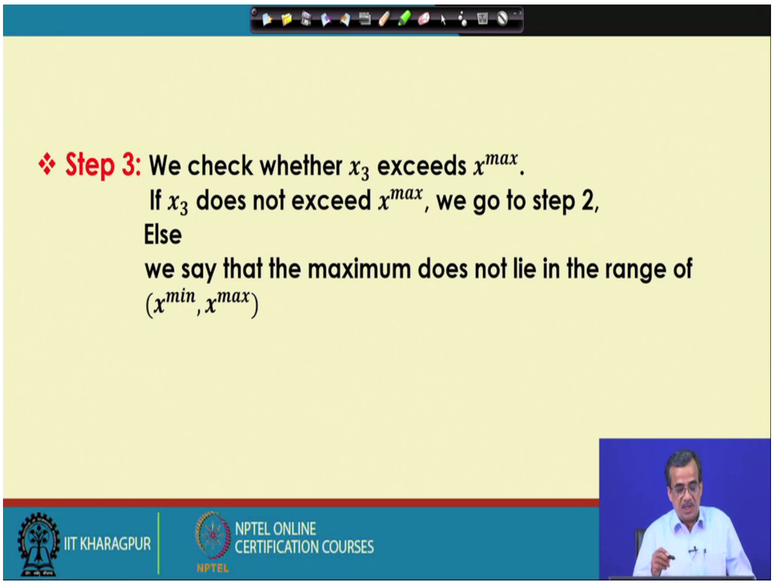
(Refer Slide Time: 06:20)



Now, step 1 we set x 1 equals to x minimum x 2 equals to x 1 plus delta x that is nothing, but x I 1 that is the first intermediate, x 3 equals to x 2 plus delta x that is x I 2 then we go for step 2 where we try to calculate the values of the function like f x 1, f x 2, f x 3 and we try to compare.

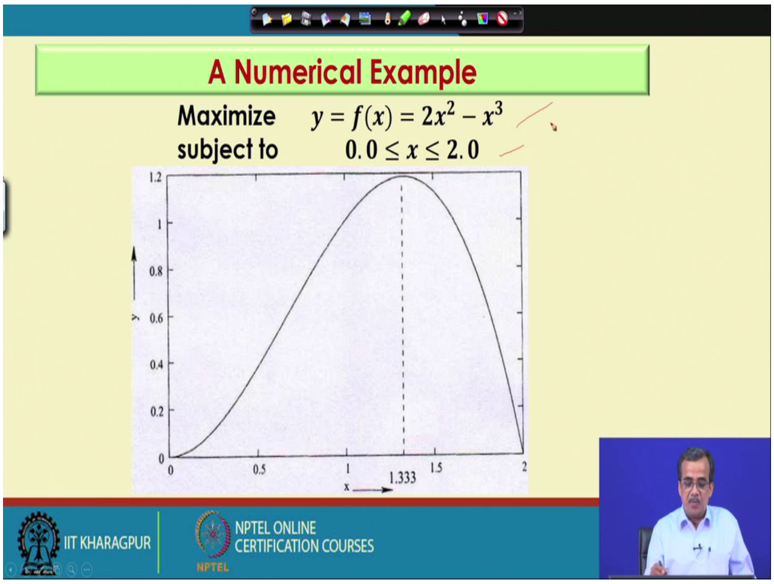
Now, if f x 1, if f x 1, if f x 1 is found to be less than equals to f x 2 and it is found to be greater than greater than equals to f x 3, then we say that the maximum point lying in the range of x 1 comma x 3 and we terminate the program, else we assign x 1 equals to x 2 previous x 2 equals to x 3 previous and x 3 equals to x 2 present plus Delta x.

(Refer Slide Time: 07:40)



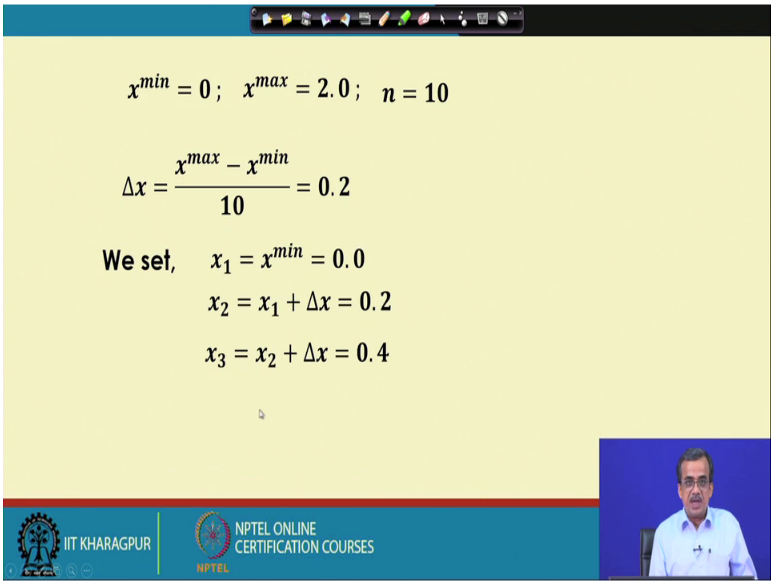
Then we go for step 3 we check whether x 3 exceeds x maximum if x 3 does not exceed x maximum we go to step 2, else we say that the maximum does not lie in the range of x 1 minimum comma x 1 maximum.

(Refer Slide Time: 08:09)



Now, I am just going to solve 1 numerical example just to explain the working principle of this particular the method. So, let me take the numerical example like this maximize y equals to f x that is 2 x square minus x cube now. So, this is actually the objective function which I will have to maximize subject to x is lying between 0 and 2. Now this is nothing, but the function plot this is a function of only 1 variable. So, y is a function of x. So, this is actually the plot of this particular the function and I will have to find out the maximum value of this particular the function.

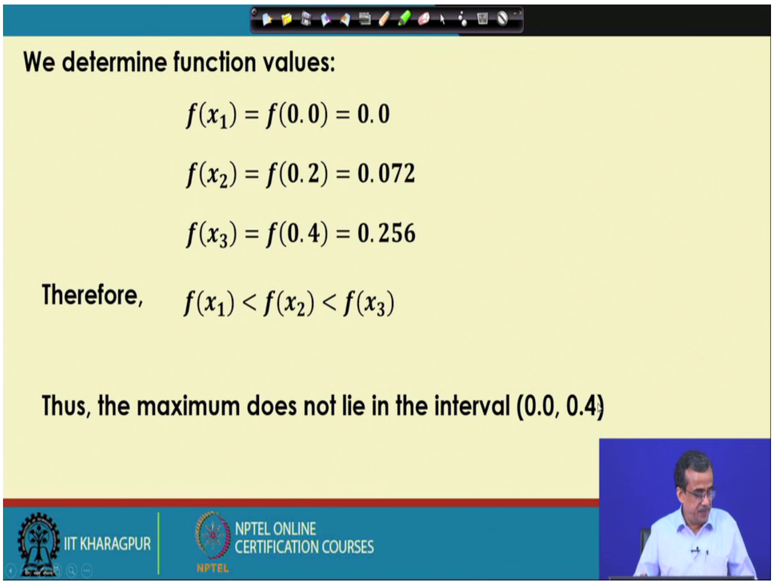
(Refer Slide Time: 09:03)



Now, here x minimum equals to 0 and x maximum equals to 2.0 and let me consider small n is equal to 10 now the change in x that is delta x is nothing, but x maximum minus x minimum divided by 10 is equals to 0.2.

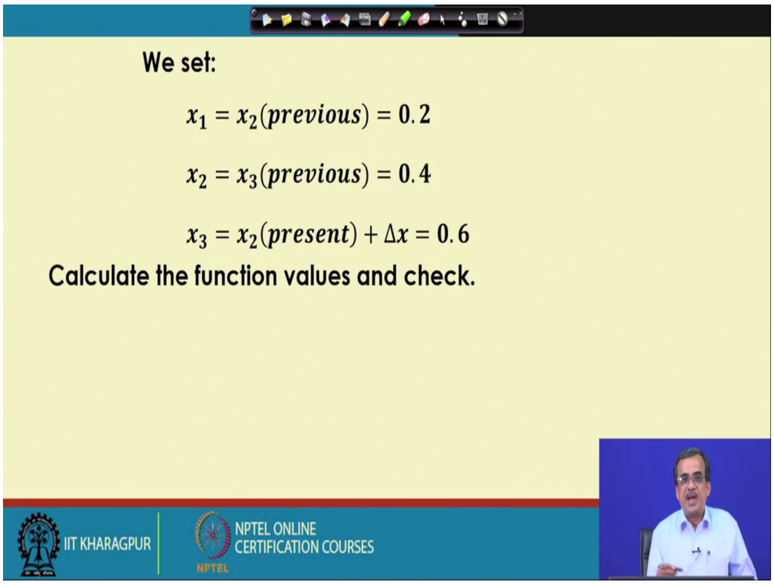
So, we set x 1 equals to x minimum that is equal to 0.0 x 2 equals to x 1 plus delta x that is equals to 0.2 x 3 equals to x 2 plus delta x and it is equals to 0.4.

(Refer Slide Time: 09:54).



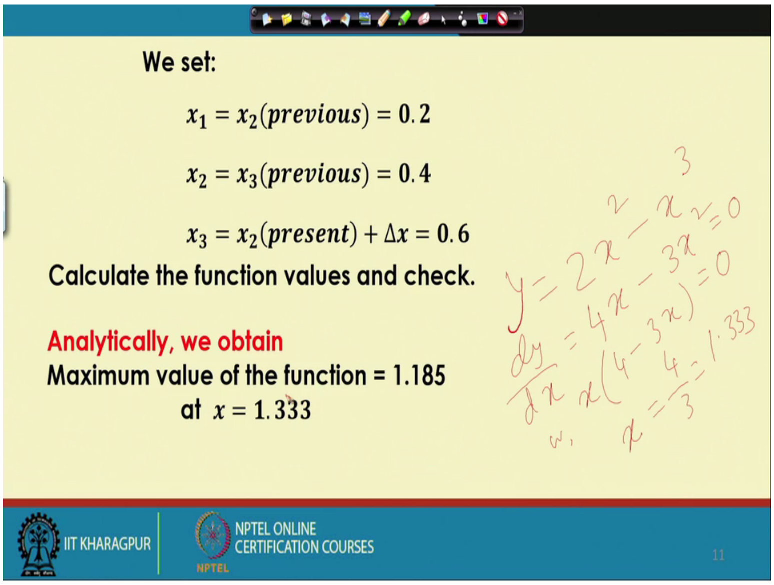
Now, we determine the function values f of x 1 is found to be equal to 0.0 f of x 2 is found to be equal to 0.0 7 2 and f of x 3 is coming to be equal to 0.2 5 6. Now here f x 2 is found to be greater than f x 1 and it is found to be less than f x 3 and we conclude that the maximum does not lie in the range of 0 comma 0.4.

(Refer Slide Time: 10:36)



Next we set x 1 equals to x 2 previous that is equal to 0.2 x 2 equals to x 3 previous that is equals to 0.4 and x 3 equals to x 2 present plus delta x that is equals to 0.6. And once again we calculate the function values and try to check whether that particular condition is fulfilled or not. Now for this particular function analytically we can find out the optimal value. Now this function is actually the mathematical form of this particular function is nothing, but 2 x square minus x cube. So, 2 square minus x cube. So, if I try to find out your the analytical solution we can find out easily.

(Refer Slide Time: 11:39)



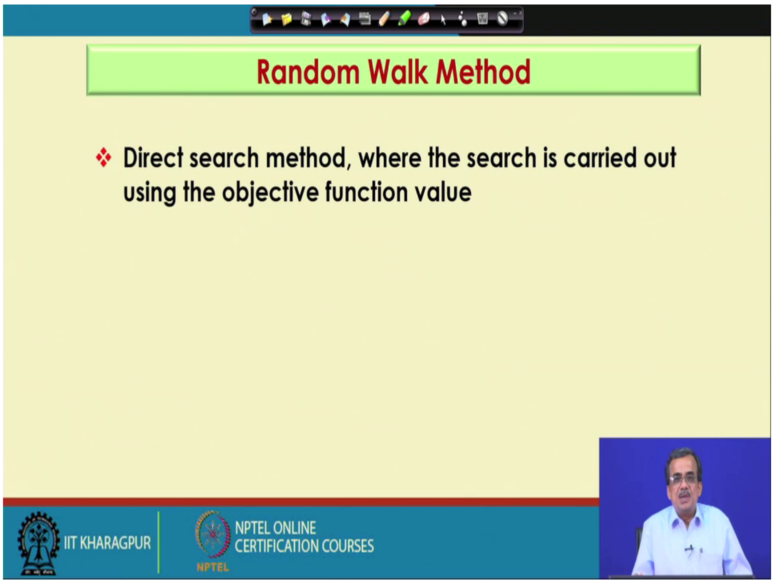
So, y is nothing, but 2 x square minus x cube. So, what I do is we try to find out the derivative that is dy dx and that is nothing, but 4 x minus 3 x square and we put this equals to 0 and we solve for x. So, this is nothing, but x 4 minus 3 x that is equals to 0.

So, either x equals to 0 or 4 minus 3 x equals to 0. So, x cannot be equal to 0. So, we can find out x is nothing, but 4 by 3 that is your 1.3 3 3. So, we can find out the maximum value of this particular function is coming at x equals to 1.3 3 3 and the maximum value of this particular function is coming as 1.1 8 5. So, analytically you can find out the solution and I can also check through this exhaustive search method and I can find out I can indicate 1 range for this particular value of x, where I am I will be getting that maximum value of this particular the function.

Now if I take a higher value of small n there is a possibility I will be getting a closer range of the value of the x where I will be getting that particular the maximum value of this particular the function. Now here I just want to mention that this particular method will be applicable only for the unimodal function for the multimodal function. So, this tool may give rise to some problem to determine the maximum or the minimum value of this particular function.

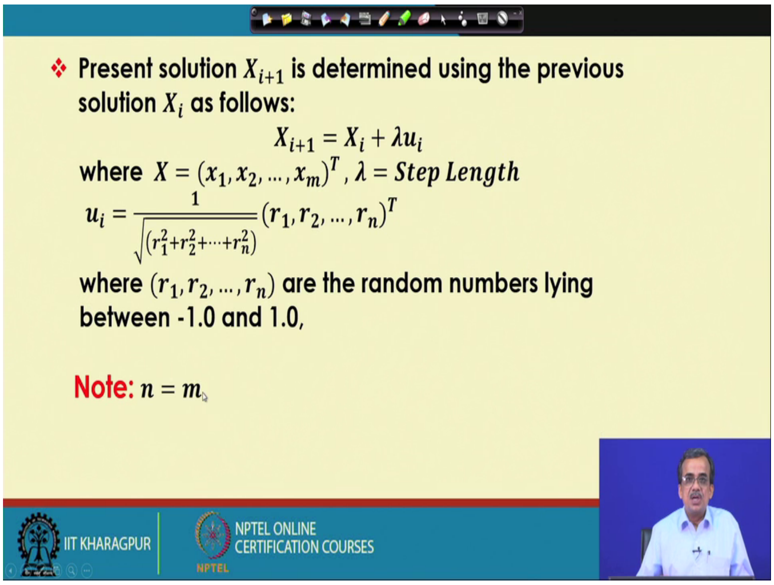
Now, I am just going for another tool.

(Refer Slide Time: 13:48)



That is known as the random walk method now this random walk method is nothing, but a direct search method here the search is carried out using the information of the objective function value; that means, we consider the numerical value of the objective function to determine what should be the search in the next iteration and in this particular method we do not use the derivative or the gradient information of the objective function.

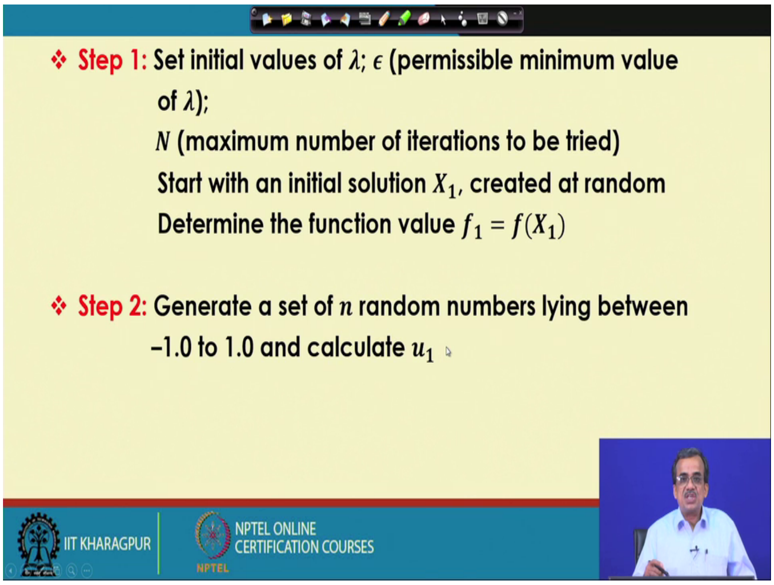
(Refer Slide Time: 14:40)



Now, according to this particular method the rule for updating is nothing, but this that is your x i Plus 1 is nothing, but X i plus lambda ui so capital x is actually a collection of all the design variables like small x 1 x 2 up to say x m, if there are m search design variables. So, capital X i is nothing, but the value of this particular the set of the design variables in earlier iteration and lambda is nothing, but the step length and u i indicates the search direction now how to find out. So, this particular search direction. So, that I am going to discuss.

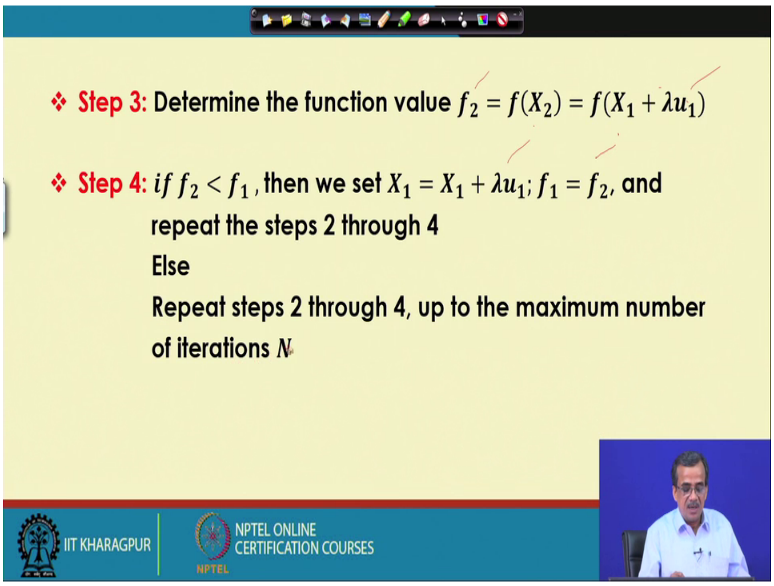
Now, as I told that capital X is nothing, but a collection of all small x values and lambda is nothing, but the step length. Now here we try to find out the search direction that is u i that is nothing, but 1 divided by square root of r 1 square plus r 2 square plus dot dot dot plus r n square multiplied by r 1 r 2 r n transpose. So, this will give you that search direction now once I have got this step length and the search direction. So, very easily I can find out what should be the values of the design variables in the next iteration, now here in this particular expression this r 1 r 2 r n are nothing, but the random numbers lying between minus 1 and plus 1 and here we consider the number of random number that is small n that is equal to the number of variables that is small m.

(Refer Slide Time: 16:39)



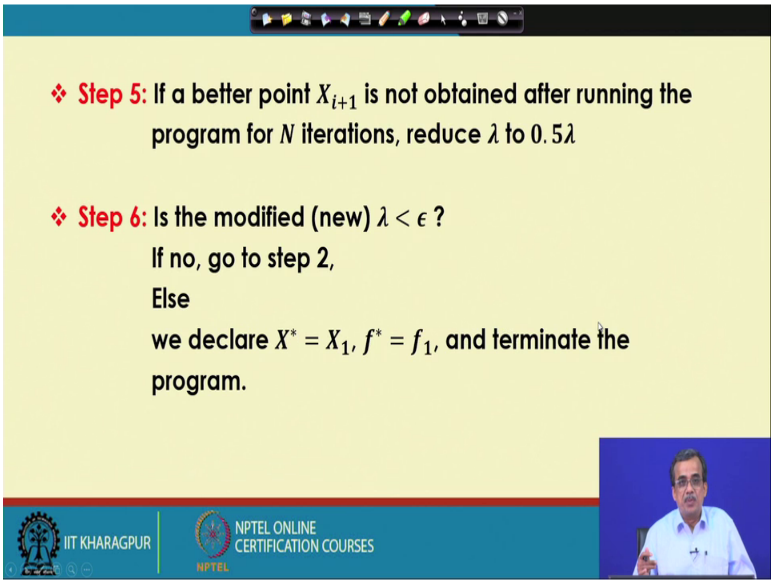
Now, let us see this particular algorithm in the form of the steps now supposing that on solving 1 minimization problem and let me concentrate on the step one. So, we set initial values of lambda that is a step length and epsilon that is the permissible minimum value of lambda and capital N indicates the maximum number of iterations to be tried. We start with an initial solution X 1 created at random and we try to determine the value of the objective function that is f 1 that is nothing, but f of X 1. Now then we go for step 2 we generate a set of n random numbers lying between minus 1 and plus 1 and we try to calculate what should be the search direction that is u 1.

(Refer Slide Time: 17:44)



Now, step 3 we determine the function value that is f 2 that is nothing, but f of X 2, f of X 2 that is your f of X 1 plus lambda u 1. So, lambda is a step length u 1 is a search direction X 1 is the previous value. So, I can find out X 2 once I have got X 2 I can find out the value of the objective function then comes step 4. Now as I told that this is a minimization problem. So, if f 2 is found to be less than f 1 then it is a good solution, then we set X 1 equals to X 1 plus lambda u 1 and we set f 1 equals to f 2 and we repeat the steps 2 through 4 else repeat steps 2 through 4 up to the maximum number of iteration that is capital N.

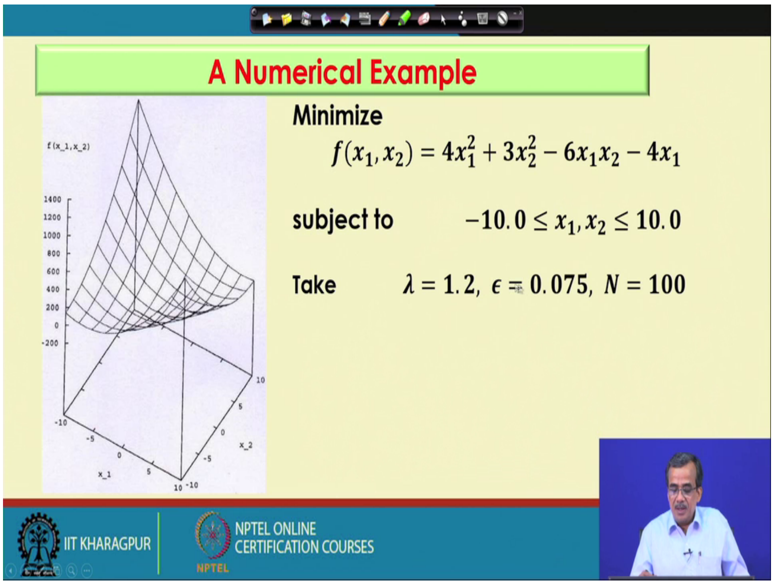
(Refer Slide Time: 18:54)



So, step 5 if a better point X i plus 1 is not obtained after running the algorithm for N iteration, we reduce lambda to 50 percent and once again we try to repeat those iterations. Now step 6 is the modified lambda now as I told we reduce lambda to 50 percent and for a particular value of lambda we run this algorithm for capital N iterations. Now we go for the reduction of the lambda through a number of iterations then we reach the minimum permissible value for this particular lambda which is nothing, but epsilon.

So, we go for step 6 for checking is the modified lambda is less than epsilon, now if no we go to step 2 else we declare that x star is equals to X 1 and f star is equals to f 1 and we terminate the program.

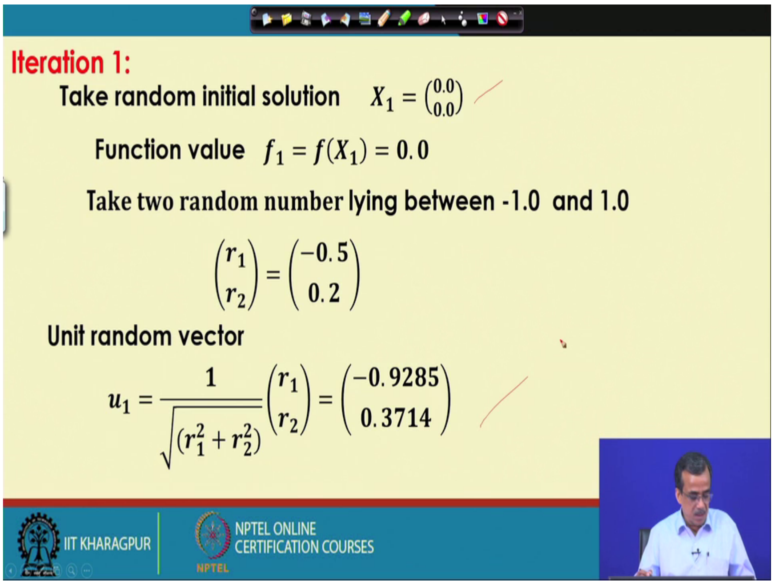
(Refer Slide Time: 20:08)



Now, this particular algorithm is very popular and I am just going to explain the working principle of this particular algorithm with the help of 1 numerical example.

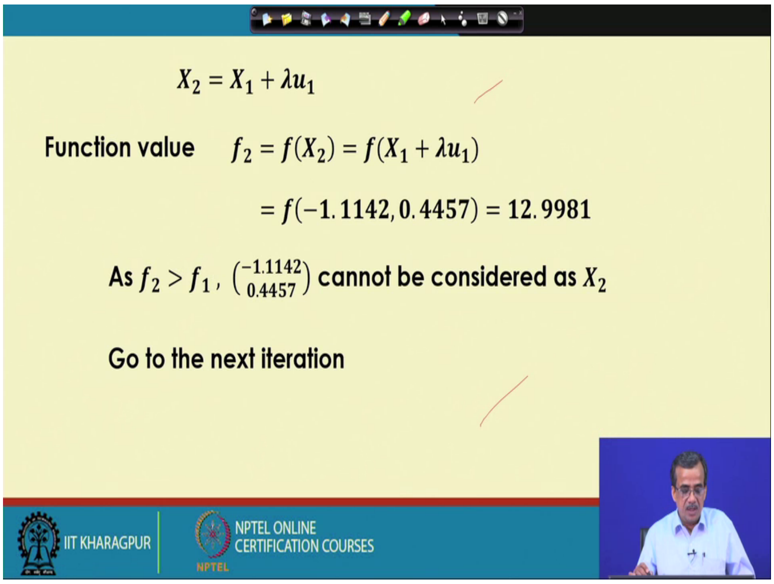
Now, here I am just going to minimize 1 function that is f of x 1 x 2 is nothing, but 4 x 1 square plus 3 x 2 square minus 6 x 1 x 2 minus 4 x 1, subject to x 1 x 2 is lying between minus 10 and plus 10 and here I am not considering any functional constraint. So, this is unconstrained optimization problem, now if we see the function plot. So, I can find out this function plot now this is a function of 2 variables x 1 and x 2. So, I will be getting this particular the plot for the function and our aim is to find out the minimum solution the minimum. Point and here we take lambda that is the step length equals to 1.2 epsilon that is the minimum permissible value for lambda and this it is equal to 0.0 7 5 and n is the number of iteration and that is equals to 100.

(Refer Slide Time: 21:41)



Now, we just go for iteration 1 we take the random initial solution denoted by capital X 1 and that is nothing, but 0 0 next we try to find out the function value that is f 1 that is f of X 1 and that is 0.0 take 2 random number lying between minus 1 and plus 1 supposing that r 1 equals to minus 0.5 and r 2 equals to 0.2 and using the value of this particular random numbers now I can find out the unit random vector, which is going to indicate the search direction. So, u 1 is nothing, but 1 divided by square root of r 1 square plus r 2 square multiplied by r 1 r 2 and if I calculate. So, I will be getting the numerical values like this I will be getting the numerical values like this the next is.

(Refer Slide Time: 22:57)

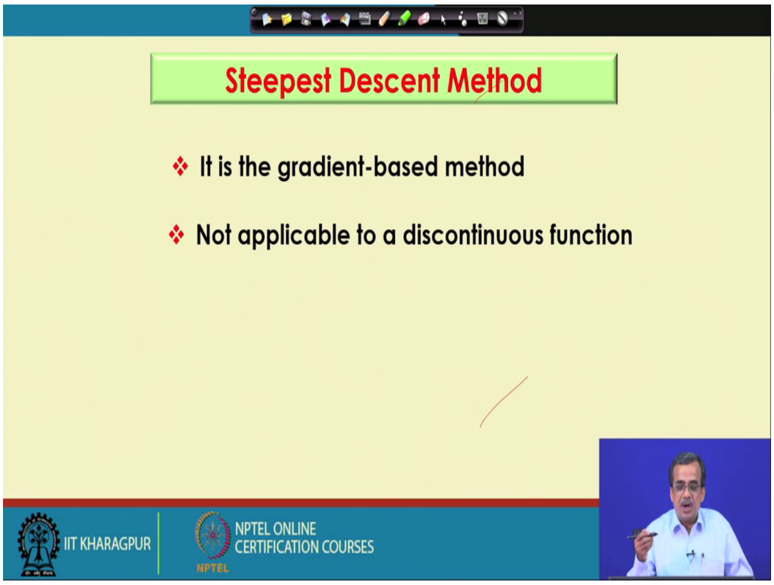


So, X 2 is nothing, but X 1 plus lambda u 1 then function value f 2 is f of X 2 and that is nothing, but f of x 1 plus lambda u 1.

Now, if I substitute the value of X 1 that is nothing, but 0 0 and lambda I have already calculated sorry I have already taken as a fixed value and u 1 is a search direction I have already calculated now I will be getting that is nothing, but f of minus 1.1 1 4 2 comma 0.4 4 5 7 and I will be getting this particular value of the objective function.

Now, as f 2 is found to be greater than f 1 f 2 was 0.0. So, f 2 is found to be greater than f one. So, this particular solution cannot be considered as X 2 because this is not a good solution. So, we go for the next iteration.

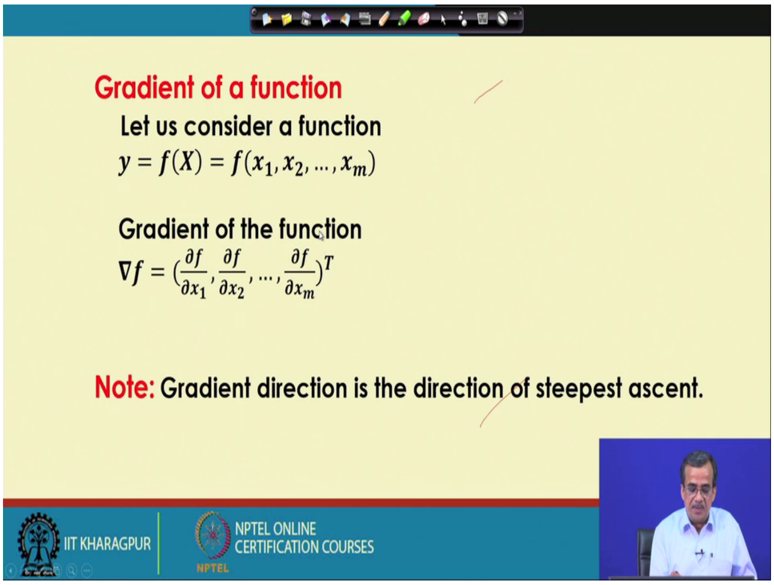
(Refer Slide Time: 24:06)



Now, this particular process will go on and on and once it reaches that particular value of capital n we reduce the value of lambda and for the reduced value of lambda, once again we run this particular algorithm for capital N number of iterations and we try to find out whether we can get that optimal solution or not. Now I am just going to discuss another very popular traditional tool for optimization, which is very popularly known as the steepest descent method. Now it is name indicates that this is a gradient based method and we try to search in a direction opposite to the gradient and on principal this can solve the minimization problem.

And let us see how does it work and before I go for the working principle of this particular method let me mention that this method is not suitable for discontinuous objective function, because the gradient does not exist for that discontinuous objective function.

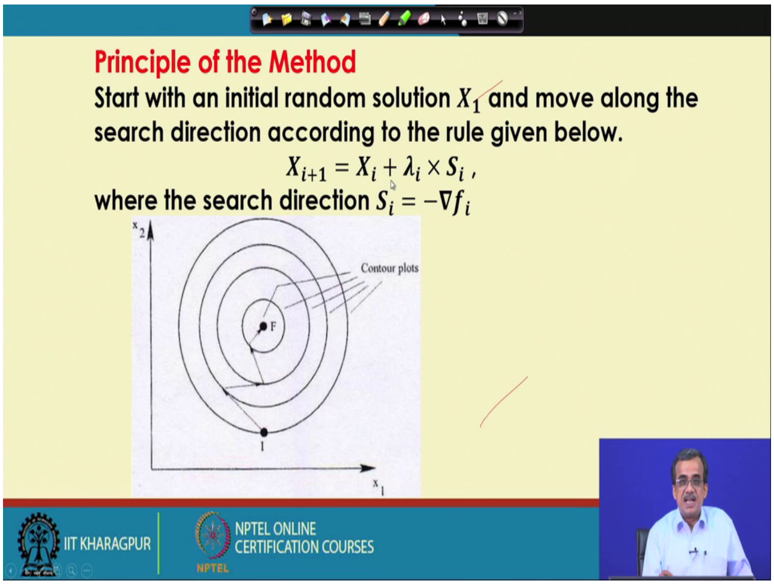
(Refer Slide Time: 25:27)



Now, to discuss this algorithm so what I will have to do is I will have to first define what do you mean by the gradient of a function. Now supposing that I am considering y is a function of capital X and that is nothing, but capital X is a collection of m number of small x values. So, this is a function of m variables.

Now, it is gradient is defined as delta f is nothing, but partial derivative of f with respect to x 1 partial derivative of f with respect to x 2 comma and the last term the partial derivative of f with respect to m transfer. So, this is nothing, but the gradient of this particular the function and gradient direction is nothing, but the direction of steepest ascent and that is why. So, we will have to move in a direction opposite to the gradient and it names indicates this is a steepest descent method an on principal it can solve the minimization problem.

(Refer Slide Time: 26:45)



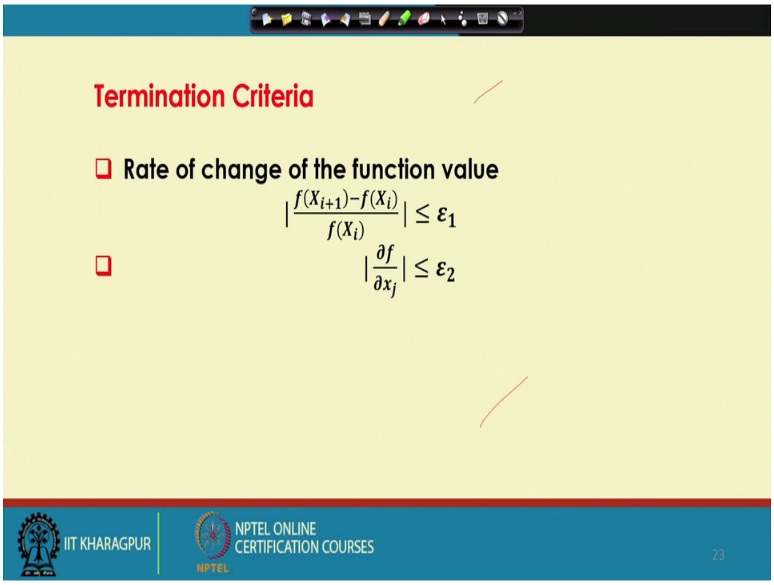
Now, the principle of this particular method we start with an initial solution X 1 and we move along the search direction according to the rule X i plus 1 is nothing, but X i plus lambda i multiplied by S i. So, S i indicates the search direction and that is nothing, but minus delta f i that is opposite to the gradient and lambda i is nothing, but the step length and this is the rule for updating, now let me take a very simple example supposing that I have got a function objective function of 2 variables X 1 and X 2.

Now, depending on the value of the y so y is a function of 2 variables X 1 and X 2. So, for a particular value of y i can find out the contour plot another value of y, another contour plot another contour plot another contour plot another contour plot I will be getting. Now this particular algorithm starts with an initial solution selected at random supposing that at the beginning the algorithm is here only.

Now, it will try to find out the search direction for the next iteration and it will try to move along the gradient direction, once again it will try to follow the gradient direction gradient direction and gradually, it is going to reach that particular the minimum solution this is the way actually this particular algorithm starting from an initial random initial solution we will try to move towards the globally optimal solution.

Now, I am just going to take 1 numerical example to solve this particular to show the working principle of this particular algorithm, but before that let me mention the termination criteria of this particular algorithm.

(Refer Slide Time: 28:46)



Now the termination criteria are as follows the rate of change of the objective function value that is nothing, but the mod value of f of X i plus 1 minus f of X i divided by f of X i. So, this particular mod value should be less than equals to some small value epsilon 1 or what I can do is I can find out the partial derivative with respect to the variables and their mode values should be less than equals to epsilon 2. So, this could be the termination criteria for this particular the algorithm.