Traditional and Non-Traditional Optimization Tools
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Lecture - 18
Scheduling GA (Contd.)

Now, let me discuss the principle of order crossover, now this order crossover is one of the most popular crossover operators, used in scheduling GA. Now, I am just going to discuss the principle of this order crossover, how to find out 2 children solution from the 2 parents.

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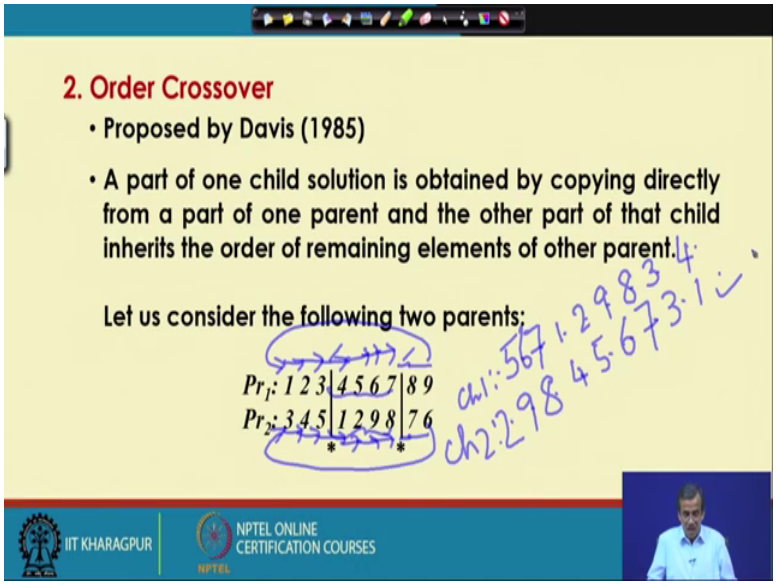

Now, the concept was proposed by Davis in the year 1985. Now, supposing that I am solving a signaling problem, say one TSP problem traveling salesperson problem, involving say 9 cities; that means, a person will have to touch all n cities once, by covering the minimum distance and supposing that. So, we have created the initial population for GA at random and using the reproduction operator. So, we have selected the mating pool and these 2 parents are going to participate in crossover.

Now, the parent 1 is 1 2 3 4 5 6 7 8 9 and parent 2 is 3 4 5 1 2 9 8 7 6. Now, let us see how to get the 2 children solution, from these 2 parents using the principle of the order crossover. Now the principle is very simple, now what I am going to do? I am just going to find out the 2 children solution.

Now, what I do is say let me write here, child 1 now to get the child 1 the first thing you have to do is, you will have to select the 2 crossover sites at random. So, this is the first crossover site, 2nd crossover site and the cities which are lying in between these 2-crossover side, first will have to consent it on that. Now, what we do is now here within these 2 crossover sites on parent 2 these cities are line like 1 2 9 8.

So, what we do first we locate 9 just position 1 2 3 4 5 6 7 8 9, now this 1 2 9 8 I am just going to directly copy from parent 2, keeping their position intact like I will start from the 4th position. So, this is the 4th position. So, let me write here 1 2 9 and 8 here, now after I selected 4 cities from parent 2, what we do is, we concentrate on the first city after the 2nd crossover site lying on parent 1.

The first city after the 2nd crossover site line on parent 1, is nothing but your city 8. Now, if I see the city 8 has already come. So, I am not going to take it once again, next I go to city 9, now city 9 is already there. So, I cannot take it again, next we continuous are in this particular direction. So, I will come back to the leftmost city lying on parent 1 that is 1, one is also lying here. So, I cannot take.

Next is city 2, 2 is already there. So, I cannot take, the next is your city 3, 3 is not there. So, I can select city 3 here, the next is city 4, 4 is also not here. So, let me select city 4 here. The next is city 5, now city 5 is actually I can is not yet copied here. So, I can put it here.

Now, then comes your city 6, now city 6 is once again not here. So, let me put city 6 here, and next is city 7, now 7 is also not here. So, let me put 7 here, now this completes your the child 1. Now, let me just try to check whether I have consider all the cities once, city 1 2 3 4 5 6 7 8 9. So, all the cities have been considered once and there is no missing city here. So, this is a valid child. So, this is your the child 1.

Now, let us see how to find out the child 2, now to find out the child to once again let me first locate the 9 positions 1 2 3 4 5 6 7 8 9. Now what we do is, the cities which are lying between the 2 crossover sites on parent 1, that is your 4 5 6 7. So, those things are directly copied, keeping their position information intact. So, what we do is, we copy 4 here, then comes 5 here 6 here and 7 here and now, I consented on the first city after the 2nd crossover site, which is lying on parent 2 and that is nothing but city 7.

Now, city 7 is actually already copied here. So, I do not want to copy once again. So, next I go for city 6, 6 is already there. So, I cannot copy it again, then I go to the starting of parent 2, now at the starting of parent 2 you have got city 3, 3 is not here. So, I can copy 3 here, the next is city 4, 4 is already there. So, I cannot copy it again, the next is 5, 5 is already there. So, I cannot copy it again.

The next is city 1. So, one is not here. So, let me put 1 here, the next is city 2, 2 is also not there. So, let me put 2 here next is city 9. So, 9 is also not there. So, let me put 9 here, then comes city 8, 8 is also not here. So, let me put city 8 here, now with this I complete actually the child 2.

Now, let me check whether all the cities have been considered once or not, now city 1 2 3 4 5 6 7 8 9. So, all the cities have been considered once. So, this is a valid child. So, using the principle of this order cross over, actually we can find out 2 valid children starting from the 2 parents.

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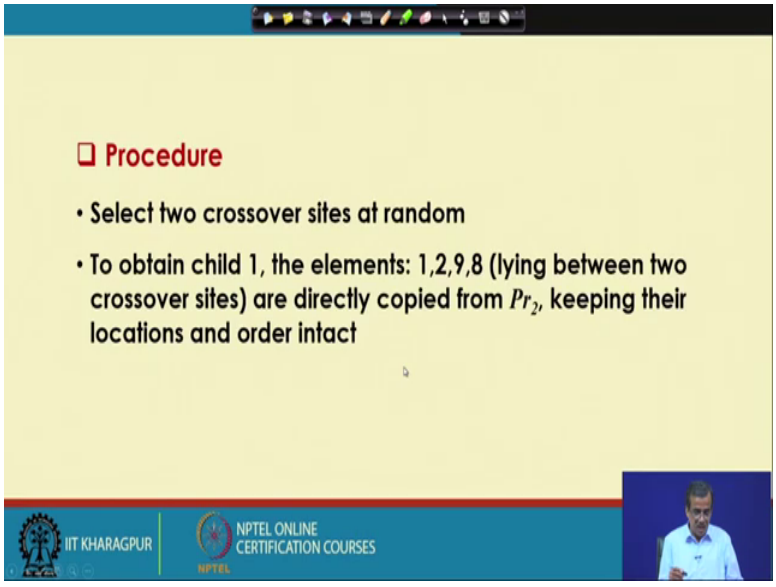

Now, whatever I have discussed the method which I have discussed, the same thing actually I have written it here, let me have a quick look. Now, the procedure is as follows, we select 2 crossover sites at random, which I have already discussed. To obtain child 1 the elements 1 2 9 8 lying between the through crossover sites, are directly copied from parent 2, keeping their location and order intact, this I have already discussed,

actually in the last slide. So, to get this child 1 we first copy with this particular 1 2 9 8 and then we concentrate here, now let us see that particular the method.

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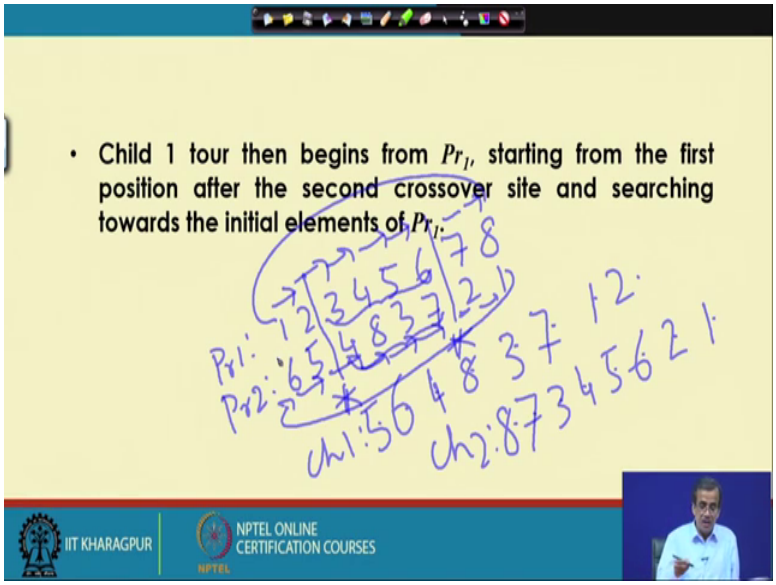

Child 1 2 then begins from parent 1, starting from the first position after the 2nd crossover site and searching towards the initial elements of parent 1 and this process will go on, and go on till I come I select all the cities once and the same procedure is followed, to get this particular the child 2. So, using this particular the crossover operator the order crossover operator, we can find out to valid children starting from the 2 parents.

Now, let me do one thing, let me take another example, just to show you the principle once again, now what I do is. So, I am going to take another example, the similar type of example say, let me consider 1 TSP involving say 8 cities. So, I am solving another example. So, supposing that the parent 1 is something like this 1 2 3 4 5 6 7 8. So, it is having 8 cities and parent 2 is nothing but say 6 5 4 8 3 7 2 1. So, these 2 parents are going to participate in order cross over, and let us see how to find out the children solution.

So, I am a redoing this particular example, now supposing that the first crossover site is here, which is selected at random. The 2nd crossover site is here, which is also selected at random and I am just going to find out the child 1. So, once again let me locate 8 position 1 2 3 4 5 6 7 8, now what I do is. So, to get child 1. So, these cities are copied fast, keeping their position information intact. So, 4 8 3 7 next I concentrate here.

So, here we have got 7, 7 is already copied next is 8, 8 is also copied. So, you come back next is 1. So, I put 1 here next is 2. So, I put 2 here, next is 3, 3 is already there, next is 4, 4 is also already there, next is 5, 5 is not there. So, let me put 5 here, next is 6, 6 is not there. So, let me put 6 here, this completes child 1.

Now, similarly I can find out the child 2 also. So, 1 2 3 4 5 6 7 8, 8 position let me locate now to get child 1. So, these cities I copy fast like your 3 4 5 6, now I consented here. So, it is 2, 2 is not there. So, let me copy to here, next is 1, 1 is not there. So, let me put 1 here, next I start from here, now here actually we have got 6, 6 is already there, next is 5, 5 is also there, next is 4, 4 is also there next is 8, 8 is not there. So, let me put 8 here.

The next is 3, 3 is already there next is 7. So, 7 I can put here. So, 1 2 3 4 5 6 7 8, 1 2 3 4 5 6 7 8. So, both the children are the valid children. So, starting from these 2 parents, using the principle of order cross over. So, we can find out the children solution very easily using this particular the principle.

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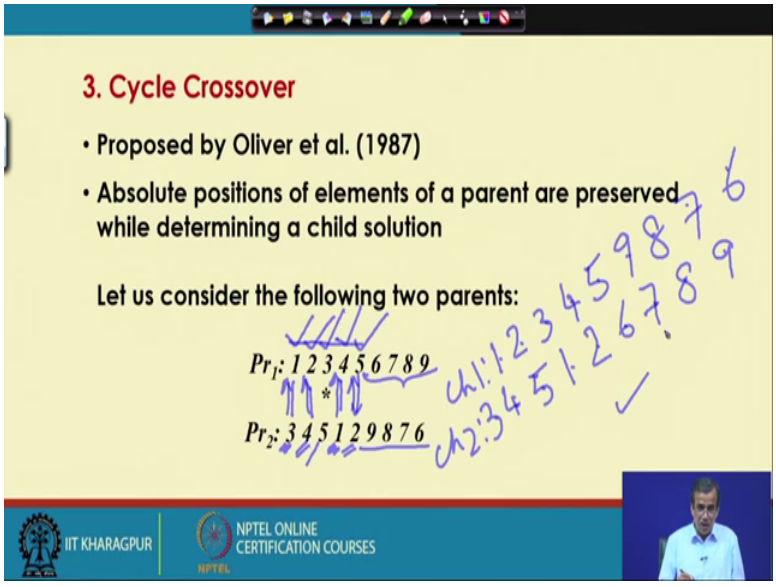

Now, I am just going to start with, a another very popular crossover operator that is called the cycle crossover. Now, the idea of this particular cycle crossover, that was introduced by Oliver et al. In the year 1987, now here actually what we do is, we start with a particular point, a particular city, lying on the parents now supposing that. So, these 2 parents are going to participate in cycle crossover.

So, the parent one is 1 2 3 4 5 6 7 8 9. So, this is one 2nd a TSP involving 9 cities and the parent 2 is 3 4 5 1 2 9 8 7 6, now let us see using the principle of cycle crossover. So, how to find out the children solution? Now what they do is. So, we first select the starting point of the cycle at random. So, I have got a 9 locations. So, out of 9 any one I can select as the starting city and year. So, at random I have selected. So, the third position as the starting, now using this particular parents parent 1 and parent 2.

Let us see how to find out child 1 and child 2? So, let me write here child 1, but before that the 9-position let me locate 1 2 3 4 5 6 7 8 9, now here actually what we do is, we start from this particular the location. So, that is the third location. So, to get the child 1 what we do is we start with the city 3, which is lying at the third position of parent 1.

So, let me put the city 3 here at the third position and once I put the city 3 at third position, now I will try to find out the location of city 3 on parent 2. Now this is the location of city 3 on parent 2 that is the first location and the city, corresponding to location 1 lying on parent 1 is nothing but city 1. So, what I do is. So, at this particular position. So, I put city 1 here.

The next once I copied 1 here, I will try to find out the position of city 1 on parent 2. So, this is the position and at this particular position, the city lying on parent 1 is 4. So, what you do is. So, here I am just going to put 4 and once a copied for, next I try to find out the location of city 4 on parent 2. So, this is the location and it is corresponding city is city 2.

So, let me put city 2 here and once I have selected city 2. So, what I do is, I try to find out the position of city 2 on parent 2. So, this is the position and it is corresponding city lying on parent 1 is 5. So, let me put 5 here and once I selected 5, I try to find out the position of city 5 on parent 2. So, this is the position and it corresponds to city 3, lying on parent 1, but 3 I have already considered. So, that completes this particular the cycle.

Now, you see. So, till now I have taken 5 cities for child 1, 1 2 3 4 5 and all 5 cities have been taken from this particular your parent 1 and till now, I have not considered anything from the parent 2, now what I do is the remaining thing I just copy from the parent 2, that is 9 8 7 6. So, let me put here 9 8 7 6.

Now, let me check. So, city 1 2 3 4 5 6 7 8 9. So, all the cities I have consider once and there is no missing city, there is no repetition. So, this is a ballad child 1, now following the same principle actually, I am just going to show how to find out the child 2.

So, child 2 let me first locate the 9 location 1 2 3 4 5 6 7 8 9 and let me once again consent it on, this third position that is the starting of the cycle and let me concentrate on your this 5. So, at this particular starting position, I have got city 5 on parent 2. So, let me let me start with that. So, if I start with city 5 at third position. So, let me put city 5 here, and a once I have selected city 5, I will try to find out the position of city 5 on parent 1. So, this is the position of city 5 on parent 1 and at this particular position it is corresponding city is city 2.

So, what I do is your the 5th position. So, 1 2 3 4 5. So, 5th position I am going to put 2, now once I have selected 2, I try to find out the possible location of city 2 on parent 1 this is this particular position that is the 2nd position, and at the 2nd position the corresponding city lying on parent 2 is 4. So, what we do is, at the 2nd position we put city 4 and once I have selected 4. So, I just try to find out the position of city 4 on parent 1.

So, this is the position and it is corresponding city on parent 2 is 1. So, at 4th position I will have to put city 1, now once I have selected 1, I will try to locate the position of city 1 on parent 1. So, this is the position that is the first position and it is corresponding city, lying on parent 2 is your city 3. So, at the first position, I am just going to put city 3 and once I have selected 3, now I am just going to mark city 3 on parent 1 and it is corresponding cities 5, which I have already taken, that completes actually this particular the cycle.

And you see all the city is still now, on child 2 I have selected from the parent 2 like 3 4 5 1 2 nd till now I have not selected anything from parent 1. So, the remaining vacant positions should be filled up from this particular the parent 1, that is this particular city should come here 6 7 8 9. So, let me let me write here 6 7 8 9 and now let me check, whether I have considered all the cities or not and a particular city, should not repeat and there should not be any missing city.

So, let me take city 1 2 3 4 5 6 7 8 9. So, all the cities have been considered 1. So, this is a valid child. So, using the principle of this particular the cycle cross over. So, we can

find out 2 valid children starting from the 2 parents, now before I just proceed to the next.

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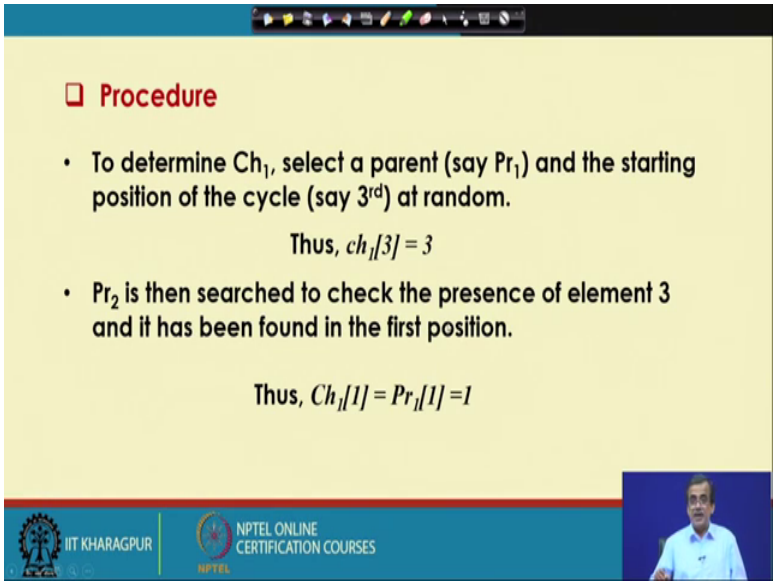

Let me whatever I discuss let me just show you, the way I have written it here. The procedure to determine child 1, select a parents a parent 1 and the starting position of the cycle say third at random. So, child 1 third position is nothing but 3, the way I discussed a few minutes ago. So, this was actually the starting for the child 1 the next is actually Pr 2 is then checked, to check the presence of element 3 and it has been found in the first position.

So, what do you do is. So, this particular the 3, we try to find out the post possible location of 3 on parent 2. So, this is the location and corresponding to this particular location, I have city 1 on parent 1. So, that has been copied and this particular procedure has been repeated. So, this is the way actually, we can find out the 2 children solutions from the 2 parents. Now let me do one thing, let me try to solve another numerical example and to solve it, let me just try to do one thing. So, so let me consider another numerical example and to solve it, let me do one thing the same parent I will consider, the only thing I will do is I am just going to change the starting position of this particular your the city.

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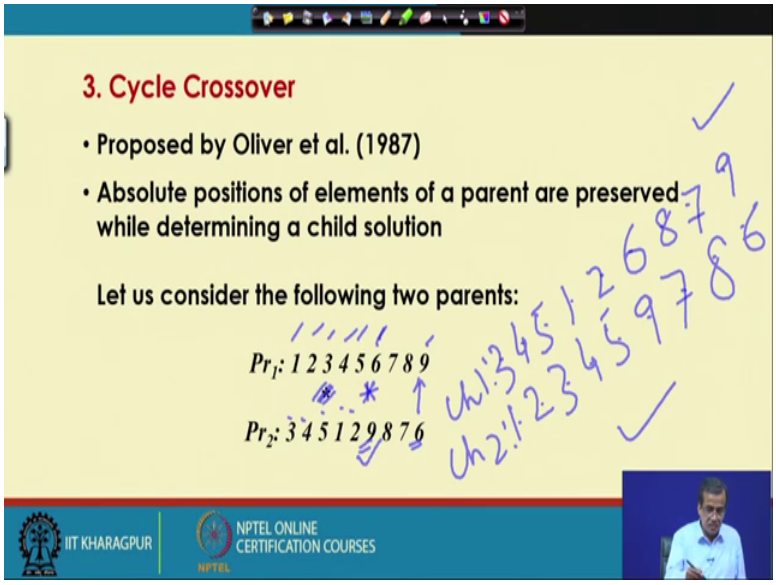

Now, if I consider the same one and if I just consider the starting, supposing that I am just going to consider. So, this is the starting position instead of these. So, this is my starting position that is the 6th position, 6th position is the starting position. So, this is not the starting position now, now let us see how to find out the children solution. So, once again I am just going to use the cycle crossover, just to solve this problem by making a little bit of change, in the starting position of the cycle.

Now, if this is the starting position that is the 6th position from the left, let us try to find out the child solution. So, child one once again 1 2 3 4 5 6 7 8 9, now let us see the 6th position, I should copy 6 1 2 3 4 5 6 I am writing 6 here and now I try to find out the possible location of 6 on parent 2. So, this is the location and it is corresponding city is 9.

So, I am just going to put 9 here, now I try to find out the location of city 9 on parent 2. So, they see the location and it is corresponding city is, city 6 6 I have already considered. So, that completes the cycle and once you have completed the cycle. So, rest of the thing I will have to take from the parent 2, these 2 I have taken from parent 1. So, if I took the other things from parent 2. So, I have to write 3. So, these 3 then 4 then comes your 5 then 1 then comes 2, 2 I have copied.

Now, you see here I have got. So, this I have already this position is filled up. So, 8 7. So, I will have to put 8 and 7 and let me check, city 1 2 3 4 5 6 7 8 9. So, all the 9 cities have come only once, there is no missing city there is no repetition. So, this is nothing but a valid child, and following the same principle I can also find out the child 2.

Child 2 So, once again 9 position 1 2 3 4 5 6 7 8 9 and let me put. So, this particular the starting 1 on parent 2 is 9. So, let me put the 9 here, at the 6th position 1 2 3 4 5 6. So, let me put 9 here and once you have got this particular 9, I try to find out the possible location of city 9 on parent 1. So, this is the location and it is corresponding city is city 6.

So, let me put city 6 here, now once I have selected 6, now I try to find out the location of 6 on parent 1. So, this is the location and it is corresponding city is 9, 9 has been considered. So, that completes the cycle, at the remaining vacant position ought to be filled up, from this particular the parent 1, now if I take the city 1 will come here, then 2 will come here, then comes 3 then comes your 4 then comes 5, this is filled up then 7 8 7 8.

Now, let me check 1 2 3 4 then comes 5 6 7 8 9. So, all the cities have come. So, this is actually a valid child, that is child 2. So, starting from 2 parents, how to find out the children solution using cycle crossover, I have discussed with the help of 2 examples.

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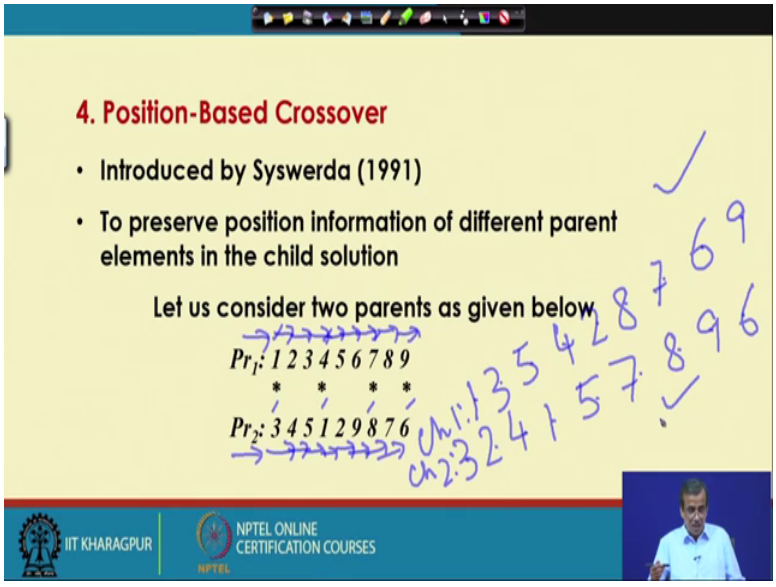

Now, I am just going to start with another crossover operator, that is called the position-based crossover. Now, this particular concept was proposed in the year 1991 by Sis Varga, now here actually the principle is something like this, we select a few positions using the random number generator at random, now supposing that we have got parent 1 like 1 2 3 4 5 6 7 8 9 and parent 2, 3 4 5 1 2 9 8 7 6.

Now, these 2 parents are going to participate in position-based crossover, now let us see how to get the children solution? Now to get this particular children solution what we do is, we select this position at random and here I have selected only 4 position, like the first position the 4th position, 7th position and the 9th position at random, I can also select 3 position or 5 position at random.

Now, here I have selected 4 position, now let us see how to find out child 1 and child 2, now child 1 now first I locate 9 position 1 2 3 4 5 6 7 8 9, now what do you do is, you consented on parent 1, now to get this child 1, the star position of parent 1 should be copied first. So, this will be copied at location 1, then 4 will be copied at location 4 then 7 will be copied at location 7 and this 9 will be copied at the 9th position.

Now, what you do is. So, all such things have been taken from parent 1, now I will have to take something from parent 2 and what do you do, we consented here now if I concentrate here, the first is city 3 3 is not considered. So, this is the first vacant position counting from the left leftmost side. So, 3 should come here, next is city 4, 4 is already there. So, I cannot copy it again, next is 5 5 is not here. So, let me write 5 here, next is 1, 1 is already there next is 2, 2 let me put 2 here next is 9, 9 is already there next is 8, 8 is not there.

So, let me put 8 here, next is 7, 7 is already there. So, next is 6, 6 is not there. So, I can write 6 here. So, let me check whether all the cities have come once or not, city 1 2 3 4 5 6 7 8 9. So, this is a valid child, similarly I can find out the child 2 also, now let me locate 1 2 3 4 5 6 7 8 9 position. Now, let us consent it on parent 2. So, this is the star position this is the star position, the star position and the star position. So, the star position I should copy first, at the first position I have got city 3.

Then if the 4th position I have got, city 1 then at the 7th position I have got 8. So, I have got 8 here and at the 9^{th} position, I have got 6, now all such things have taken from parent 2 like 3 1 8 6 are taken from parent 2. Now, I consent it on parent 1, now here at the past position we have got one city 1 I have already taken. So, I cannot retake then city 2, 2 is not there. So, I can select 2, next is 3, 3 is already there. So, I cannot take 4 4 is not here. So, let me take 4 here, next is 5, 5 is also not here. So, let me put 5 here.

Next is 6, 6 is already there, then comes your 7, now 7 is not here. So, let me put 7 here, next is 8, 8 is already there next is 9. So, let me put 9 here and now let me check,

whether I have considered all the cities only once, 1 2 3 4 5 6 7 8 9. So, this is actually a valid child and using the principle of this position-based crossover. So, we can find out 2 valid children starting from the 2 parents.

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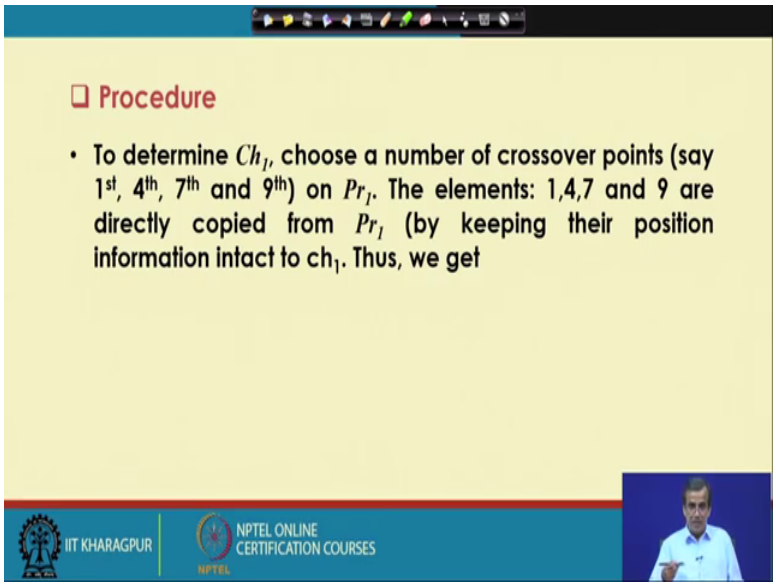

Now so, this particular principle, let me try to explain once again, but before that let me the principle which I discussed. So, let me read it from the slide little bit, just to show you the principle. To determine child 1 chose a number of crossover points say, first 4th 7th and 9th on parent 1. So, this I have already done the elements 1 4 7 and 9 are directly copied from parent 1. So, this I have already copied from parent 1, by keeping their position information intact to child 1.

So, we get a few cities copied in child 1 and the remaining cities will have to take from the parent 2. So, which I have already discussed, now what I am going to do is, the same numerical example, I am just going to change the positions at random and let us see, how can we solve it? Now let me let me remove this particular position whatever I have considered previously.

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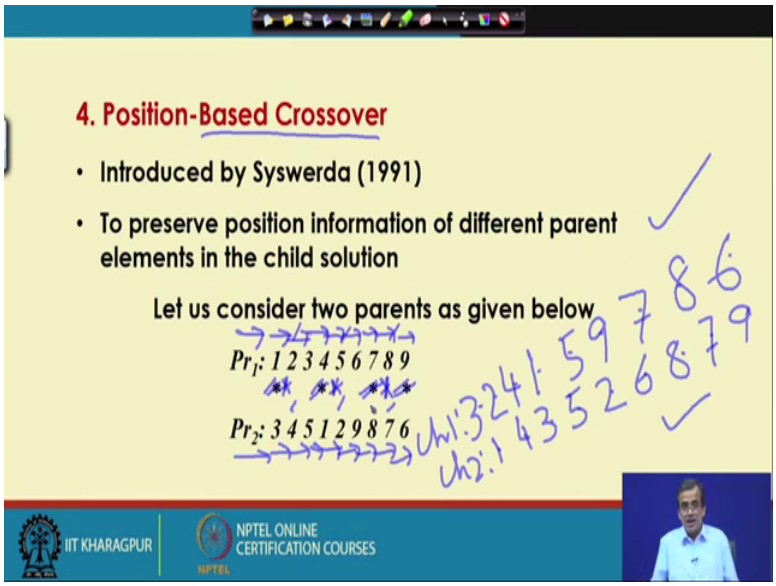

So, this is removed the star position is removed and let me reposition the star. So, let me put 1 star here at the 2nd position, let me put 1 star here at the 5th position and let me put 1 star here at the 8th position. So, previously consider the star position I am removing and now I am just putting star here, at the 2nd position the 5th position and the 8th position and let us see how to find out the children solution from the further, from the 2-pair same to parents.

Now, if I want to find out child 1, once again first I locate the 9 position 4th 5th 6th 7th 8th 9th, 2nd position I have got 2. So, let me put 2 here, the 5th position I have got 5. So, I have got 5 here and the 8th position I have got 8, now I will start from the starting city there is a leftmost city lying on parent 2. So, the first one is city 3, 3 is not considered.

So, let me put 3 here, next is 4, 4 is not considered. So, let me put 4 here, next is 5, 5 is already considered. So, I cannot retake next is 1, now one is not here. So, let me put one here, the next is 2, 2 is already there. So, I cannot take next is city 9, 9 is not there. So, let me put 9 here, next is 8, 8 is already there next is 7. So, 7 is not here. So, let me put 7 and the last is 6. So, let me put such 6 here, now you see 1 2 3 4 5 6 7 8 9. So, this is a valid child.

Now, if we compare the child 1 which I got earlier and this particular child what, they are different because we have changed the star position. Now similarly, we can find out this child 2 and once again 9 position let me locate 1 2 3 4 5 6 7 8 9, now the star position

will have to copy first, from parent 2 that is the 2nd position then comes, the 5th position and the 8th position.

The 2nd position let me put 4 here, the 5th position let me put 2 here, and the 8th position let me put 7 here, and now I will start my search from parent 1. So, the first is city 1, 1 is not considered. So, let me put 1 here, next is 2, 2 is considered already. So, I cannot retake, then city 3, 3 is not here. So, let me put 3 here, then comes 4 4 is already there. So, I cannot retake next is 5.

So, let me put 5 here, next is 6. So, let me put 6 here, next is 7, 7 is already taken next is 8. So, 8 is not here. So, let me put 8 here, next is 9. So, let me put 9 here, now let me check city 1 2 3 4 5 6 7 8 9. So, all the cities have been considered once. So, this is actually a valid child. So, starting from the 2 parents; so we can find out the valid children solution, using the principle of this position-based crossover.

And here I have solved 2 examples and you can see that, an example 1 and example 2 the children solutions which we got, even considering the same set of parents are different, the reason is your the star position, previously I considered 4-star position and in the 2nd example I consider only 3 star position, and I am getting the different children solution. So, this is the way actually we can find out, the children solution from the 2 parents.

Thank you.